

FORMULAS ESTRUCTURALES

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**DERECHOS RESERVADOS CONFORME
A LEY**

IMPRESO EN EL PERU

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PROLOGO

En primer término, debemos reconocer que, con referencia a los problemas de pórticos, existe una gran variedad de métodos para las soluciones, los que en ocasiones resultan incomprensibles para los estudiantes o simplemente difíciles para optar por determinado método. Por otro lado, no siempre las diferentes formas de solución se encuentran reunidas en un sólo libro, sino que, para los diversos estilos de pórticos debemos consultar varios, con la consiguiente pérdida de esfuerzo y tiempo.

En consideración a lo explicado, en el presente trabajo, se trata de presentar una cuidadosa selección de fórmulas prácticas de las diferentes formas de pórticos, más generalizados.

Los datos provienen principalmente de los tratados de Mecánica Racional, Resistencia de Materiales y Estructuras, escritos por consagrados maestros en la materia, tanto del Japón como de Alemania, tales como los Doctores Okamura Masao, Umemura Hazime, Takabeya Fukuhei, Kleionlogel, Gehler y el Dr. Müller-Breslau.

Si mediante el presente trabajo, se evita la inconveniencia de la búsqueda de determinadas fórmulas a través de las consultas de diferentes libros, y se facilita a los estudiantes, la solución de los pórticos de una manera sencilla y práctica, todos nuestros esfuerzos estarán compensados.

Finalmente, hay muchas personas a quienes deseo testimoniar mi agradecimiento, pero por el temor de omitir algunos, dado lo extenso de la lista, prefiero que el tiempo me permita testimoniárselo a ellas, personalmente.

Ing. Fernando Oshiro Higa
Lima, del día en que florecieron las dalias

*Primero se llevaron a los comunistas
pero a mí no me importó
porque yo no era.*

*Enseguida se llevaron a unos obreros
pero a mí no me importó
porque yo tampoco era.*

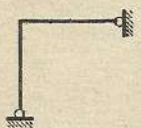

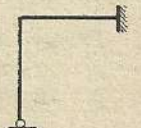
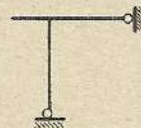
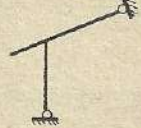
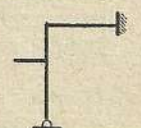
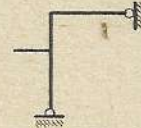
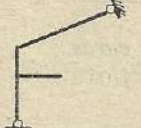
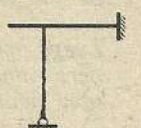
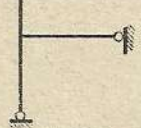

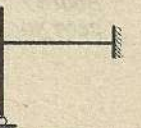

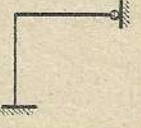

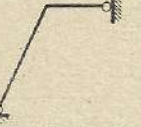
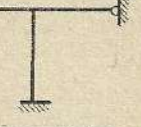
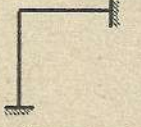
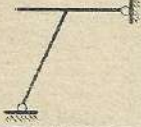
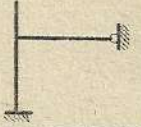
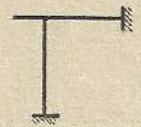
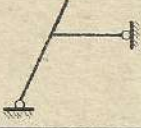
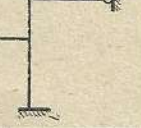
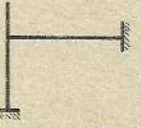
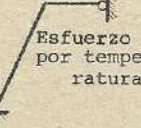


*Después detuvieron a los sindicalistas
pero a mí no me importó
porque yo no soy sindicalista.*

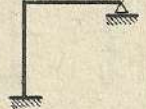
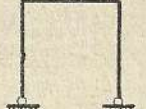
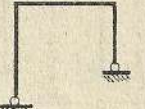
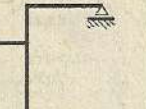
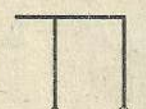

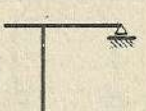
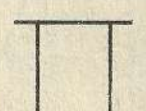
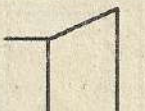

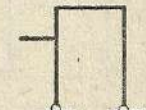
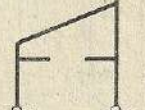
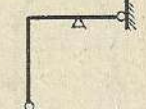
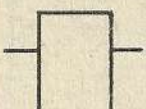

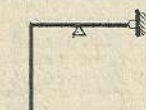
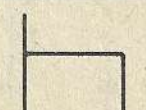

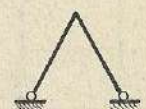
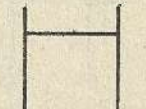

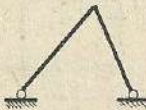
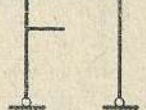


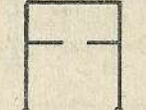
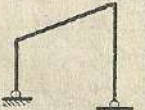


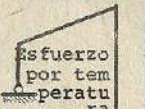
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pero como yo no soy religioso
tampoco me importó.*

*Ahora me llevan a mí
pero ya es tarde.*

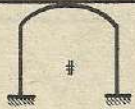
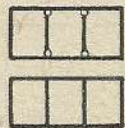
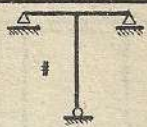
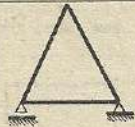
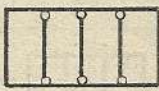
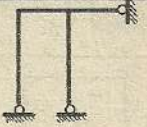
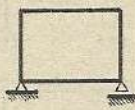
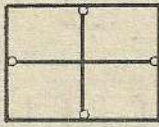
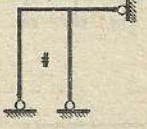
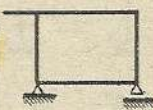
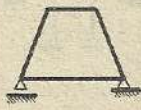
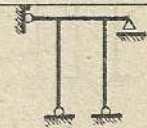
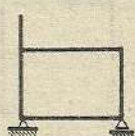

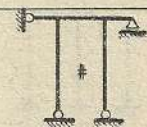
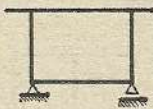

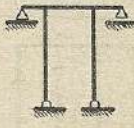
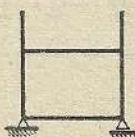
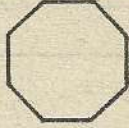
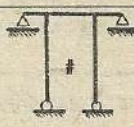
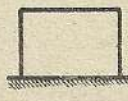
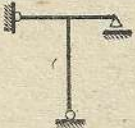
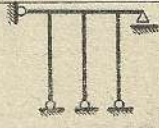

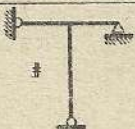
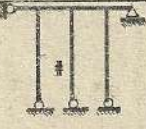
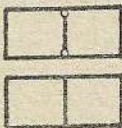
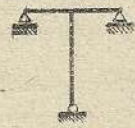
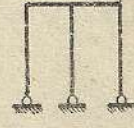
Bertolt Brecht

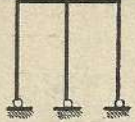
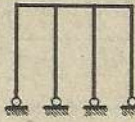
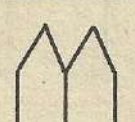
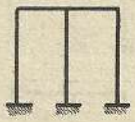
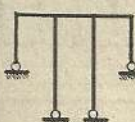
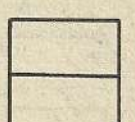
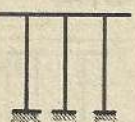
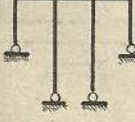
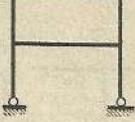
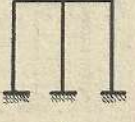
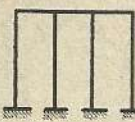
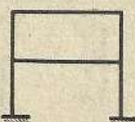
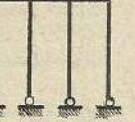

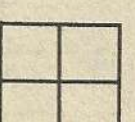
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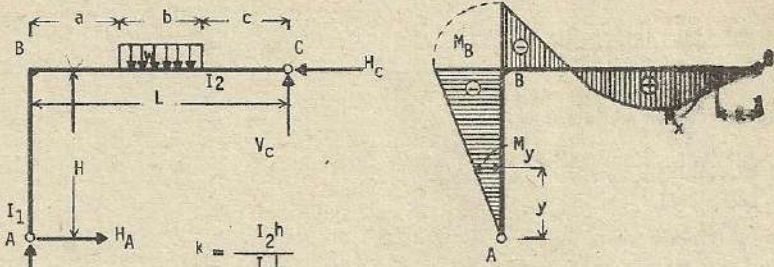
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: Esfuerzos por temperatura.

(1)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wb(2c+b)(2L^2 - 2c^2 - 2bc - b^2)}{8hL^2(k+1)}$$

$$V_A = \frac{wb(2c+b)(4L^2(k+1) + 2L^2 - 2c^2 - 2bc - b^2)}{8L^3(k+1)}$$

$$V_C = \frac{wb}{L} \left(a + \frac{b}{2}\right) - \frac{wb(2c+b)(2L^2 - 2c^2 - 2bc - b^2)}{8L^3(k+1)}$$

$$M_B = -\frac{wb(2c+b)(2L^2 - 2c^2 - 2bc - b^2)}{8L^2(k+1)}$$

Momento de flexión en la columna

$$M_y = -\frac{wb(2c+b)(2L^2 - 2c^2 - 2bc - b^2)}{8hL^2(k+1)} y$$

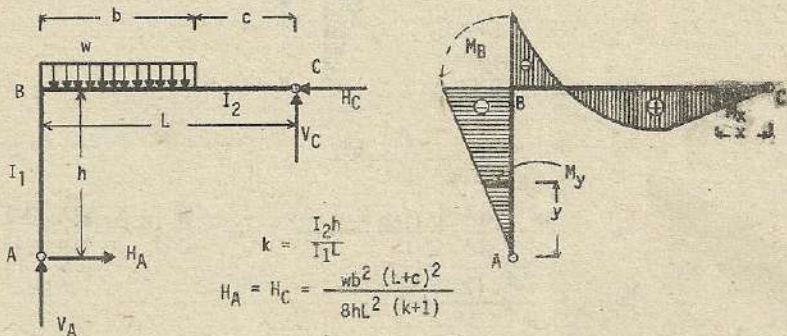
Momento de flexión en la viga

$$M_x = V_C x \quad x \leq c$$

$$M_x = V_C x - \frac{w}{2} (x-c)^2 \quad c \leq x \leq b+c$$

$$M_x = V_C x - wb \left(x - \frac{b}{2} - c\right) \quad x \geq b+c$$

(2)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wb^2(L+c)^2}{8hL^2(k+1)}$$

$$V_A = \frac{wb(L+c)(4L^2(k+1) + b(L+c))}{8L^3(k+1)}$$

$$V_C = \frac{wb^2 \{4L^2 (k+1) - (L+c)^2\}}{8L^3 (k+1)}$$

$$M_B = -\frac{wb^2 (L+c)^2}{8L^2 (k+1)}$$

Momento de flexión en la columna

$$M_y = -\frac{wb^2 \cdot (L+c)^2}{8hL^2 (k+1)} \cdot y$$

Momento de flexión en la viga

$$M_x = \frac{wb^2 \{4L^2 \cdot (k+1) - (L+c)^2\}}{8L^3 (k+1)} x$$

$$x \leq c$$

$$M_x = \frac{wb^2 \{4L^2 \cdot (k+1) - (L+c)^2\}}{8L^3 (k+1)} x - \frac{w}{2} (x-c)^2$$

$$x \geq c$$

En caso de $b = \frac{L}{2}$; $H_A = H_C = \frac{9}{128 h (k+1)} wL^2$

$$V_A = \frac{3(16k+19)}{128 (k+1)} wL ; \quad V_C = \frac{(16k+7)}{128 (k+1)} wL ; \quad M_B = -\frac{9}{128 (k+1)} wL^2$$

Momento de flexión en la columna :

$$M_y = -\frac{9wL^2}{128h (k+1)} y$$

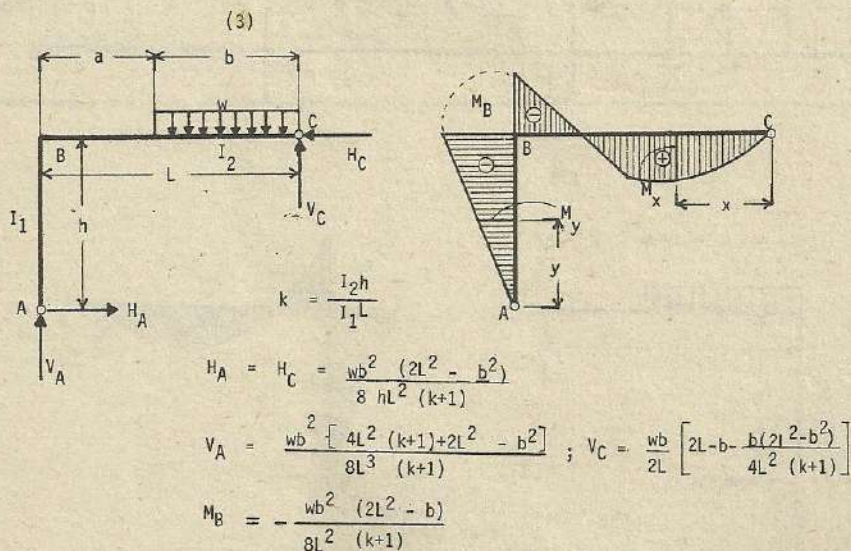
Momento de flexión en la viga :

$$M_x = \frac{16k+7}{128 (k+1)} wLx$$

$$x \leq \frac{L}{2}$$

$$M_x = \frac{16k+7}{128 (k+1)} wLx - \frac{w}{2} \left(x - \frac{L}{2}\right)^2$$

$$x \geq \frac{L}{2}$$



Momento de flexión en la columna : $M_y = -\frac{wb^2 (2L^2 - b^2)}{8hL^2 (k+1)} y$

Momento de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2} \quad x \leq b$$

$$M_x = V_C x - wb \left(x - \frac{b}{2} \right) \quad x \geq b$$

En caso donde $b = \frac{L}{2}$; $H_A = H_C = \frac{7}{128h(k+1)} wL^2$

$$V_A = \frac{(16k+23)}{128(k+1)} wL \quad ; \quad V_C = \frac{48k+41}{128(k+1)} wL \quad ; \quad M_B = \frac{7}{128(k+1)} wL^2$$

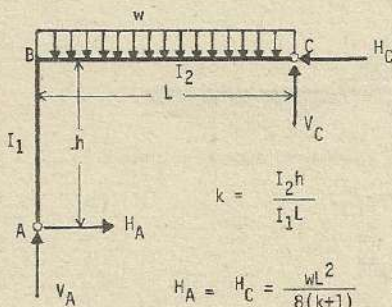
Momento de doblado o flexión en la columna : $M_y = -\frac{7}{128h(k+1)} wL^2$

Momento de flexión en la viga :

$$M_x = \frac{48k+41}{128(k+1)} wLx - \frac{wx^2}{2} \quad x \leq \frac{L}{2}$$

$$M_x = \frac{48k+41}{128(k+1)} wLx - \frac{wL}{2} \left(x - \frac{L}{4} \right) \quad x \geq \frac{L}{2}$$

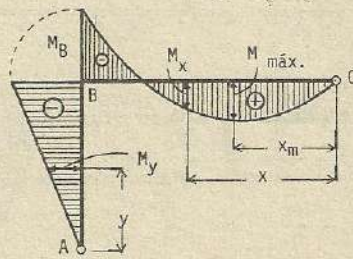
(4)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wL^2}{8(k+1)}$$

$$V_C = \frac{4k+3}{8(k+1)} wL$$



$$V_A = \frac{4k+5}{8(k+1)} wL$$

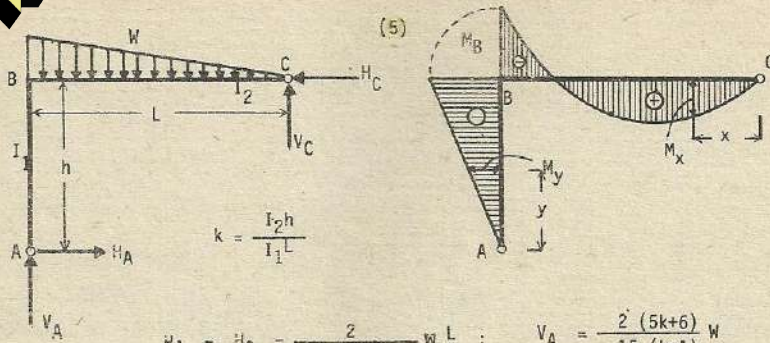
$$M_B = -\frac{1}{8(k+1)} wL^2$$

Momento de flexión en la columna : $M_y = -\frac{wL^2 y}{8(k+1)h}$

Momento de flexión en la viga :

$$M_x = \frac{4k+3}{8(k+1)} wLx - \frac{wL^2}{2}$$

$$x_m = \frac{4k+3}{8(k+1)} L \quad ; \quad M_{m\acute{a}x.} = \frac{(4k+3)^2}{128(k+1)^2} wL^2$$



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{2}{15(k+1)} w \frac{L}{h} : V_A = \frac{2(5k+6)}{15(k+1)} w$$

$$V_C = \frac{5k+3}{15(k+1)} w : M_B = -\frac{2}{15(k+1)} wL$$

Momento de flexión en la columna :

$$M_y = -\frac{2}{15(k+1)} wL y$$

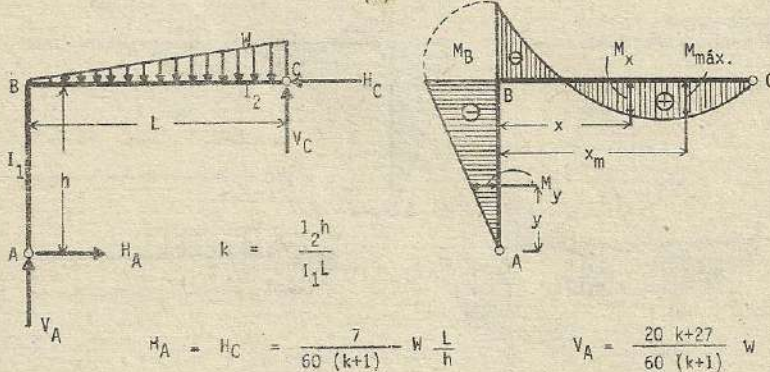
Momento de flexión en la viga :

$$M_x = \frac{5k+3}{15(k+1)} w x - \frac{w x^3}{3L^2}$$

$$x_m = L \sqrt{\frac{5k+3}{15(k+1)}}$$

$$M_{\max} = \frac{2(5k+3)}{45(k+1)} \sqrt{\frac{5k+3}{15(k+1)}} wL$$

(6)



$$k = \frac{1}{2} \frac{h}{I_1 L}$$

$$H_A = H_C = \frac{7}{60(k+1)} w \frac{L}{h} : V_A = \frac{20k+27}{60(k+1)} w$$

$$V_C = \frac{40k+33}{60(k+1)} w : M_B = -\frac{7}{60(k+1)} wL$$

Momento de flexión en la columna :

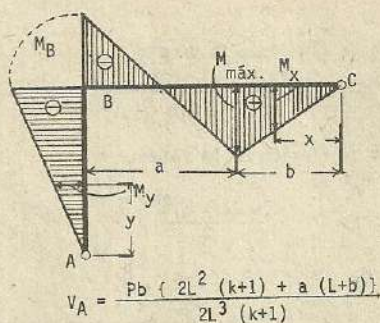
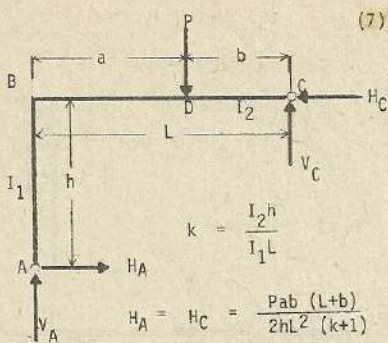
$$M_y = -\frac{7}{60(k+1)} wL \frac{y}{h}$$

Momento de flexión en la viga :

$$M_x = w \frac{(20k+27)x - 7L}{60(k+1)} - \frac{w x^3}{3L^2}$$

$$x_m = L \sqrt{\frac{20k+27}{60(k+1)}}$$

$$M_{\max} = \frac{wL}{3} \left[\frac{(20k+27)}{30(k+1)} - \sqrt{\frac{20k+27}{60(k+1)}} - \frac{7}{20(k+1)} \right]$$



$$V_C = \frac{Pa \{ 2L^2(k+1) - b(L+b) \}}{2L^3(k+1)} ; M_B = - \frac{Pab(L+b)}{2L^2(k+1)}$$

Momento de flexión en la columna : $M_y = - \frac{Pab(L+b)}{2hL^2(k+1)} y$

Momento de flexión en la viga :

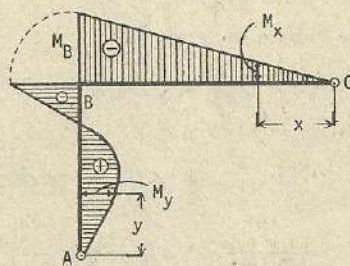
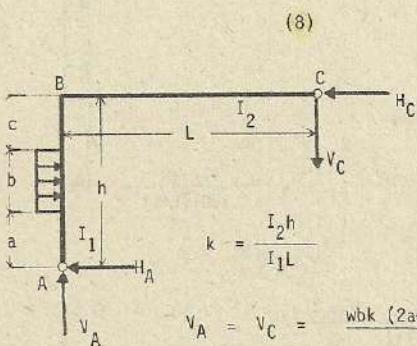
$$M_x = \frac{Pb \{ 2L^2(k+1) - a(L+b) \} x}{2L^3(k+1)} \quad x < b$$

$$M_x = \frac{Pb \{ 2L^2(k+1) - a(L+b) \}}{2L^3(k+1)} x - P(x-b) \quad x > b$$

$$M_{m\acute{a}x.} = \frac{pab(2kL^2 + 3al - a^2)}{2L^3(k+1)} \quad x_m = b$$

Si $a = b = \frac{L}{2}$; $H_A = H_C = \frac{3PL}{16(k+1)h}$; $V_A = \frac{8k+11}{16(k+1)} P$

$V_C = \frac{8k+5}{16(k+1)} P$; $M_B = - \frac{3}{16(k+1)} PL$; $M_{m\acute{a}x.} = \frac{8k+5}{32(k+1)} \cdot PL$



$$V_A = V_C = \frac{wbk(2a+b)(2h^2 - 2a^2 - 2ab - b^2)}{8h^2 L(k+1)}$$

$$H_A = - \frac{wbk(2a+b)(2h^2 - 2a^2 - 2ab - b^2)}{8h^3(k+1)} + \frac{wb}{h} \left(c + \frac{b}{2} \right)$$

$H_C = wb - H_A$; $M_B = - \frac{wbk(2a+b)(2h^2 - 2a^2 - 2ab - b^2)}{8h^2(k+1)}$

Handwritten note:
 $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

Momento de flexión en la viga :

$$M_x = - \frac{wbk(2a+b)(2h^2 - 2a^2 - 2ab - b^2)}{8h^2L(k+1)} x$$

Momento de flexión en la columna :

$$M_y = H_A \cdot y$$

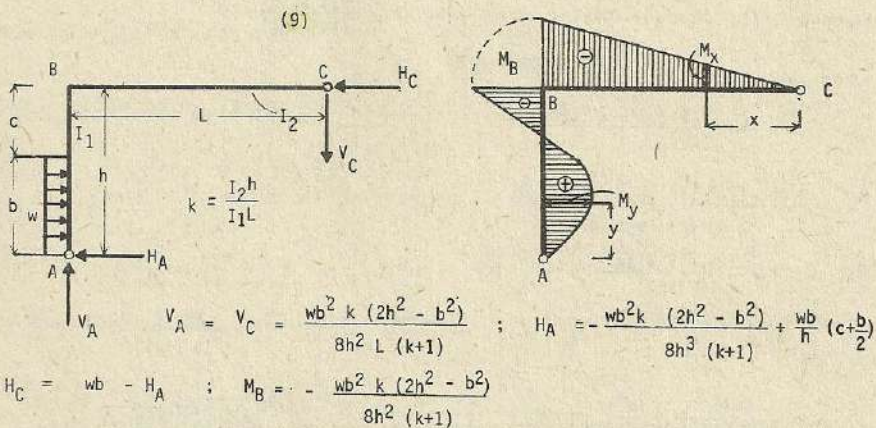
$$M_y = H_A \cdot y - \frac{w(y-a)^2}{2}$$

$$y \leq a$$

$$a \leq y \leq a+b$$

$$M_y = H_A \cdot y - wb(y-a - \frac{b}{2})$$

$$y \geq a+b$$



Momento de flexión en la viga : $M_x = - \frac{wb^2k(2h^2 - b^2)}{8h^2L(k+1)} x$

Momento de flexión en la columna :

$$M_y = H_A \cdot y - \frac{wy^2}{2} \quad y \leq b$$

$$M_y = H_A \cdot y - wb(y - \frac{b}{2}) \quad y \geq b$$

Si $b = c = \frac{h}{2}$; $V_A = V_C = \frac{7k}{12(8k+1)} \cdot \frac{wh^2}{L}$; $H_A = \frac{41k+48}{128(k+1)} \cdot wh$

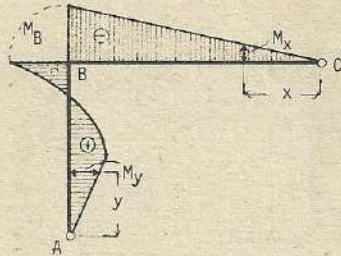
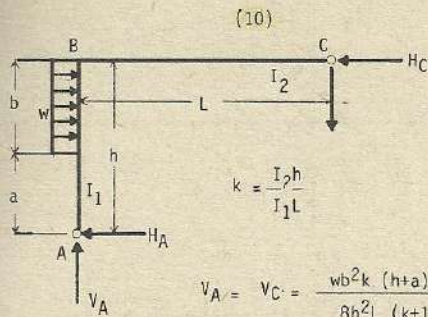
$$H_C = \frac{23k+16}{128(k+1)} \cdot wh \quad M_B = -\frac{7k}{128(k+1)} \cdot wh^2$$

Momento de flexión en la viga : $M_x = -\frac{7k}{128(k+1)} \cdot \frac{wh^2}{L} x$

Momento de flexión en la columna :

$$M_y = \frac{41k+48}{128(k+1)} \cdot why - \frac{wy^2}{2} \quad y \leq b$$

$$M_y = \frac{41k+48}{128(k+1)} \cdot why - \frac{wL}{2}(y - \frac{L}{4}) \quad y \geq b$$



$$H_C = wb - H_A \quad ; \quad M_B = \frac{wb^2k(h+a)^2}{8h^2(k+1)}$$

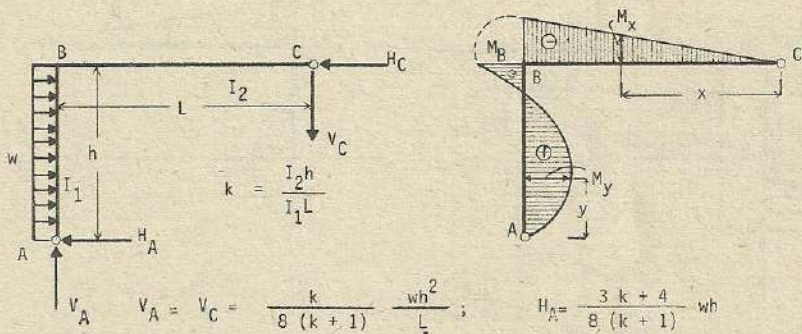
Momento de flexión en la viga : $M_x = -\frac{wb^2k(h+a)^2}{8h^2L(k+1)}x$

Momento de flexión en la columna :

$$M_y = H_A \cdot y \quad y < a$$

$$M_y = H_A \cdot y - \frac{w(y-a)^2}{2} \quad y > a$$

(11)



$$H_C = \frac{5k+4}{8(k+1)} wh \quad ; \quad M_B = -\frac{k}{8(k+1)} wh^2$$

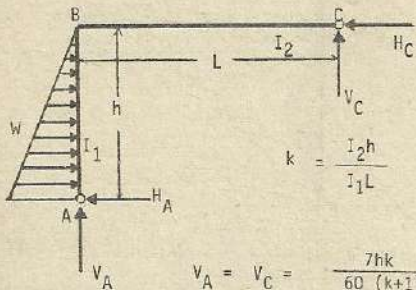
Momento de flexión en la viga : $M_x = -\frac{k}{(8k+1)L} wh^2x$

Momento de flexión en la columna :

$$M_y = \frac{(4(h-y) + k(3h-4y))}{8(k+1)} wy \quad ; \quad y_m = \frac{3k+4}{8(k+1)} h$$

$$M_{m\acute{a}x} = \frac{(3k+4)^2}{128(k+1)^2} \cdot wh^2$$

(12)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{7hk}{60(k+1)} W$$

$$H_A = \frac{33k + 40}{60(k+1)} W$$

$$H_C = \frac{27k + 20}{60(k+1)} W$$

$$M_B = -\frac{7k}{60(k+1)} Wh$$

Momento de flexión en la viga :

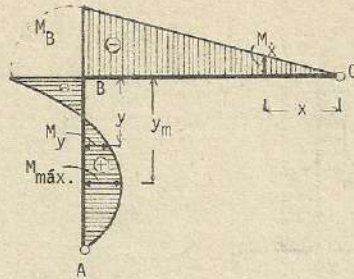
$$M_x = -\frac{7k}{60(k+1)} Whx$$

Momento de flexión en la columna :

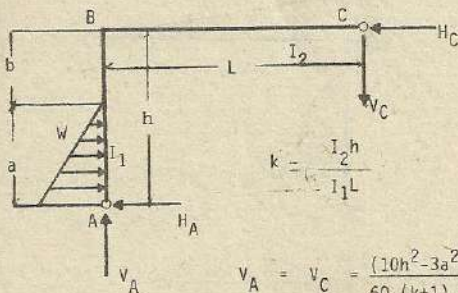
$$M_y = H_C y - V_C L - \frac{Wy^3}{3h^2}$$

$$y_m = h \sqrt{\frac{3k+5}{15(k+1)}}$$

$$M_{\max.} = \frac{2(3k+5)}{45(k+1)} \sqrt{\frac{3k+5}{15(k+1)}} Wh$$



(13)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{(10h^2 - 3a^2) ak}{60(k+1) h^2 L} W ; H_A = \frac{(3b+2a)}{3h} W - V_C \frac{L}{h}$$

$$H_C = -\frac{a}{3h} W + V_C \frac{L}{h} ; M_B = -\frac{ak(10h^2 - 3a^2)}{60h^2(k+1)} W$$

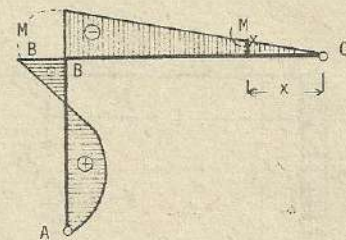
Momento de flexión en la viga :

$$M_x = -\frac{(10h^2 - 3a^2) ak}{60(k+1) h^2 L} Wx$$

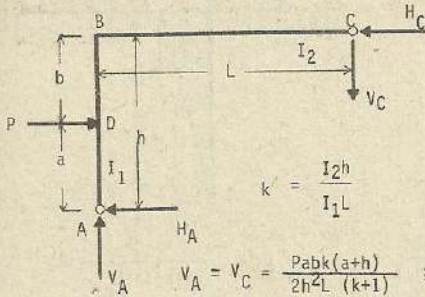
Momento de flexión en la columna :

$$M_y = H_C y - V_C L \quad y \leq b$$

$$M_y = H_C y - V_C L - \frac{W(y-h)^3}{3a^2} \quad y \geq b$$



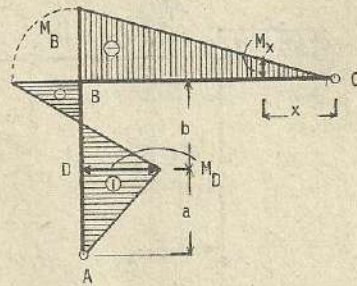
(14)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{Pabk(a+h)}{2h^2L(k+1)}$$

$$H_C = Pa \cdot \frac{2h^2(k+1) + bk(a+b)}{2h^3(k+1)}$$



$$H_A = Pb \frac{2h^2(k+1) - ak(a+h)}{2h^3(k+1)}$$

$$M_B = \frac{Pabk(a+h)}{2h^2(k+1)}$$

Momento de flexión en la viga :

$$M_x = \frac{Pabk(a+h)}{2h^2L(k+1)} \cdot x$$

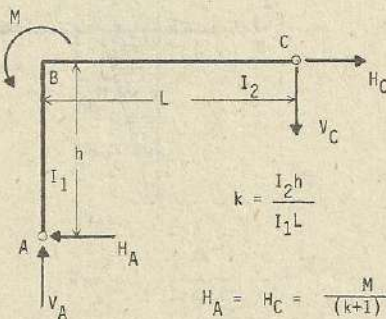
Momento de flexión en la columna :

$$M_y = H_A y \quad y \leq a$$

$$M_y = H_A y - P(y-a) \quad y \geq a$$

$$M_{\max} = \frac{Pab(2h^2 + bk(3h-b))}{3h^3(k+1)}$$

(15)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{M}{(k+1)h}$$

$$M_{BA} \text{ (col.)} = \frac{M}{(k+1)}$$

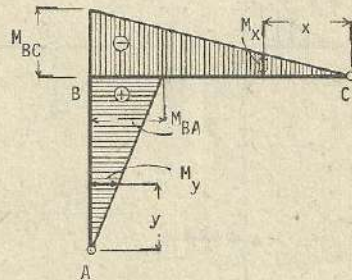
$$M_{BC} \text{ (viga)} = \frac{Mk}{(k+1)}$$

Momento de flexión en la columna :

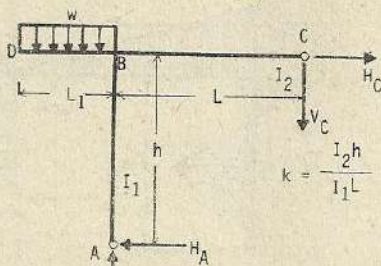
$$M_y = \frac{M}{(k+1)} \cdot \frac{y}{h}$$

Momento de flexión en la viga :

$$M_x = -\frac{Mk}{(k+1)} \cdot \frac{x}{L}$$



(16)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wL_1^2}{2(k+1)h} ; V_A = \frac{wL_1^2 k}{2(k+1)L} + wL_1$$

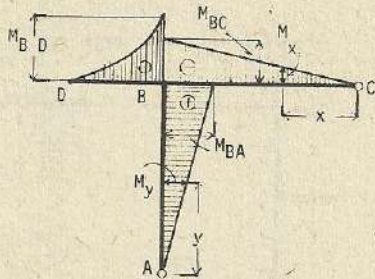
$$V_C = \frac{wL_1^2 k}{2(k+1)L} ; M_{BA} \text{ (col.)} = \frac{wL_1^2}{2(k+1)} ; M_{BD} \text{ (Cantiliver)} = \frac{wL_1^2}{2}$$

$$M_{BC} \text{ (viga)} = \frac{wL_1^2 k}{2(k+1)} ; \text{Momento de flexión en la columna :}$$

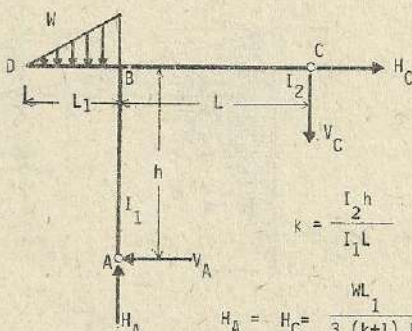
$$M_y = \frac{wL_1^2}{2(k+1)h} \cdot y$$

Momento de flexión en la viga :

$$M_x = -\frac{wL_1^2 k}{2(k+1)L} \cdot x$$



(17)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wL_1}{3(k+1)h}$$

$$V_A = \frac{wL_1 k}{3(k+1)L} + w$$

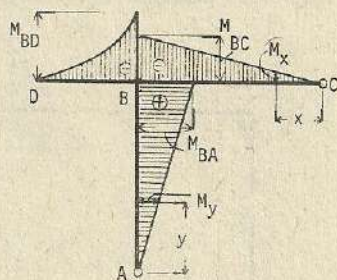
$$V_C = \frac{wL_1 k}{3(k+1)L} ; M_{BA} = \frac{wL_1}{3(k+1)} ; M_{BD} = -\frac{wL_1}{3} ; M_{BC} = \frac{wL_1 k}{3(k+1)}$$

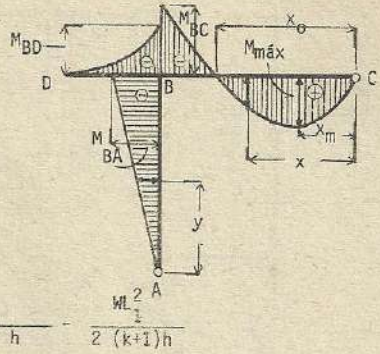
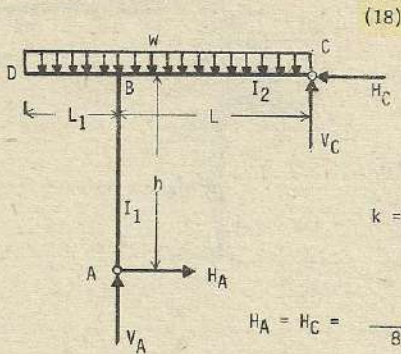
Momento de flexión en la viga :

$$M_x = -\frac{wL_1 kx}{3(k+1)L}$$

Momento de flexión en la columna :

$$M_y = \frac{wL_1}{3(k+1)h} \cdot y$$





$$k = \frac{I_2 h}{I_1 L}$$

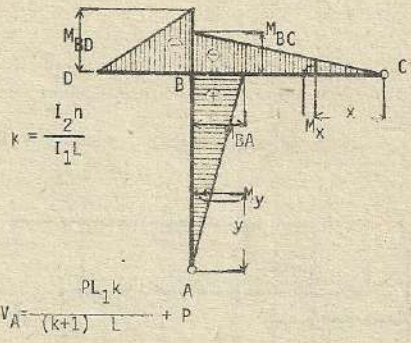
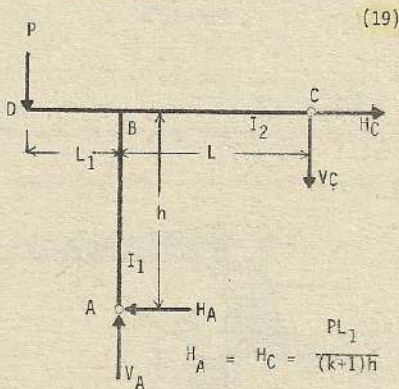
$$H_A = H_C = \frac{wL^2}{8(k+1)h} - \frac{wL_1^2}{2(k+1)h}$$

$$V_A = \frac{(4k+5)}{8(k+1)} wL + \frac{k}{2(k+1)L} wL_1^2 + wL_1 ; \quad V_C = w(L+L_1) - V_A$$

$$M_{BD} = -w \frac{L_1^2}{2} ; \quad M_{BA} = -H_A h ; \quad M_{BC} = V_C L - w \frac{L^2}{2}$$

$$x_m = \frac{V_C}{w} ; \quad M_{\max} = V_C x_m - w \frac{x_m^2}{2}$$

Momento de flexión en la viga : $M_x = V_C x - \frac{wx^2}{2} ; \quad x_0 = \frac{2V_C}{w}$



$$k = \frac{I_2 h}{I_1 L}$$

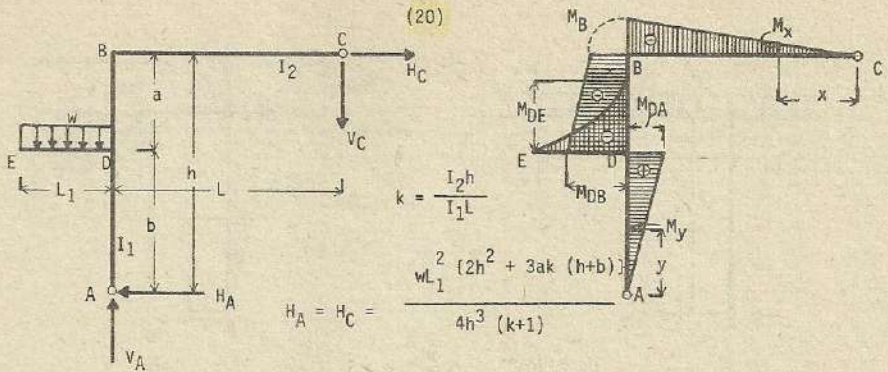
$$H_A = H_C = \frac{PL_1}{(k+1)h} ; \quad V_A = \frac{PL_1 k}{(k+1)L} + P$$

$$V_C = \frac{PL_1 k}{(k+1)L} ; \quad M_{BA} = -\frac{PL_1}{(k+1)} ; \quad M_{BD} = PL_1 ; \quad M_{BC} = \frac{PL_1 k}{(k+1)}$$

Momento de flexión en la columna :

$$M_y = \frac{PL_1 y}{(k+1)h}$$

Momento de flexión en la viga : $M_x = -\frac{PL_1 kx}{(k+1)L}$



$$V_A = \frac{w L_1^2 k (3b^2 - h^2)}{4h^2 L (k+1)} + w L_1 ; \quad V_C = \frac{w L_1^2 k (3b^2 - h^2)}{4h^2 L (k+1)}$$

$$M_B = -\frac{w L_1^2 k (2h^2 - 3a(h+b))}{4h^2 (k+1)} ; \quad M_{DE} = -\frac{w L_1^2}{2} ; \quad M_{DA} = \frac{w L_1^2 (2h^2 + 3ka(h+b))}{4h^3 (k+1)}$$

$$M_{DB} = -w L_1^2 \left[1 + \frac{b (2h^2 + 3ka(h+b))}{2h^3 (k+1)} \right]$$

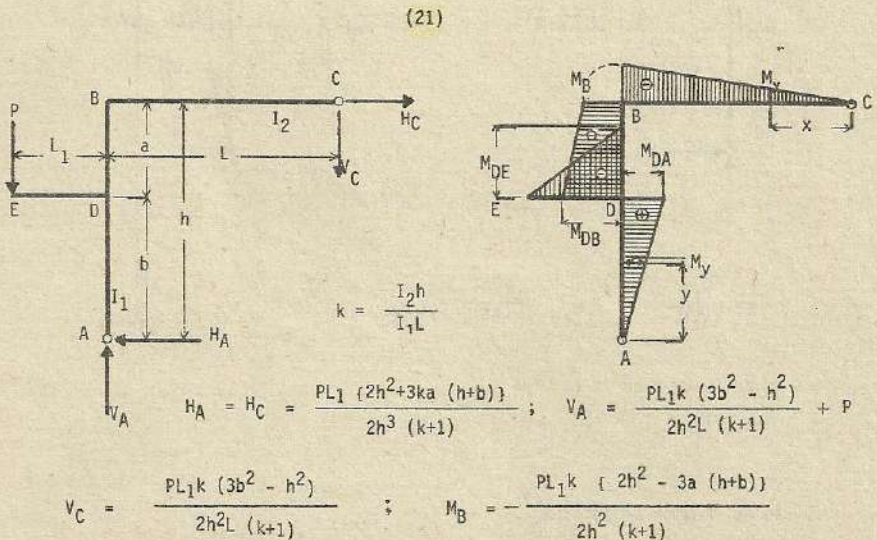
Momento de flexión en la viga :

$$M_x = -\frac{w L_1^2 k (3b^2 - h^2)}{4 h^2 L (k+1)} x$$

Momento de flexión en la columna :

$$M_y = H_A y \quad y < b$$

$$M_y = H_A y - \frac{w L_1}{2} x \quad y \geq b$$



$$M_{DE} = PL_1 \quad ; \quad M_{DB} = -PL_1 \left[1 + \frac{b(2h^2 + 3ka(h+b))}{2h^3(k+1)} \right]$$

$$M_{DA} = \frac{PL_1 b (2h^2 + 3ka(h+b))}{2h^3(k+1)}$$

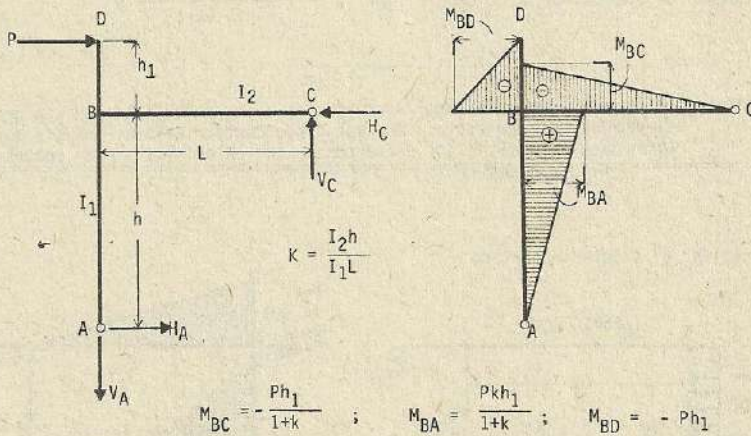
$$\text{Esfuerzo de momento en la viga : } M_x = - \frac{PL_1 k (3b^2 - h^2)}{2h^2 L (k+1)} x$$

Momento de flexión en la columna :

$$M_y = H_A y \quad y < b$$

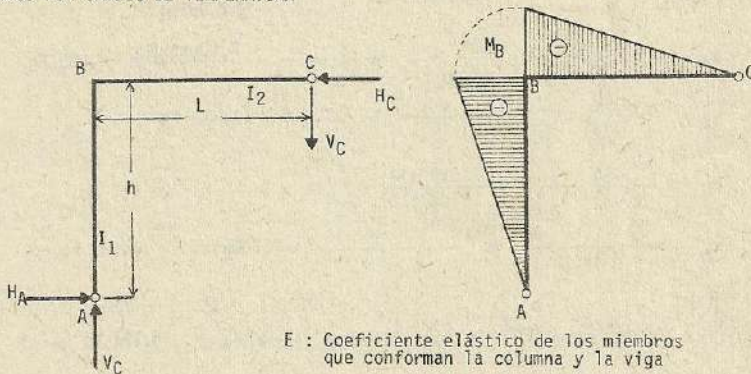
$$M_y = H_A y - PL_1 \quad y > b$$

(22)



(23)

ESFUERZOS POR CAMBIO DE TEMPERATURA



E : Coeficiente elástico de los miembros que conforman la columna y la viga

e : Coeficiente de expansión

t : Grado de variación de la temperatura

I_1 = Momento de inercia de la columna

I_2 = Momento de inercia de la viga

h = Altura de la columna

L = Luz a largo de la viga

$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3EetI (h^2 + L^2)}{L^2 h^2 (k+1)} \quad ; \quad V_A = V_C = \frac{3EetI (h^2 + L^2)}{L^3 h (k+1)}$$

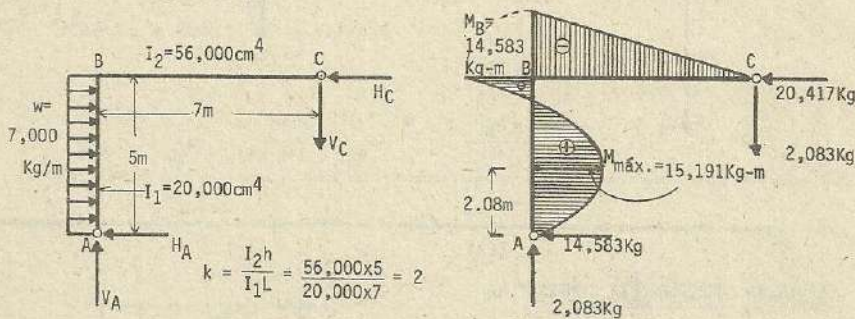
$$M_B = - \frac{3EetI (h^2 + L^2)}{L^2 h (k+1)}$$

$$\text{Momento de flexión en la viga : } M_x = - \frac{3EetI (h^2 + L^2)}{L^3 h (k+1)} x$$

$$\text{Momento de flexión en la columna : } M_x = - \frac{3EetI (h^2 + L^2)}{L^2 h^2 (k+1)} y$$

Nota : Los resultados presentados corresponden a los casos donde existe una elevación de temperatura. En caso donde contrariamente, exista descenso de temperatura, los esfuerzos se anotan con los signos contrarios.

Resolver el siguiente pórtico



$$k = \frac{I_2 h}{I_1 L} = \frac{56,000 \times 5}{20,000 \times 7} = 2$$

$$V_A = V_C = \frac{k}{8(k+1)} \cdot \frac{wh^2}{L} = \frac{2}{8(2+1)} \cdot \frac{7,000 \times 5 \times 5}{7} = 2,083 \text{ Kg.}$$

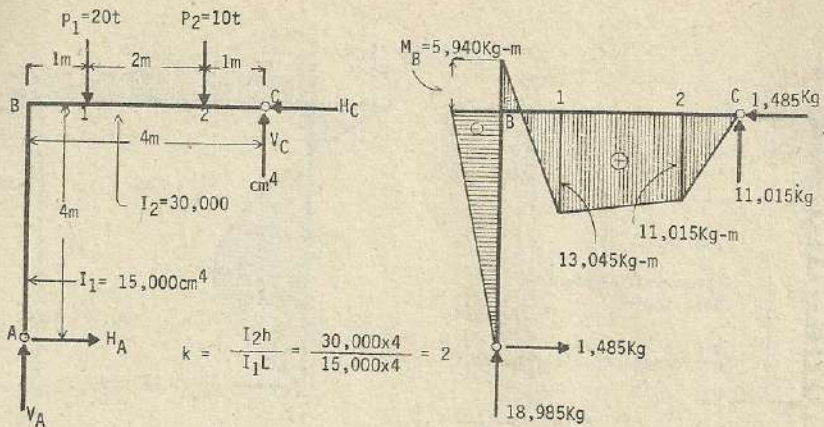
$$H_A = \frac{3k+4}{8(k+1)} wh = \frac{3 \times 2 + 4}{8(2+1)} \times 7,000 \times 5 = 14,583 \text{ Kg.}$$

$$H_C = \frac{5k+4}{8(k+1)} \cdot wh = \frac{5 \times 2 + 4}{8(2+1)} \times 7,000 \times 5 = 20,417 \text{ Kg.}$$

$$M_B = - \frac{k}{8(k+1)} \cdot wh^2 = - \frac{2}{8(2+1)} \times 7,000 \times 5 \times 5 = -14,583 \text{ Kg -m.}$$

$$M_{\max} = - \frac{(3k+4)^2}{128(k+1)^2} 2wh^2 = - \frac{(3 \times 2 + 4)^2}{128(2+1)^2} \times 7,000 \times 5 \times 5 = 15,191 \text{ Kg -m.}$$

$$y_m = \frac{(3k+4)}{8(k+1)} \cdot h = \frac{3 \times 1 + 4}{8(2+1)} \times 5 = 2.08 \text{ m.}$$



Debido a (P_1) ; $H_A' = H_C' = \frac{P_1 a b (L+b)}{2hL^2 (k+1)} = \frac{20 \times 1 \times 3 (4+3)}{2 \times 4 \times 4 \times 4 (2+1)} = 1,094 \text{ Kg.}$

Debido a (P_2) ; $H_A'' = H_C'' = \frac{P_2 a b (L+b)}{2hL^2 (k+1)} = \frac{10 \times 3 \times 1 (4+1)}{2 \times 4 \times 4 \times 4 (2+1)} = 391 \text{ Kg.}$

$\therefore H_A = H_C = 1,094 + 391 = 1,485 \text{ Kg.}$

$V_A = \frac{Pb}{L} + \frac{H_A h}{L}$ (cuando $\Sigma M_C = 0$)

Debido a (P_1) ; $V_A' = \frac{P_1 b}{L} + \frac{H_A' h}{L} = \frac{20 \times 3}{4} + \frac{1,094 \times 4}{4} = 16,094 \text{ Kg.}$

Debido a (P_2) ; $V_A'' = \frac{P_2 b}{L} + \frac{H_A'' h}{L} = \frac{10 \times 1}{4} + \frac{0,391 \times 4}{4} = 2,891 \text{ Kg.}$

$\therefore V_A = 16,094 + 2,891 = 18,985 \text{ Kg.}$

$V_C = \frac{P a}{L} - \frac{H_A h}{L}$ (cuando $\Sigma M_B = 0$)

Debido a (P_1) ; $V_C' = \frac{P_1 a}{L} - \frac{H_A' h}{L} = \frac{20 \times 1}{4} - \frac{1,094 \times 4}{4} = 3,906 \text{ Kg.}$

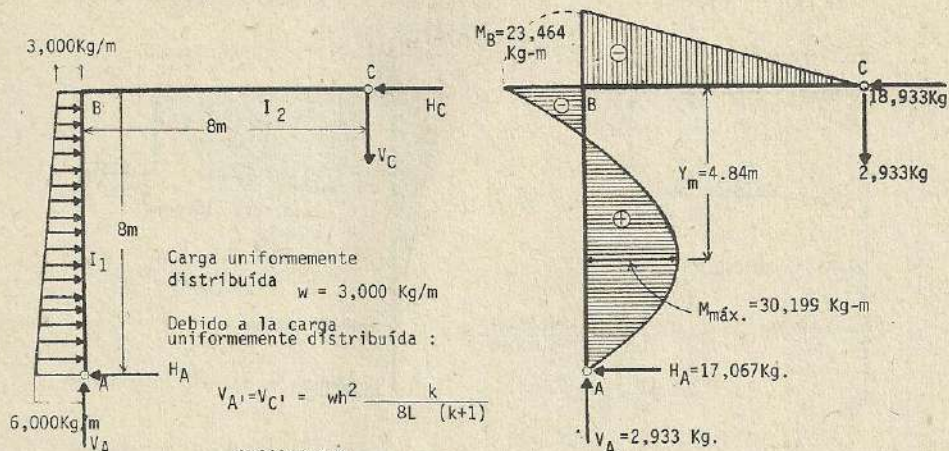
Debido a (P_2) ; $V_C'' = \frac{P_2 a}{L} - \frac{H_A'' h}{L} = \frac{10 \times 3}{4} - \frac{0,391 \times 4}{4} = 7,109 \text{ Kg.}$

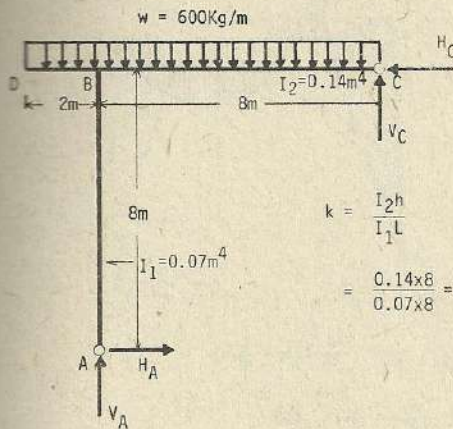
$\therefore V_C = 3,906 + 7,109 = 11,015 \text{ Kg.}$

$M_B = -H_A h = -1,485 \times 4 = -5,940 \text{ Kg -m}$

$M_1 = -M_B + V_A a = -5,940 + 18,985 \times 1 = 13,045 \text{ Kg -m (} M_{\text{máx}})$

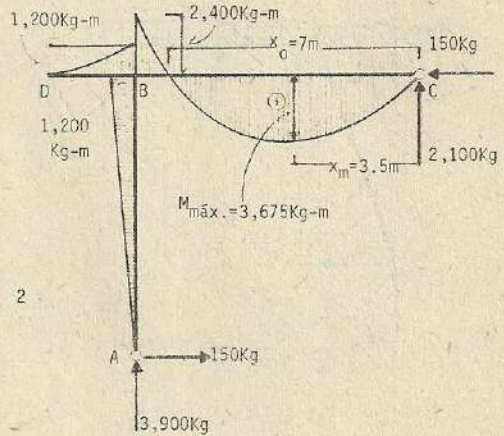
$M_2 = V_C b = 11,015 \times 1 = 11,015 \text{ Kg -m.}$





$$k = \frac{I_2 h}{I_1 L}$$

$$= \frac{0.14 \times 8}{0.07 \times 8} = 2$$



$$H_A = H_C = \frac{wL^2}{8(k+1)h} - \frac{wL_1^2}{2(k+1)h} = \frac{600 \times 8 \times 8}{8(2+1) \times 8} - \frac{600 \times 2 \times 2}{2(2+1) \times 8} = 150 \text{ Kg.}$$

$$V_A = \frac{(4k+5)}{8(k+1)} \cdot wL + \frac{k}{2(k+1)L} \cdot wL_1^2 + wL_1 = \frac{8+5}{8 \times (2+1)} \times 600 \times 8 + \frac{2}{2(2+1) \times 8} \times 600 \times 2 \times 2 + 600 \times 2$$

$$= 3,900 \text{ Kg.}$$

$$V_C = w(L+L_1) - V_A = 600(8+2) - 3,900 = 2,100 \text{ Kg.}$$

$$M_{BD} = -w \frac{L_1^2}{2} = -600 \times \frac{2 \times 2}{2} = -1,200 \text{ Kg-m} ; \quad M_{BA} = -H_A h = -150 \times 8 = -1,200 \text{ Kg-m}$$

$$M_{BC} = V_C L - \frac{wL^2}{2} = 2,100 \times 8 - 600 \times \frac{8 \times 8}{2} = -2,400 \text{ Kg-m}$$

Considerando como (x_m) la distancia del punto (C) al punto de momento máximo.

$$V_C - wx_m = 0 \rightarrow x_m = \frac{V_C}{w} = \frac{2,100}{600} = 3.5 \text{ m}$$

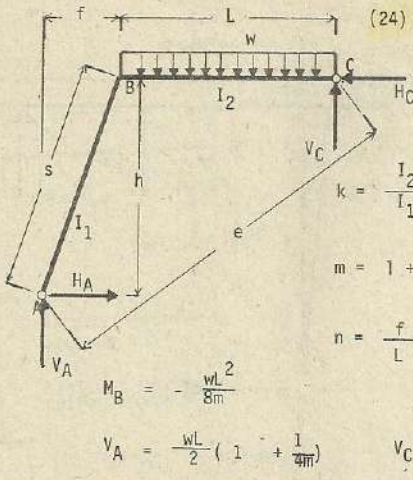
$$M_{\text{máx.}} = V_C x_m - w \frac{x_m^2}{2} = 2,100 \times 3.5 - 600 \times \frac{3.5 \times 3.5}{2} = 3,675 \text{ Kg-m}$$

Si hacemos como (x_0) la distancia que media entre el punto (C), al punto donde el momento de flexión es cero :

$$V_C x_0 - \frac{wx_0^2}{2} = 0$$

$$x_0 \cdot \left(V_C - \frac{wx_0}{2} \right) = 0$$

$$x_0 = 0 ; \quad x_0 = \frac{2V_C}{w} = \frac{2 \times 2,100}{600} = 7 \text{ m.}$$



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

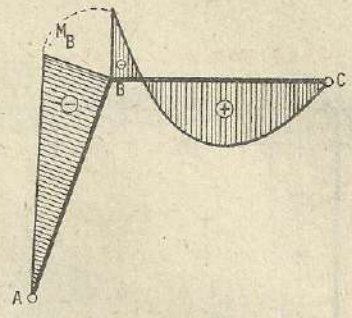
$$n = \frac{f}{L}$$

$$M_B = -\frac{wL^2}{8m}$$

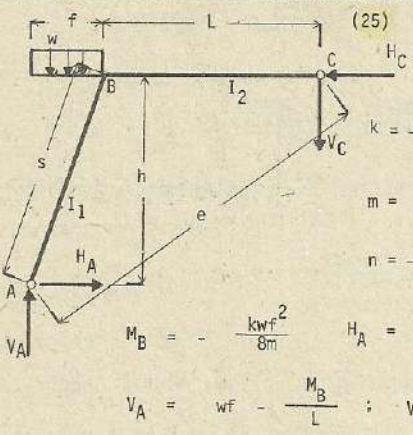
$$H_A = H_C = \frac{wL^2}{2n} \left(n + \frac{1+n}{4m} \right)$$

$$V_A = \frac{wL}{2} \left(1 + \frac{1}{4m} \right)$$

$$V_C = \frac{wL}{2} \left(1 - \frac{1}{4m} \right)$$



$$e^2 = (L + f)^2 + h^2$$



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

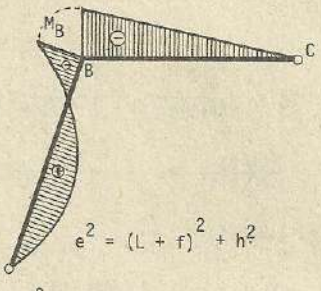
$$n = \frac{f}{L}$$

$$M_B = -\frac{k w f^2}{8m}$$

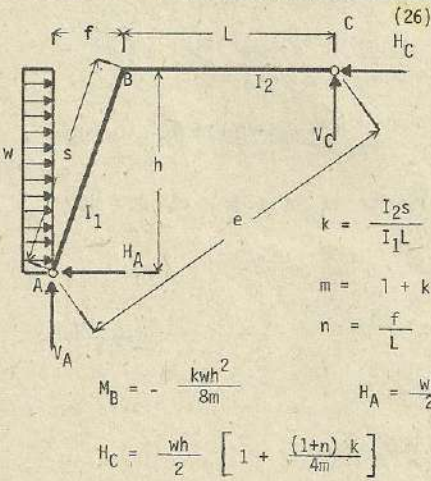
$$H_A = H_C = \frac{w f^2}{2h} - \frac{1+n}{h} M_B$$

$$V_A = w f - \frac{M_B}{L}$$

$$V_C = -\frac{M_B}{L}$$



$$e^2 = (L + f)^2 + h^2$$



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

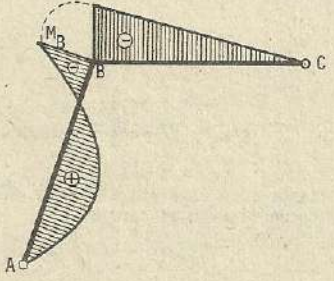
$$n = \frac{f}{L}$$

$$M_B = -\frac{k w h^2}{8m}$$

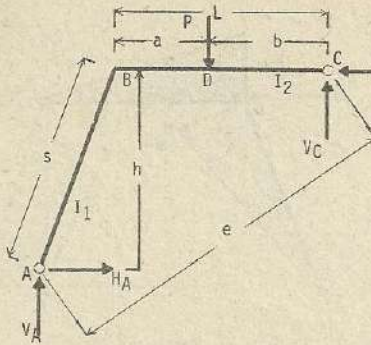
$$H_A = \frac{w h}{2} \left[1 - \frac{(1+n) k}{4m} \right]$$

$$H_C = \frac{w h}{2} \left[1 + \frac{(1+n) k}{4m} \right]$$

$$V_A = V_C = -\frac{M_B}{L}$$



$$e^2 = (L + f)^2 + h^2$$



(27)

$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

$$n = \frac{f}{L}$$

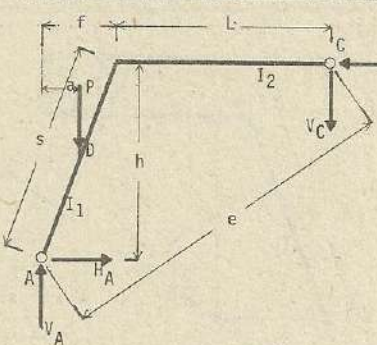
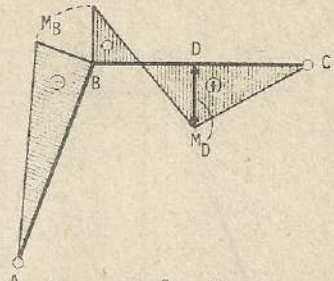
$$e^2 = (L + f)^2 + h^2$$

$$M_B = - \frac{Pab}{2 L^2 m} (L + b)$$

$$H_A = H_C = \frac{n P b - (1 + n) M_B}{h}$$

$$V_A = \frac{P b - M_B}{L}$$

$$V_C = \frac{P a + M_B}{L}$$



(28)

$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

$$n = \frac{f}{L}$$

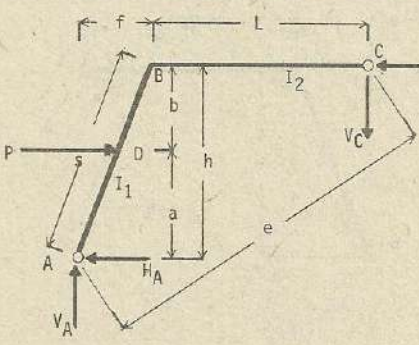
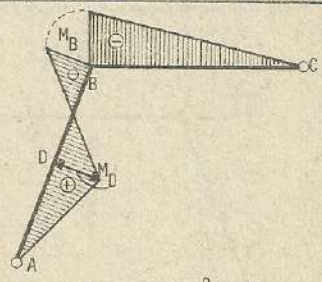
$$e^2 = (L + f)^2 + h^2$$

$$M_B = - \frac{Pak}{2m} \left[1 - \left(\frac{a}{L} \right)^2 \right]$$

$$H_A = H_C = \frac{Pa - (1 + n) M_B}{h}$$

$$V_A = P - \frac{M_B}{L}$$

$$V_C = - \frac{M_B}{L}$$



(29)

$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

$$n = \frac{f}{L}$$

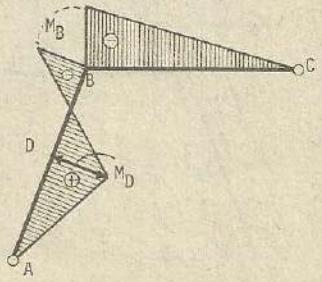
$$e^2 = (L + f)^2 + h^2$$

$$M_B = - \frac{Pabk}{2h^2 m} (h + a)$$

$$H_A = \frac{Pb + (1 + n) M_B}{h}$$

$$V_A = V_C = - \frac{M_B}{L}$$

$$H_C = \frac{Pa - (1 + n) M_B}{h}$$



(30)

$k = \frac{I_2 s}{I_1 L}$
 $m = 1 + k$
 $n = \frac{f}{L}$

$e^2 = (L + f)^2 + h^2$

$M_{BA} = \frac{M}{m}$; $M_{BC} = - \frac{kM}{m}$
 $H_A = \frac{(1 + n) M_{BA} - nM}{h}$ $V_A = V_C = - \frac{M_{BC}}{L}$

(31)

$k = \frac{L}{s}$

$M_B = - \frac{wa^2}{8} \frac{2 - \left(\frac{a}{L}\right)^2}{1 + k}$

(32)

$k = \frac{L}{s}$

$M_B = - \frac{wb^2}{8} \frac{k}{1+k} \left[2 - \left(\frac{b}{L}\right)^2 \right]$

(33)

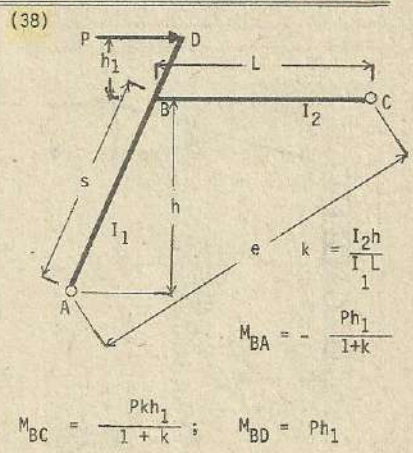
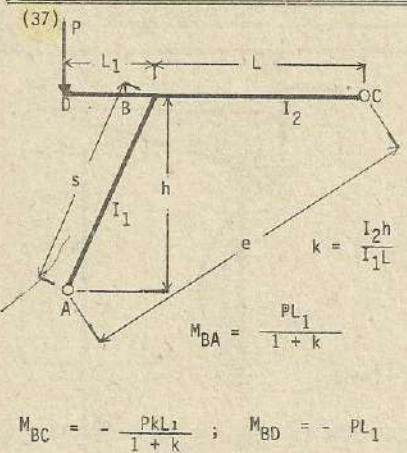
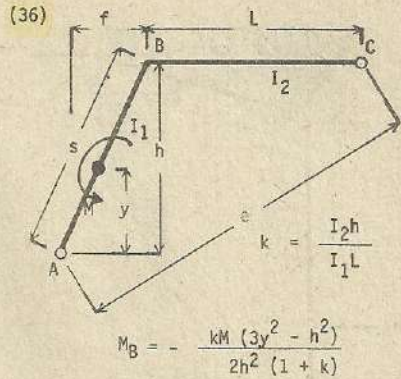
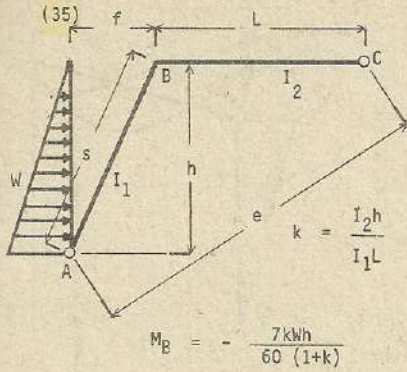
$k = \frac{L}{s}$

$M_B = - \frac{M}{2(1+k)}$

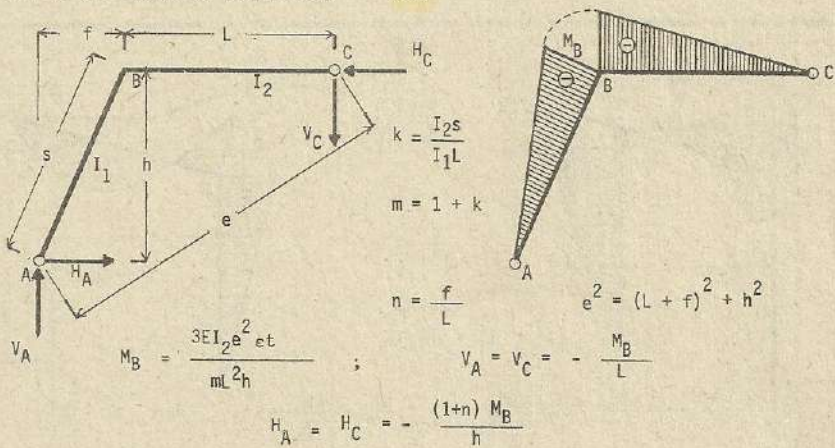
(34)

$k = \frac{L}{s}$

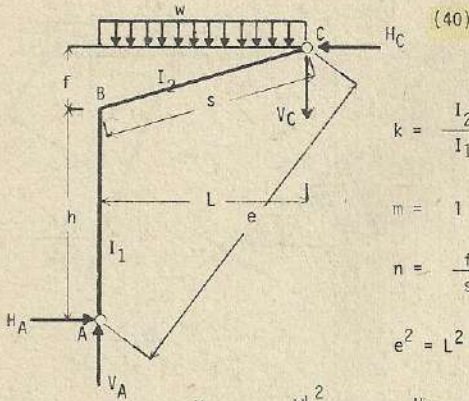
$M_B = - \frac{kM}{2(1+k)}$



Esfuerzos por cambio de temperatura (39)



Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios.



(40)

$$k = \frac{I_2 h}{I_1 s}$$

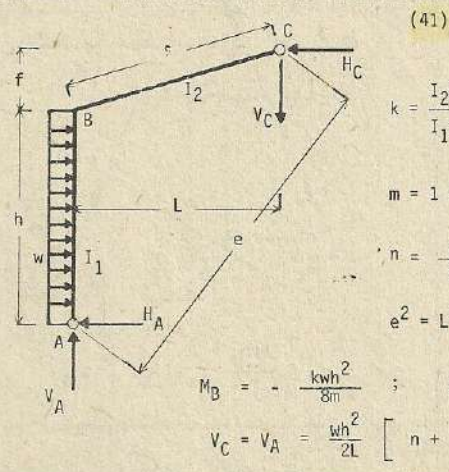
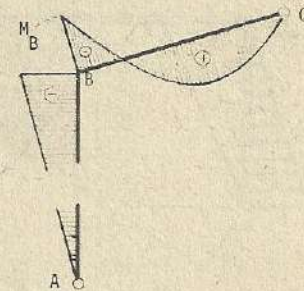
$$m = 1 + k$$

$$n = \frac{f}{s}$$

$$e^2 = L^2 + (h+f)^2$$

$$M_B = -\frac{wL^2}{8m} ; \quad H_A = \frac{wL^2}{8mh} ; \quad V_A = \frac{wL}{2} \left(1 + \frac{1+n}{4m}\right)$$

$$V_C = \frac{wL}{2} \left(1 - \frac{1+n}{4m}\right)$$



(41)

$$k = \frac{I_2 h}{I_1 s}$$

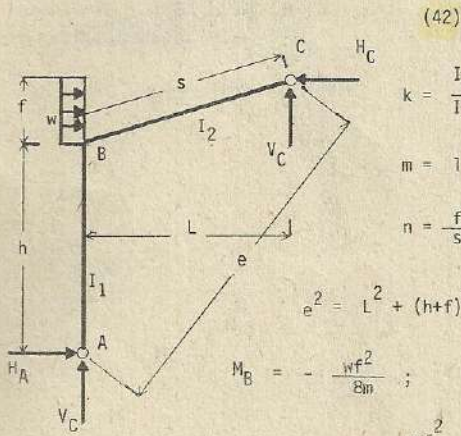
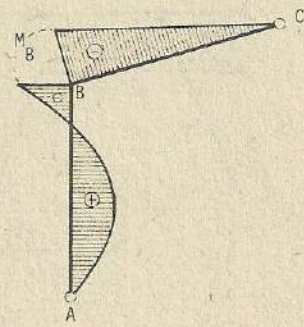
$$m = 1 + k$$

$$n = \frac{f}{s}$$

$$e^2 = L^2 + (h+f)^2$$

$$M_B = -\frac{kwh^2}{8m} ; \quad H_A = \frac{wh}{2} \left(1 - \frac{k}{4m}\right) ; \quad H_C = \frac{wh}{2} \left(1 + \frac{k}{4m}\right)$$

$$V_C = V_A = \frac{wh^2}{2L} \left[n + \frac{k(1+n)}{4m} \right]$$



(42)

$$k = \frac{I_2 h}{I_1 s}$$

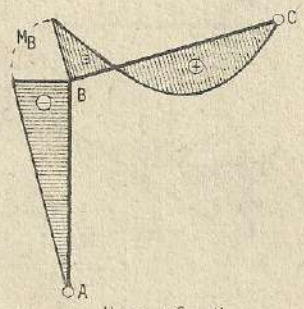
$$m = 1 + k$$

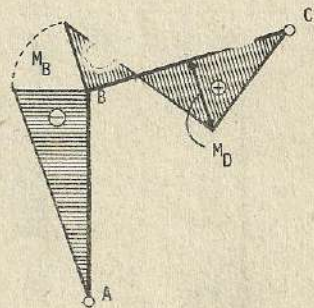
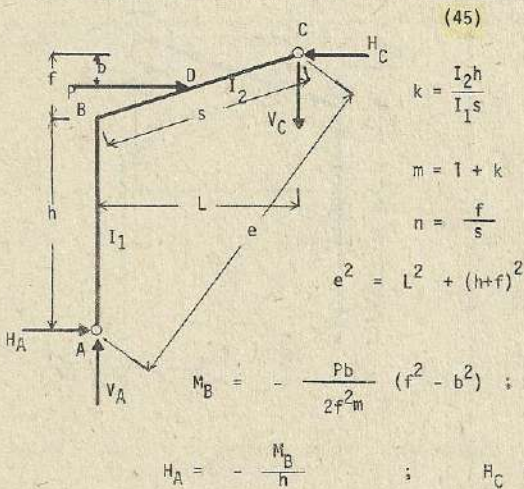
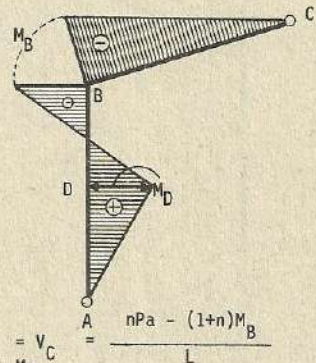
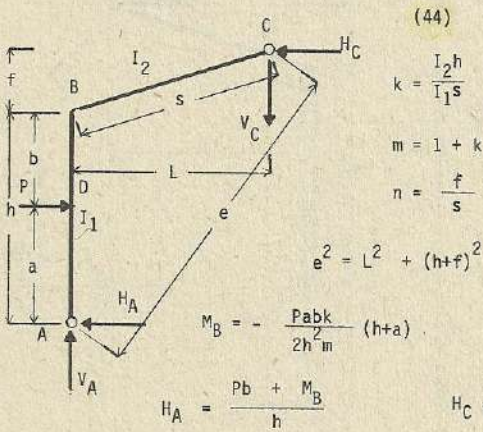
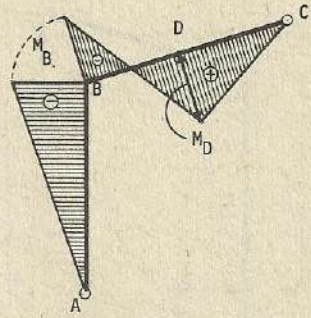
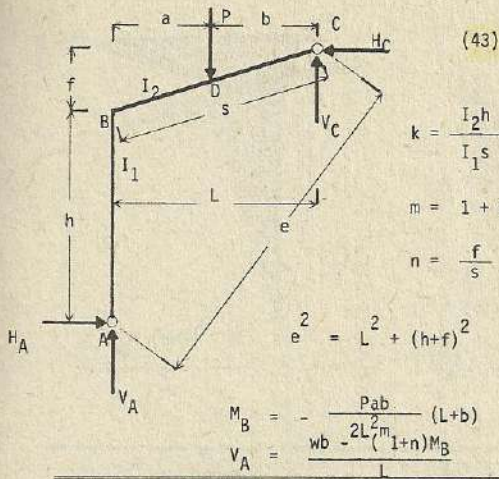
$$n = \frac{f}{s}$$

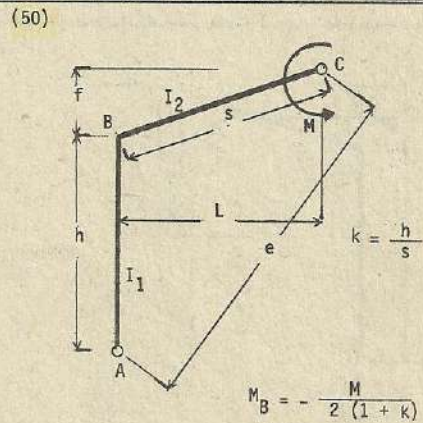
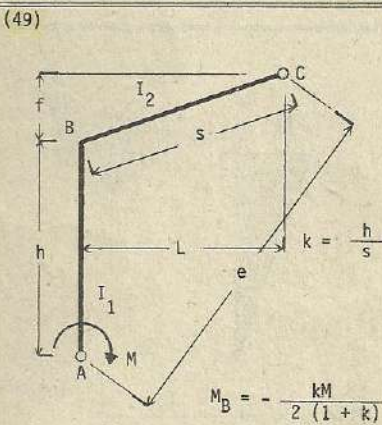
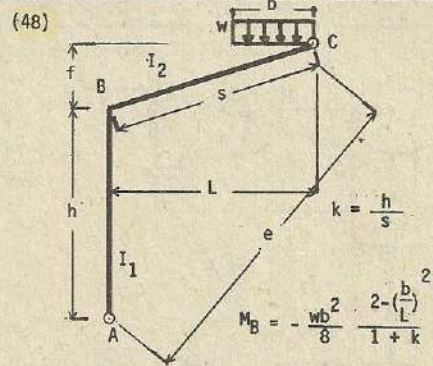
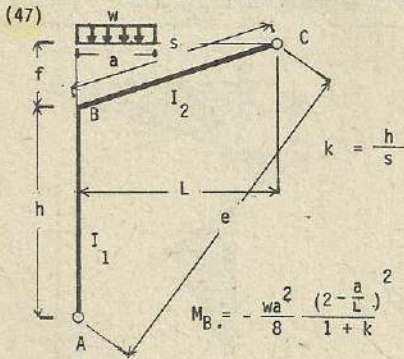
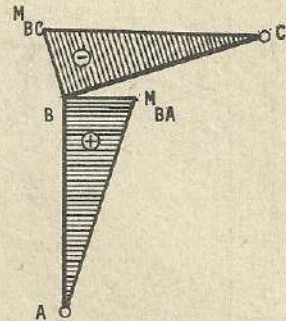
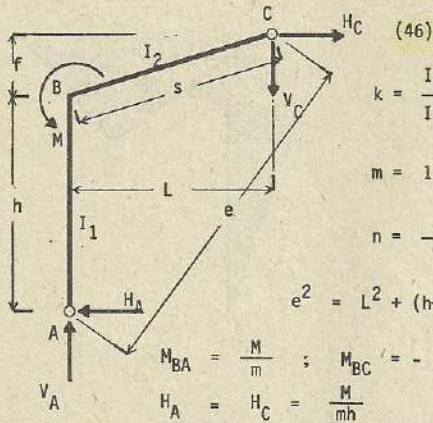
$$e^2 = L^2 + (h+f)^2$$

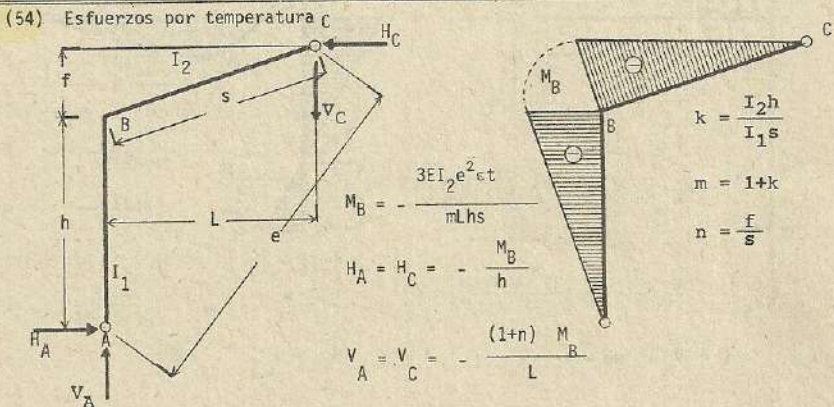
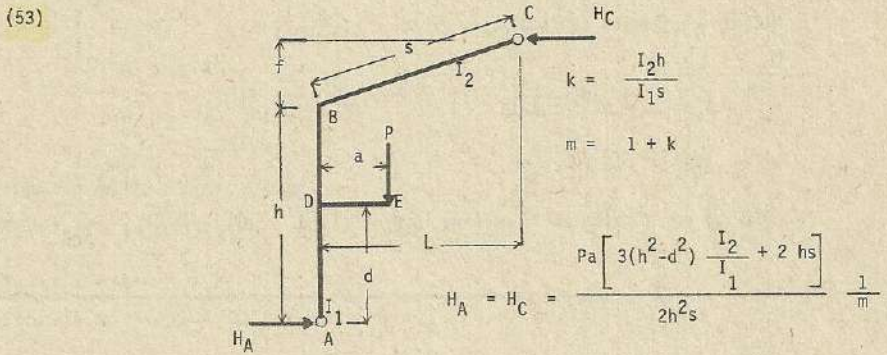
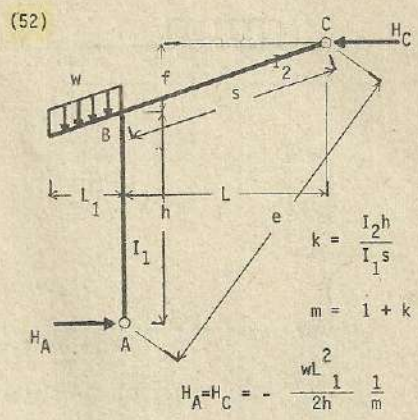
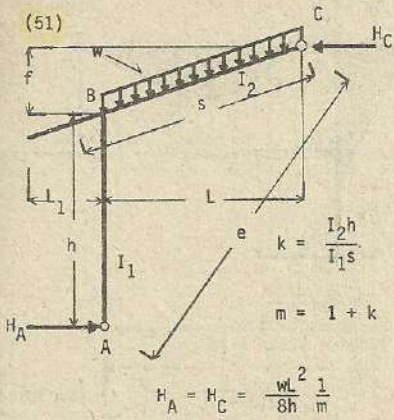
$$M_B = -\frac{wf^2}{8m} ; \quad H_A = \frac{wf^2}{8mh} ; \quad H_C = wf - H_A$$

$$V_C = \frac{wf}{2} \left(1 + \frac{1+n}{4m}\right)$$

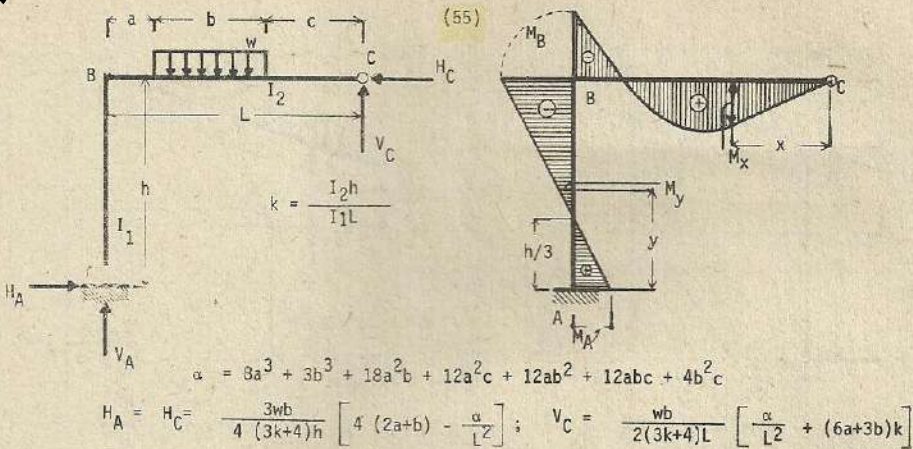








Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios.

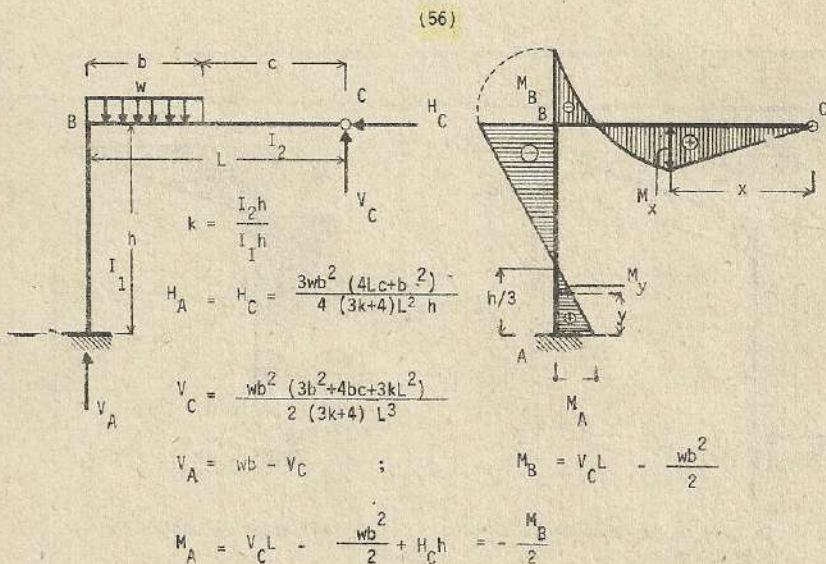


$$M_x = V_C x \quad x < c$$

$$M_x = V_C x - \frac{w(x-c)^2}{2} \quad c \leq x \leq b+c$$

$$M_x = V_C x - wb(x-c - \frac{b}{2}) \quad x > b+c$$

Esfuerzo por flexión en la columna : $M_y = V_C L - \frac{wb}{2}(2a+b) + H_C h$

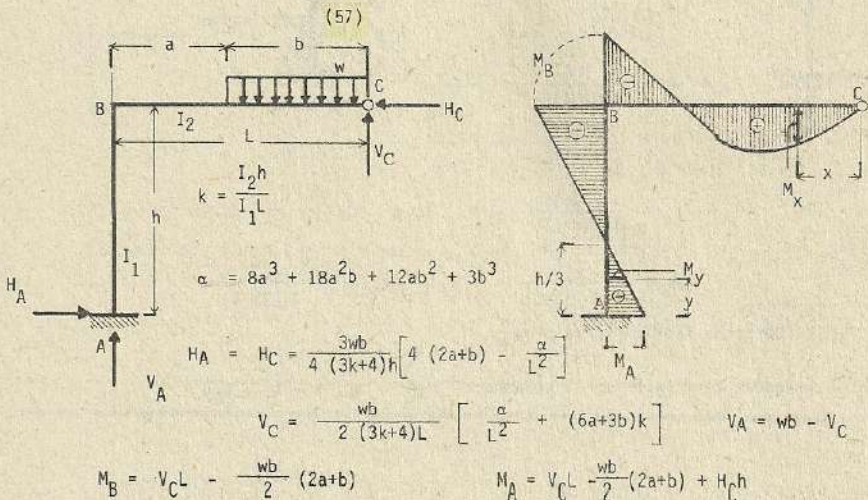


Esfuerzos por doblado o flexión en la viga :

$$M_x = V_C x \quad x \leq c$$

$$M_x = V_C x - \frac{w(x-c)^2}{2} \quad x \geq c$$

Esfuerzos por flexión en la columna : $M_y = V_C L - \frac{wb^2}{2} + H_C y$



Esfuerzo de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2} \quad x \leq \frac{L}{2}$$

$$M_x = V_C x - wb \left(x - \frac{b}{2} \right) \quad x \geq \frac{L}{2}$$

Esfuerzo de flexión en la columna : $M_y = V_C L - \frac{wb}{2}(2a+b) + H_C y$

En caso que $a = b = \frac{L}{2}$; $\alpha = \frac{41}{8} L^3$; $H_A = H_C = \frac{21wL^2}{64(3k+4)h}$

$$V_C = \frac{36k+41}{32(3k+4)} \cdot wL \quad ; \quad V_A = \frac{12k+23}{32(3k+4)} wL$$

$$M_B = - \frac{7}{32(3k+4)} wL^2 \quad ; \quad M_A = \frac{7}{64(3k+4)} wL^2$$

Esfuerzos de flexión en la viga :

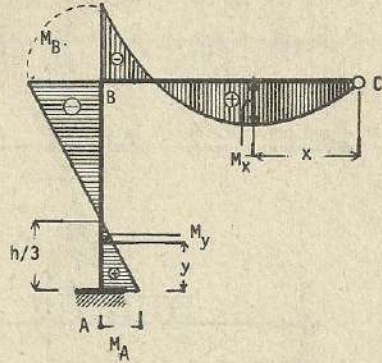
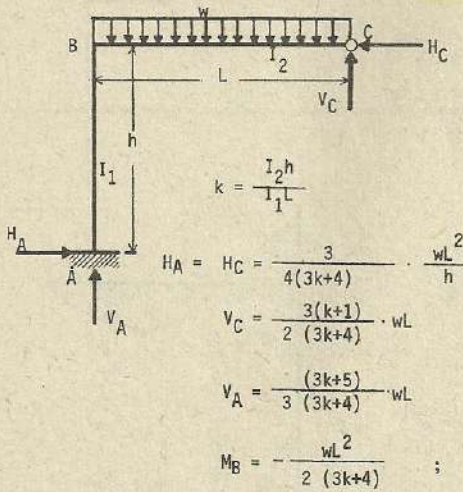
$$M_x = \frac{36k+41}{32(3k+4)} wLx - \frac{wx^2}{2} \quad x \leq \frac{L}{2}$$

$$M_x = \frac{36k+41}{32(3k+4)} wLx - \frac{wL}{2} \left(x - \frac{L}{4} \right) \quad x \geq \frac{L}{2}$$

Esfuerzos de flexión en la columna :

$$M_x = \frac{36k+41}{32(3k+4)} wL^2 - \frac{3wL^2}{8} + \frac{21wL^2 y}{64(3k+4)h}$$

(58)



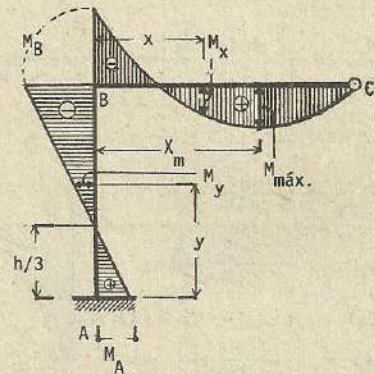
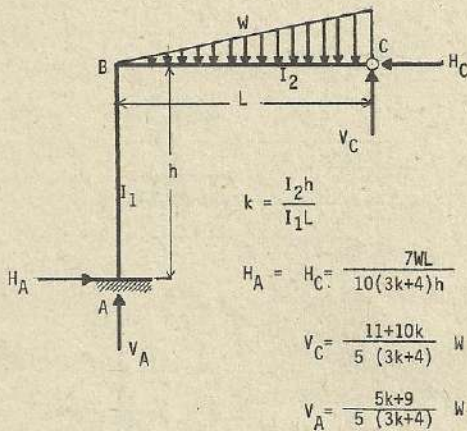
Esfuerzo de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2}$$

Esfuerzo de flexión en la columna :

$$M_y = V_C L - \frac{wL^2}{2} + H_C y$$

(59)

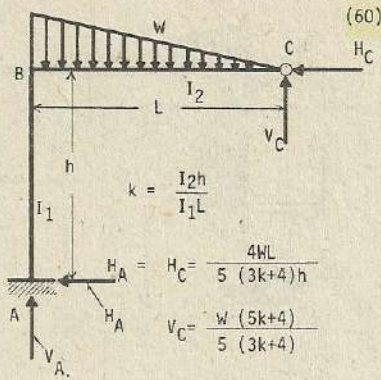


Esfuerzo por flexión en la viga :

$$M_x = M_B + V_A x - \frac{Wx^3}{3L^2}$$

Esfuerzo por flexión en la columna :

$$M_y = M_A - H_A y$$

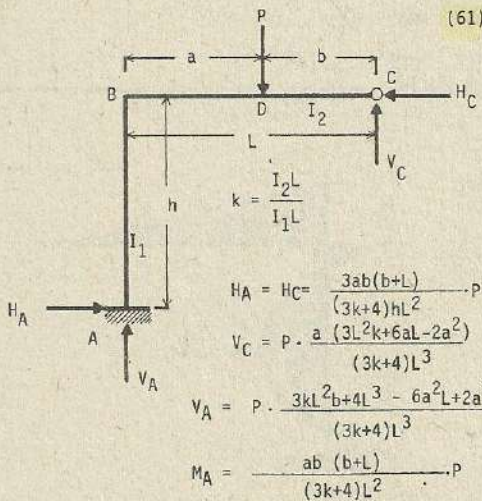
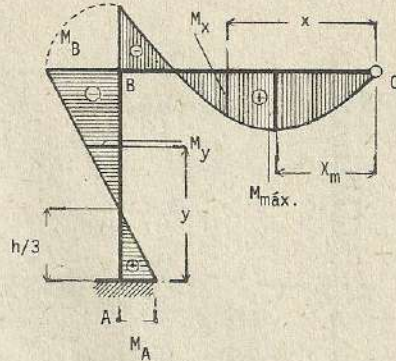


$$V_A = \frac{2W(5k+8)}{5(3k+4)} \quad M_B = -\frac{3WL}{15(3k+4)} \quad M_A = \frac{4WL}{15(3k+4)}$$

Esfuerzo de flexión en la columna : $M_y = M_A - H_A y$

Esfuerzo de flexión en la viga : $M_x = V_C x - \frac{Wx^3}{3L^2} \quad x_m = L \sqrt{\frac{5k+4}{5(3k+4)}}$

$$M_{\text{máx.}} = \frac{2WL(5k+4)}{15(3k+4)} \quad \sqrt{\frac{5k+4}{5(3k+4)}}$$

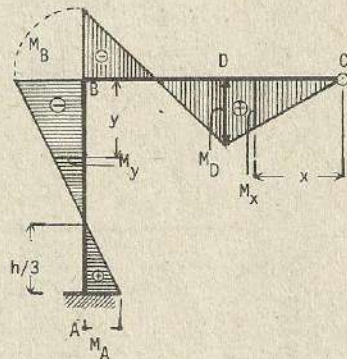


$$H_A = H_C = \frac{3ab(b+L)}{(3k+4)hL^2} P$$

$$V_C = P \cdot \frac{a(3L^2k+6aL-2a^2)}{(3k+4)L^3}$$

$$V_A = P \cdot \frac{3kL^2b+4L^3-6a^2L+2a^3}{(3k+4)L^3}$$

$$M_A = \frac{ab(b+L)}{(3k+4)L^2} P$$



$$M_B = -\frac{2ab(b+L)}{(3k+4)L^2} P$$

$$M_D = \frac{Pab(3L^2k+6aL-2a^2)}{(3k+4)L^2}$$

Esfuerzo de flexión en la viga : $M_x = V_C x \quad x \leq b$

$M_x = V_C x - P(x-b) \quad x \geq b$

Esfuerzo de flexión en la columna : $M_y = V_C L - Pa + H_C y$

Si $a = b = \frac{L}{2}$; $H_A = H_C = \frac{9}{8(3k+4)} \cdot \frac{PL}{h}$; $V_C = \frac{6k+5}{4(3k+4)} \cdot P$

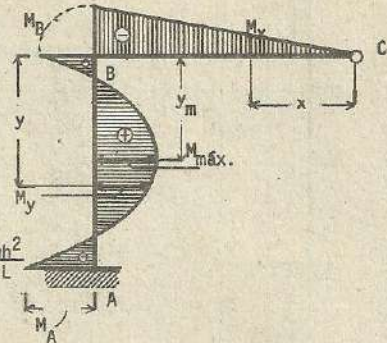
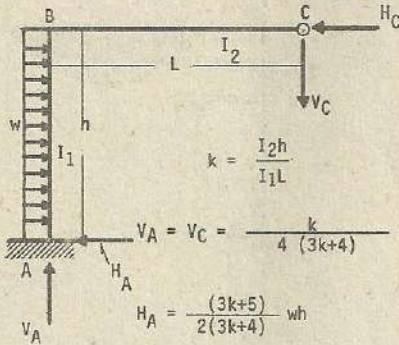
$$V_A = \frac{6k+11}{4(3k+4)} \cdot P$$

$$M_B = -\frac{3}{4(3k+4)} PL$$

$$M_A = \frac{3}{8(3k+4)} \cdot PL$$

$$M_D = \frac{6k+5}{8(3k+4)} \cdot PL$$

(62)



$$H_C = \frac{3(k+1)}{2(3k+4)} wh$$

$$M_A = -\frac{k+2}{4(3k+4)} wh^2$$

$$M_B = -\frac{k}{4(3k+4)} wh^2$$

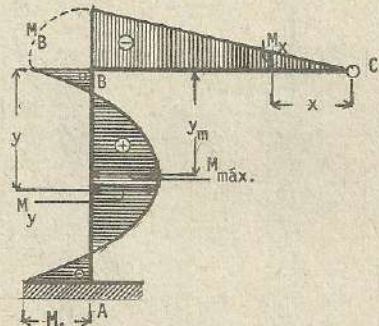
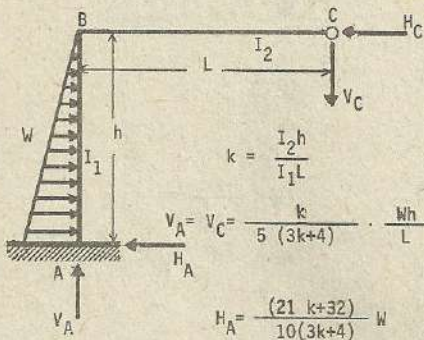
Esfuerzo de flexión en la viga : $M_x = -\frac{k}{4(3k+4)} \cdot \frac{wh^2}{L} \cdot x$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{k}{4(3k+4)} wh^2 + \frac{3(k+1)}{2(3k+4)} why - \frac{wy^2}{2}$$

$$y_m = \frac{3(k+1)}{2(3k+4)} \quad M_{\max} = \frac{(9+10k+3k^2)}{8(3k+4)^2} wh^2$$

(63)



$$H_C = \frac{(9k+8)}{10(3k+4)} W$$

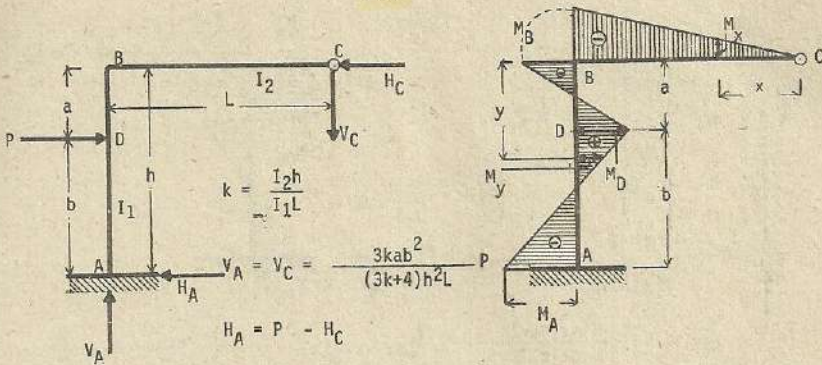
$$M_B = -\frac{k}{5(3k+4)} Wh ; M_A = -\frac{(9k+16)}{30(3k+4)} Wh$$

Esfuerzo de flexión en la viga : $M_x = -\frac{k}{5(3k+4)} \cdot \frac{Wh}{L} \cdot x$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{k}{5(3k+4)} Wh + \frac{(9k+8)}{10(3k+4)} wy - \frac{wy^3}{3h^2}$$

(64)



$$H_C = \frac{(3k(2a+h) + 2(a+2h))}{(3k+4)h^3} Pb^2; \quad M_A = -\frac{3ka + 2(a+h)}{(3k+4)h^2} Pab$$

$$M_B = -\frac{3kab^2}{(3k+4)h^2} P; \quad M_D = \frac{(6ka + 2(a+2h))}{(3k+4)h^3} Pab^2$$

Esfuerzo de flexión en la viga :

$$M_x = -\frac{3kab^2}{(3k+4)h^2L} Px$$

Esfuerzo de flexión en la columna :

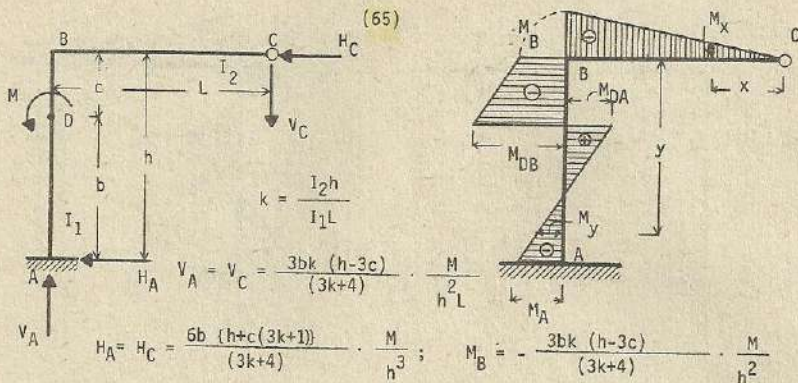
$$M_y = -\frac{3kab^2}{(3k+4)h^2} P + \frac{(3k(2a+b) + 2(a+2h))}{(3k+4)h^3} Pb^2 y \quad y < a$$

$$M_y = -\frac{3kab^2}{(3k+4)h^2} P + \frac{(3k(2a+b) + 2(a+2h))}{(3k+4)h^3} Pb^2 y - P(y-a); \quad y > a$$

$$\text{Si } a = b = \frac{h}{2} \quad V_A = V_C = \frac{3kh}{8(3k+4)} P \quad ; \quad H_A = \frac{6k+11}{4(3k+4)} P$$

$$H_C = \frac{(6k+5)}{4(3k+4)} P$$

$$M_A = \frac{(3k+6)}{8(3k+4)} Ph; \quad M_B = -\frac{3k}{8(3k+4)} Ph; \quad M_D = \frac{(3k+5)}{8(3k+4)} Ph$$



$$M_{DB} = -\frac{3bM}{(3k+4)h^2} \left[k(h-3c) + \frac{2c(h+c(3k+1))}{h} \right]; \quad M_{DA} = M - M_{DB}$$

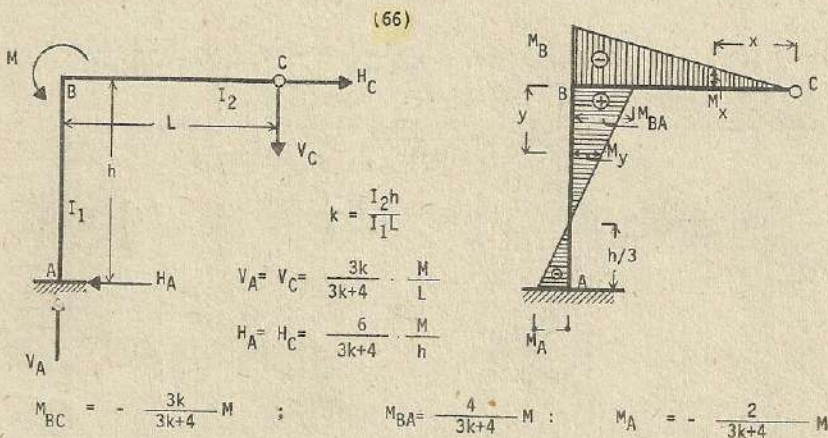
$$M_A = M - V_C L - H_C h$$

Esfuerzo de flexión en la viga : $M_x = -V_C x$

Esfuerzo de flexión en la columna:

$$M_y = -V_C L - H_C y \quad y \leq a$$

$$M_y = M - V_C L - H_C y \quad y \geq a$$

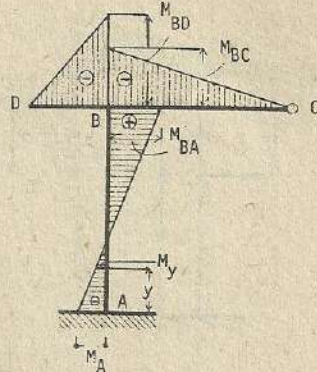
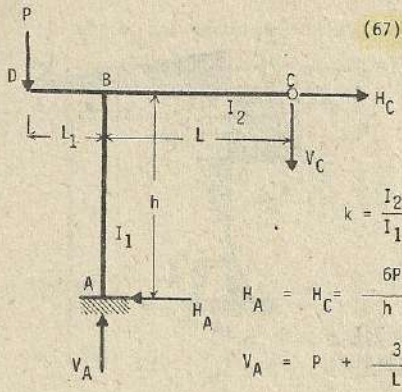


Esfuerzo de flexión en la viga :

$$M_x = -\frac{3k}{3k+4} \cdot \frac{x}{L} \cdot M$$

Esfuerzo de flexión en la columna :

$$M_y = \frac{3k}{3k+4} \cdot M - \frac{6y}{3k+4} \cdot \frac{M}{h} + M$$

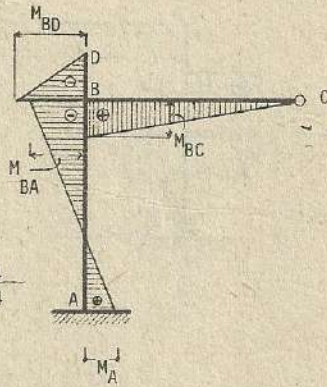
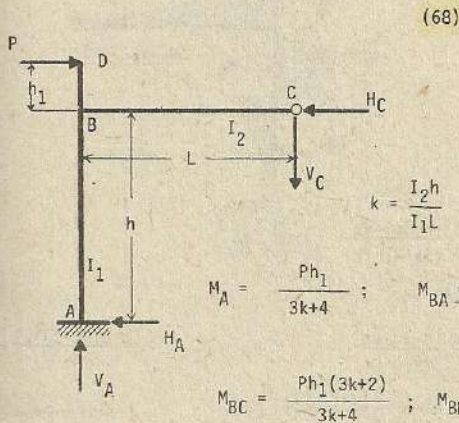


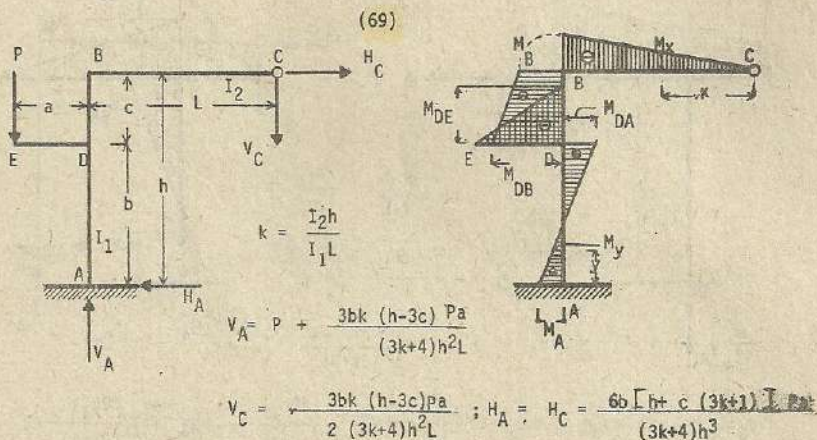
$$V_C = \frac{3PL_1 k}{L(3k+4)} ; \quad M_{BD} = -PL_1 ; \quad M_{BA} = \frac{4PL_1}{3k+4} ; \quad M_{BC} = -\frac{3PL_1 k}{(3k+4)}$$

$$M_A = -\frac{2PL_1}{3k+4} ; \quad \text{Esfuerzo de flexión en la viga : } M_x = -\frac{PL_1 k x}{(3k+4)L}$$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{2PL_1}{(3k+4)} + \frac{6PL_1 y}{(3k+4)h}$$





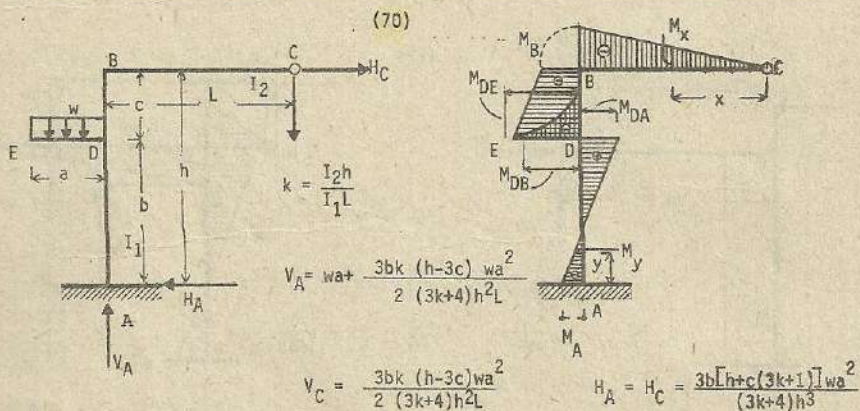
$$M_{DB} = -\frac{3bPa}{(3k+4)h^2} \left[k(h-3c) + \frac{2c(h+c(3k+1))}{h} \right]; M_B = -\frac{3bk(h-3c)Pa}{(3k+4)h^2}$$

$$M_{DA} = Pa - M_{DB}; M_A = Pa - V_C L - H_C h$$

Esfuerzo de flexión en la columna: $M_y = -V_C y - H_C y \quad y \leq c$

$M_y = Pa - V_C y - H_C y \quad y \geq c$

Esfuerzo de flexión en la viga: $M_x = -V_C x$



$$M_{DB} = -\frac{3bwa^2}{2(3k+4)h^2} \left[k(h-3c) + \frac{2c[h+c(3k+1)]}{h} \right]$$

$$M_B = -\frac{3bk(h-3c)wa^2}{2(3k+4)h^2}; M_{DA} = \frac{wa^2}{2} + M_{DB}; M_A = \frac{wa^2}{2} - V_C L - H_C h$$

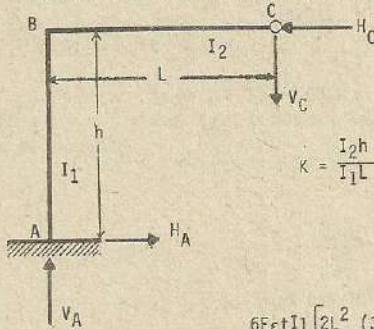
Esfuerzo de flexión en la viga: $-V_C x$

Esfuerzo de flexión en la columna: $M_y = -V_C y - H_C y \quad y \leq c$

$M_y = \frac{wa^2}{2} - V_C y - H_C y \quad y \geq c$

(71)

Esfuerzos por cambio de temperatura



$$K = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{6E\epsilon t I_1 [2L^2 (3k+1) + 3kh^2]}{(3k+4)h^3 L}$$

$$V_A = V_C = \frac{6E\epsilon t I_1 (2h^2 + 3L^2) k}{(3k+4)h^2 L^2} ; \quad M_B = -\frac{6E\epsilon t I_1 (2h^2 + 3L^2) k}{(3k+4)h^2 L}$$

$$M_A = \frac{6E\epsilon t I_1 [(3k+2) + kh^2]}{(3k+4)h^2 L} ; \quad \text{Esfuerzo de flexión en la viga:}$$

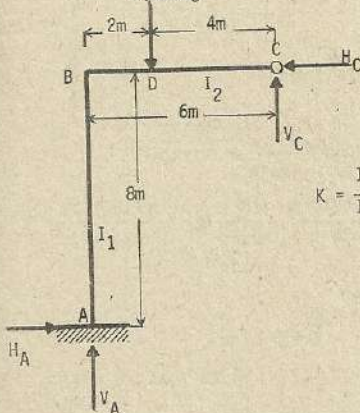
$$M_x = -V_C x$$

Momento de flexión en la columna : $M_y = M_B + H_C y$

Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios.

Solución de problemas numéricos:

4,000Kg



$$K = \frac{I_2 h}{I_1 L} = \frac{45,000 \times 8}{15,000 \times 6} = 4$$

$$I_2 = 45,000 \text{ cm}^4$$

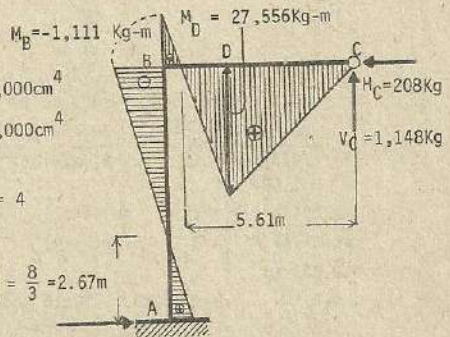
$$I_1 = 15,000 \text{ cm}^4$$

$$\frac{h}{3} = \frac{8}{3} = 2.67 \text{ m}$$

$$H_A = 208 \text{ Kg}$$

$$V_A = 2,909 \text{ Kg}$$

$$M_A = 556 \text{ Kg-m}$$



$$H_A = H_C = \frac{3ab(b+L)}{(3k+4)hL^2} p = \frac{3 \times 2 \times 4 \times (4+6)}{(3 \times 4 + 4) \times 8 \times 6 \times 6} \times 4,000 = 208 \text{ Kg.}$$

$$V_C = p \frac{a(3L^2 k + 6aL - 2a^2)}{(3k+4)L^3} = 4,000 \times \frac{2(3 \times 6 \times 6 \times 4 + 6 \times 2 \times 6 - 2 \times 2 \times 2)}{(3 \times 4 + 4) \times 6 \times 6 \times 6} = 1,148 \text{ Kg.}$$

$$V_A = P \frac{3kL^2b + 4L^3 - 6a^2L + 2a^3}{(3k+4)L^3}$$

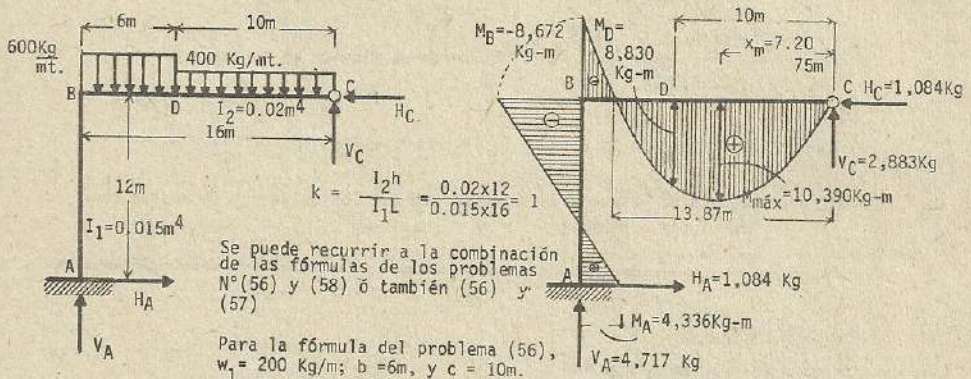
$$= 4,000 \times \frac{3 \times 4 \times 6 \times 6 \times 4 + 4 \times 6 \times 6 \times 6 - 6 \times 2 \times 2 \times 6 + 2 \times 2 \times 2 \times 2}{(3 \times 4 + 4) \times 6 \times 6 \times 6} = 2,852 \text{ Kg.}$$

$$M_B = - \frac{2ab(b+L)}{(3k+4)L^2} \cdot P = - \frac{2 \times 2 \times 4 (4+6)}{(3 \times 4 + 4) \times 6 \times 6} \times 4,000 = - 1,111 \text{ Kg-m}$$

$$M_A = \frac{ab(b+L)}{(3k+4)L^2} \cdot P = \frac{2 \times 4 \times (4+6)}{(3 \times 4 + 4) \times 6 \times 6} \times 4,000 = 556 \text{ Kg-m}$$

$$M_D = M_{\text{máx.}} = \frac{Pab(3L^2k + 5aL - 2a^2)}{(3k+4)L^2}$$

$$= \frac{4,000 \times 2 \times 4 (3 \times 6 \times 6 \times 4 + 6 \times 2 \times 6 - 2 \times 2 \times 2)}{(3 \times 4 + 4) \times 6 \times 6} = 27,556 \text{ Kg-m}$$



Para (58), $w_2 = 400 \text{ Kg/m}$.

$$H_A = H_C = \frac{3w_1b^2(4Lc + b^2)}{4(3k+4)L^2h} + \frac{3w_2L^2}{4(3k+4)h} = \frac{3 \times 200 \times 6^2 \times (4 \times 16 \times 10 + 6^2)}{4 \times (3 \times 1 + 4) \times 16^2 \times 12} + \frac{3 \times 400 \times 16^2}{4(3 \times 1 + 4) \times 12}$$

$$= 1,084 \text{ Kg.}$$

$$V_C = \frac{w_1b^2(3b^2 + 4bc + 3kL^2)}{2(3k+4)L^3} + \frac{3w_2L(k+1)}{2(3k+4)} = \frac{200 \times 6^2 \times (3 \times 6^2 + 4 \times 6 \times 10 + 3 \times 1 \times 16^2)}{2(3 \times 1 + 4) \times 16^3} + \frac{3 \times 400 \times 16(1+1)}{2(3 \times 1 + 4)}$$

$$= 2,883 \text{ Kg.}$$

$$V_A = w_1b + w_2L - V_C = 200 \times 6 + 400 \times 16 - 2,883 = 4,717 \text{ Kg.}$$

$$M_B = V_C L - w_1 \frac{b^2}{2} - w_2 \frac{L^2}{2} = 2,883 \times 16 - 200 \times \frac{6 \times 6}{2} - 400 \times \frac{16 \times 16}{2} = - 8,672 \text{ Kg-m}$$

$$M_A = M_B + H_C h = - 8,672 + 1,084 \times 12 = 4,336 \text{ Kg-m}$$

Fuerza cortante en el miembro (BC) ; $V = V_C - wx$

Si hacemos como (x_m) la distancia desde (C) al punto donde la cortante es cero (0).

$$V_C - wx_m = 0 \longrightarrow 2,883 - 400x_m = 0$$

$$x_m = 7,2075 \text{ m}$$

$$\text{Luego, } M_{\text{máx.}} = V_C x_m - w \frac{x_m^2}{2} = 2,883 \times 7.2075 - 400 \cdot \frac{7.2075^2}{2} \approx 10,390 \text{ Kg} \cdot \text{m}$$

$$M_D = V_C \times 10 - w_2 \frac{10^2}{2} = 2,883 \times 10 - 400 \times \frac{10^2}{2} \approx 8,830 \text{ Kg} \cdot \text{m}$$

La distancia aproximada (x_0) desde (C) hasta el punto donde el esfuerzo de momento es cero (0) :

$$M_x = V_C x - w_1 \frac{(x-10)^2}{2} - w_2 \cdot \frac{x^2}{2}$$

Si $x_0 > 10\text{m}$.

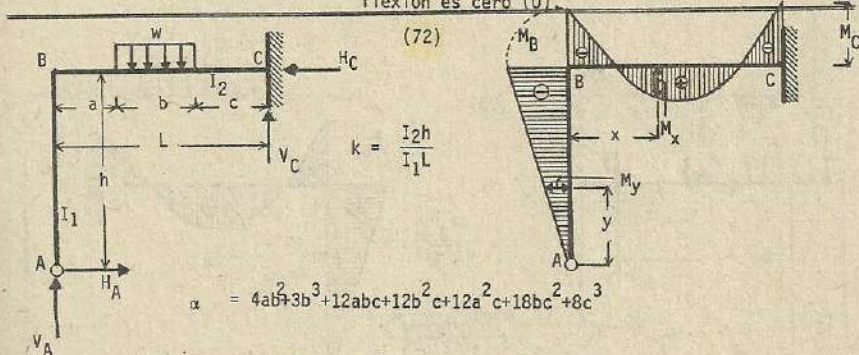
$$V_C x_0 - w_1 \frac{(x_0 - 10)^2}{2} - w_2 \cdot \frac{x_0^2}{2} = 0$$

$$2,883 x_0 - 200 x \frac{(x_0 - 10)^2}{2} - 400 \cdot \frac{x_0^2}{2} = 0$$

$$300 x_0^2 - 4,883 x_0 + 10,000 = 0$$

donde : $x_0 = 2.6$

$x_0 = 13.87$ Luego, a 13.87 m del punto (C) el momento de flexión es cero (0).



$$\alpha = 4ab^2 + 3b^3 + 12abc + 12b^2c + 12a^2c + 18bc^2 + 8c^3$$

$$H_A = H_C = \frac{wb}{4(4k+3)L^2h} [3\alpha - 8L(b^2 + 3c^2 + 3bc)]$$

$$V_A = \frac{wb}{2(4k+3)L^3} [k+3] \alpha - 6L(b^2 + 3c^2 + 3bc) ; M_C = -M_B + V_A L - \frac{wb}{2}(b+2c)$$

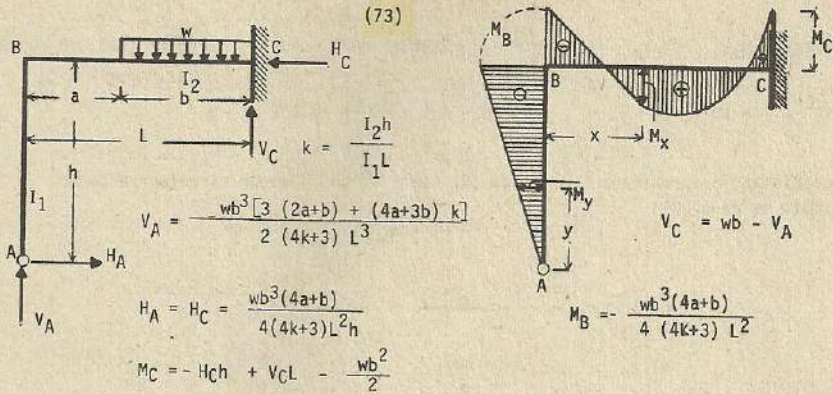
$$V_C = wb - V_A ; M_B = - \frac{wb}{4(4k+3)L^2h} [3\alpha - 8L(b^2 + 3c^2 + 3bc)]$$

Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga : $M_x = -H_A h + V_A x$ $x \leq a$

$$M_x = -H_A h + V_A x - \frac{w(x-a)^2}{2} \quad a \leq x \leq a+b$$

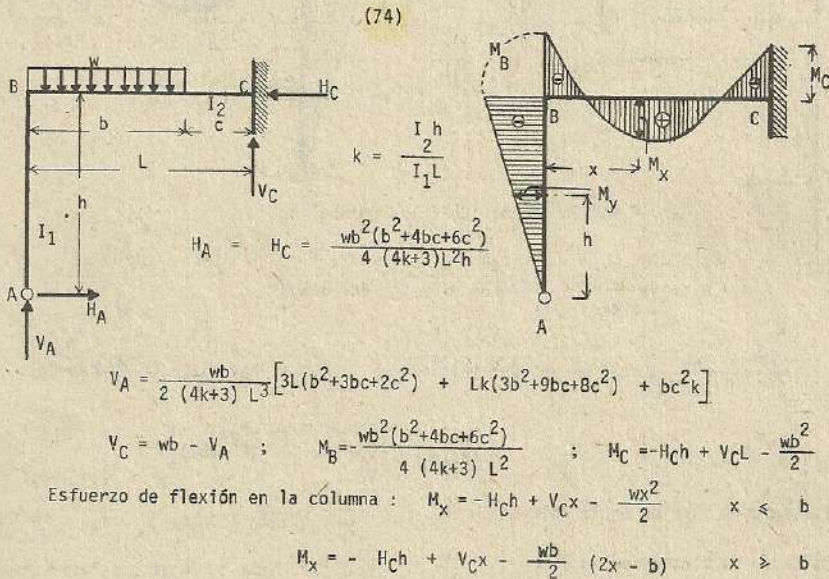
$$M_x = -H_A h + V_A x - wb(x-a - \frac{b}{2}) \quad x \geq a+b$$



Esfuerzo de flexión en la viga : $M_x = -H_C h + V_C x$ $x \leq a$

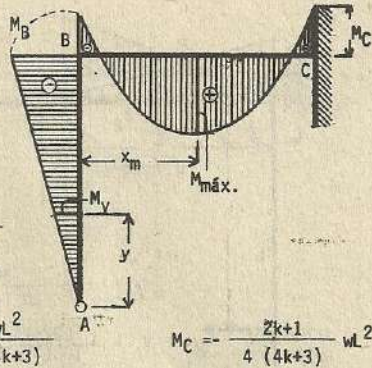
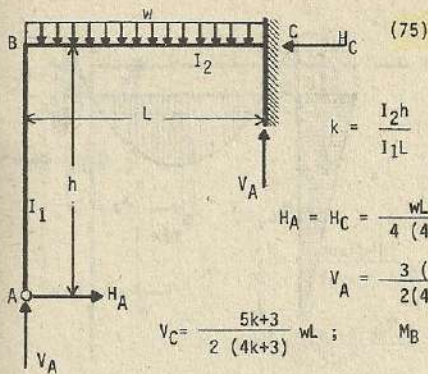
$$M_x = -H_C h + V_C x - \frac{w(x-a)^2}{2} \quad x \geq a$$

Esfuerzo de flexión en la columna : $M_y = -H_C y$



Esfuerzo de flexión en la columna :

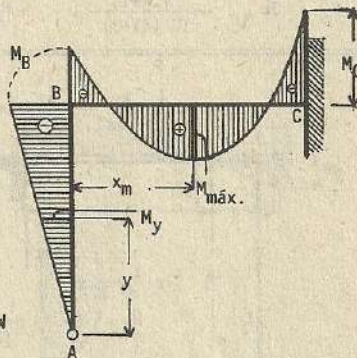
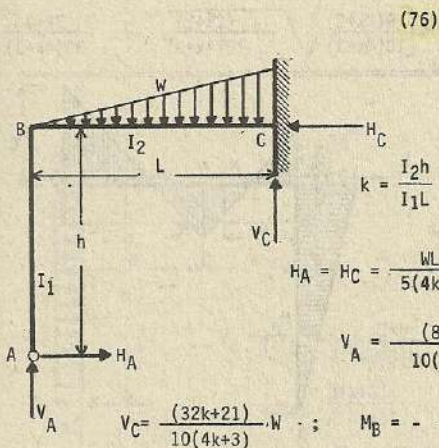
$$M_y = -H_C y$$



Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga :

$$M_x = -H_A h + V_A x - \frac{wx^2}{2} ; \quad x_m = \frac{3(k+1)L}{2(4k+3)} ; \quad M_{\text{máx.}} = \frac{(3+10k+9k^2)}{8(4k+3)^2} wL^2$$

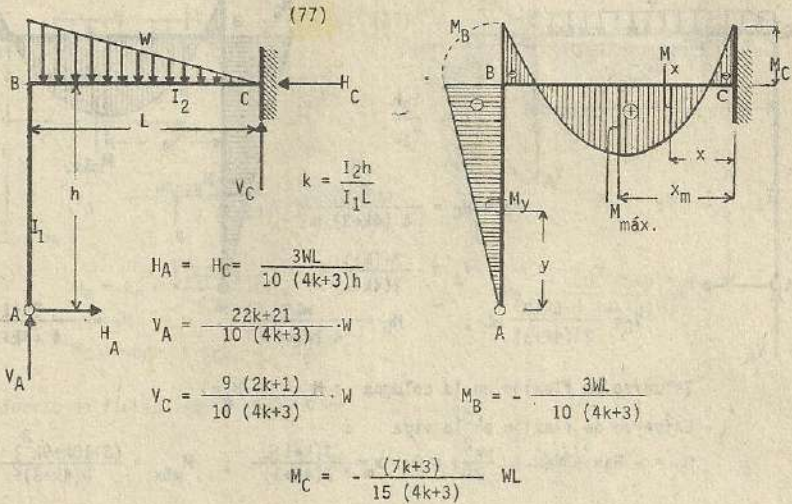


Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga :

$$M_x = -H_A h + V_A x - \frac{wx^3}{3L^2}$$

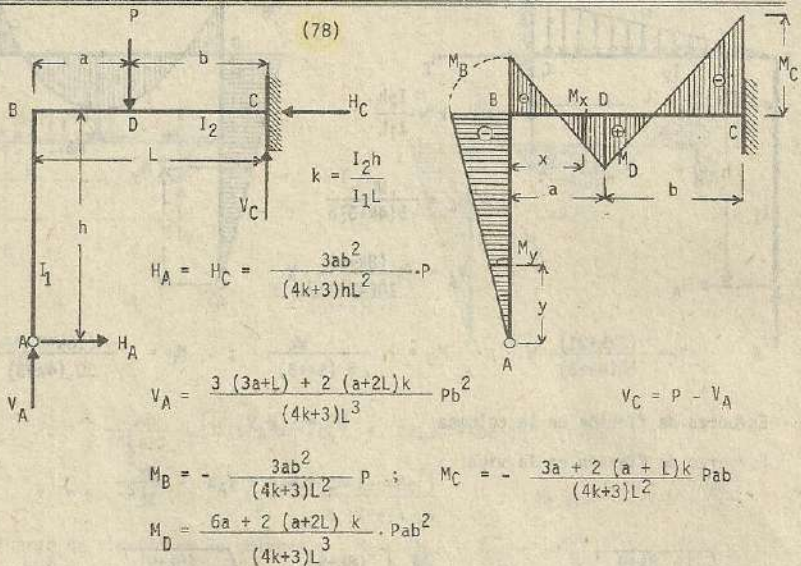
$$x_m = L \cdot \sqrt{\frac{8k+9}{10(4k+3)}} ; \quad M_{\text{máx.}} = \frac{wL}{15} \left[\frac{(8k+9)}{(4k+3)} \cdot \sqrt{\frac{(8k+9)^3}{10(4k+3)}} - \frac{3}{(4k+3)} \right]$$



Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga : $M_x = -M_C + V_C x - \frac{wx^3}{3L^2}$

$$x_m = 3L \sqrt{\frac{(2k+1)}{10(4k+3)}} ; \quad M_{m\acute{a}x.} = 2WL \left[\frac{9(2k+1)}{10(4k+3)} \sqrt{\frac{(2k+1)}{10(4k+3)}} - \frac{7k+3}{30(4k+3)} \right]$$



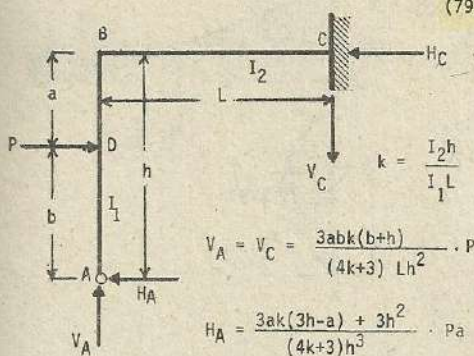
Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga :

$$M_x = -H_A h + V_A x \quad x \leq a$$

$$M_x = -H_A h + V_A x - P(x-a) \quad x \geq a$$

(79)



$$V_A = V_C = \frac{3abk(b+h)}{(4k+3)Lh^2} \cdot P$$

$$H_A = \frac{3ak(3h-a) + 3h^2}{(4k+3)h^3} \cdot Pa$$

$$M_C = \frac{abk(b+h)}{(4k+3)h^2} \cdot P$$

$$M_B = \frac{2abk(b+h)}{(4k+3)h^2} \cdot P$$

$$M_D = \frac{2ak(3h-a) + 3h^2}{(4k+3)h^3} \cdot Pab$$

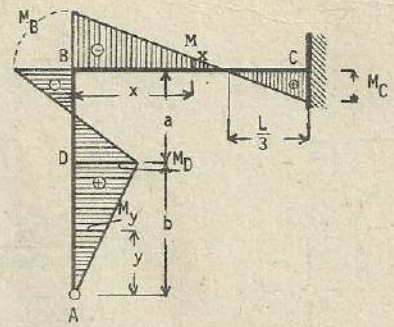
Esfuerzo de flexión en la columna :

$$M_y = H_A y \quad y \leq b$$

$$M_y = H_A y - P(y-b) \quad y \geq b$$

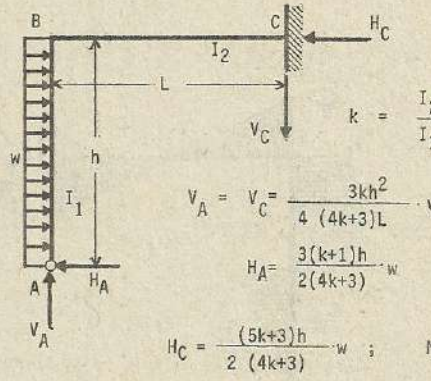
Esfuerzo de flexión en la viga :

$$M_x = H_A h - Pa + V_A x$$



$$H_C = P - H_A$$

(80)



$$V_A = V_C = \frac{3kh^2}{4(4k+3)L} \cdot w$$

$$H_A = \frac{3(k+1)h}{2(4k+3)} \cdot w$$

$$H_C = \frac{(5k+3)h}{2(4k+3)} \cdot w$$

$$M_B = -\frac{k}{2(4k+3)} \cdot wh^2$$

$$M_C = \frac{k}{4(4k+3)} \cdot wh^2$$

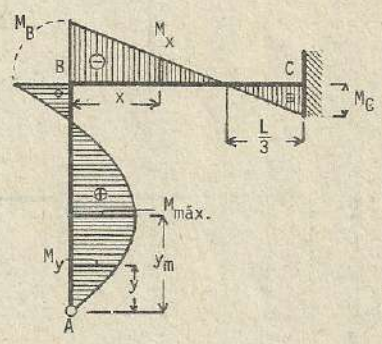
Esfuerzo de flexión en la columna :

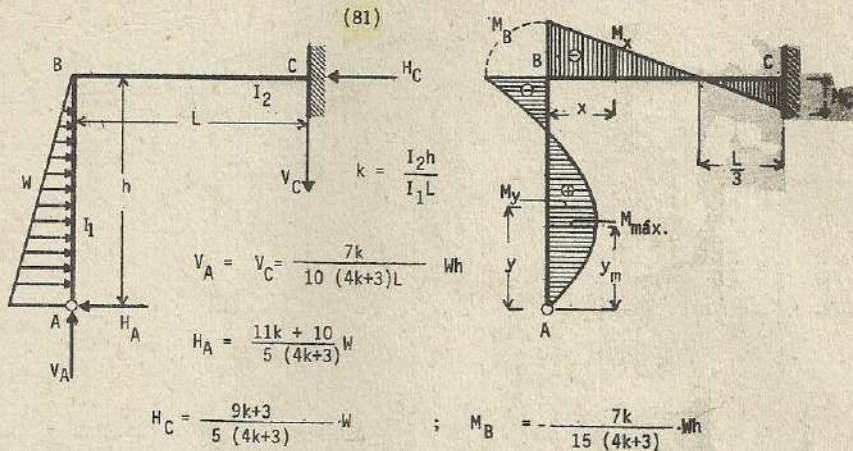
$$M_y = H_A y - \frac{wy^2}{2} \quad ; \quad y_m = \frac{3(k+1)}{2(4k+3)} \cdot h$$

$$M_{m\acute{a}x.} = \frac{9(k+1)^2}{8(4k+3)^2} \cdot wh^2$$

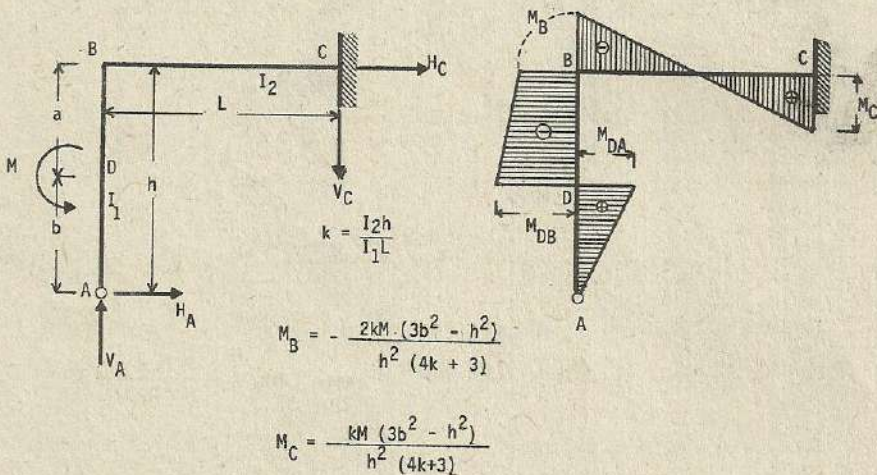
Esfuerzo de flexión en la viga :

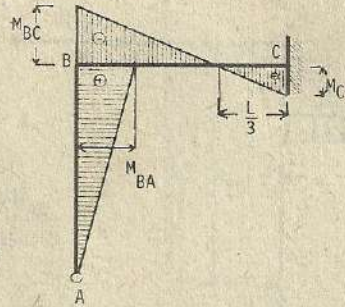
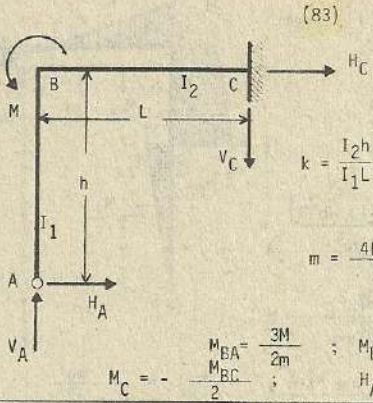
$$M_x = H_A h - \frac{wh^2}{2} + V_A x$$



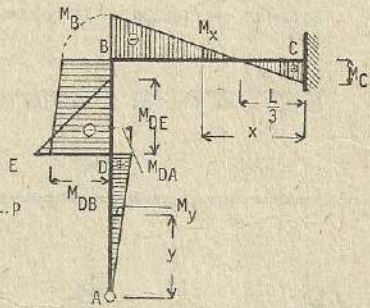
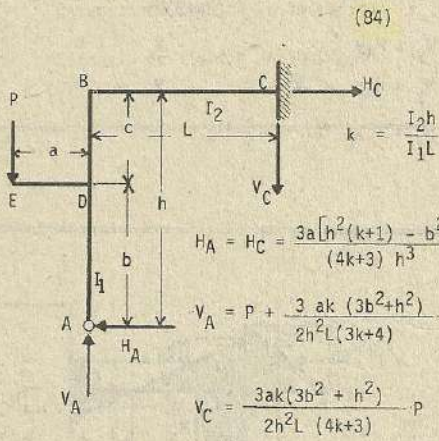


(82)





$M_C = -\frac{M_{BC}}{2}$; $M_{BA} = \frac{3M}{2m}$; $M_{BC} = -\frac{2kM}{m}$; $H_A = H_C = \frac{M_{BA}}{h}$; $V_A = V_C = \frac{3M}{L}$



$H_A = H_C = \frac{3a[h^2(k+1) - b^2k]}{(4k+3)h^3} \cdot P$

$V_A = P + \frac{3ak(3b^2+h^2)}{2h^2L(3k+4)} \cdot P$

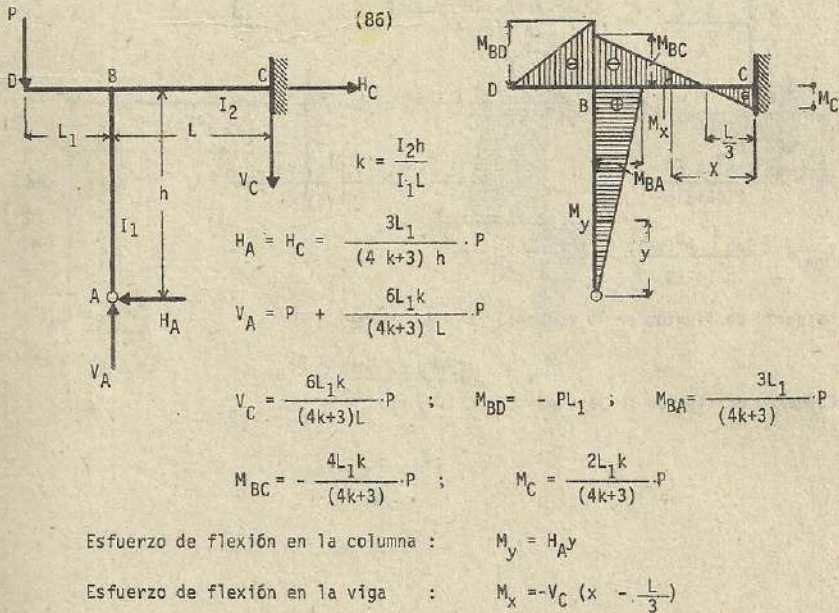
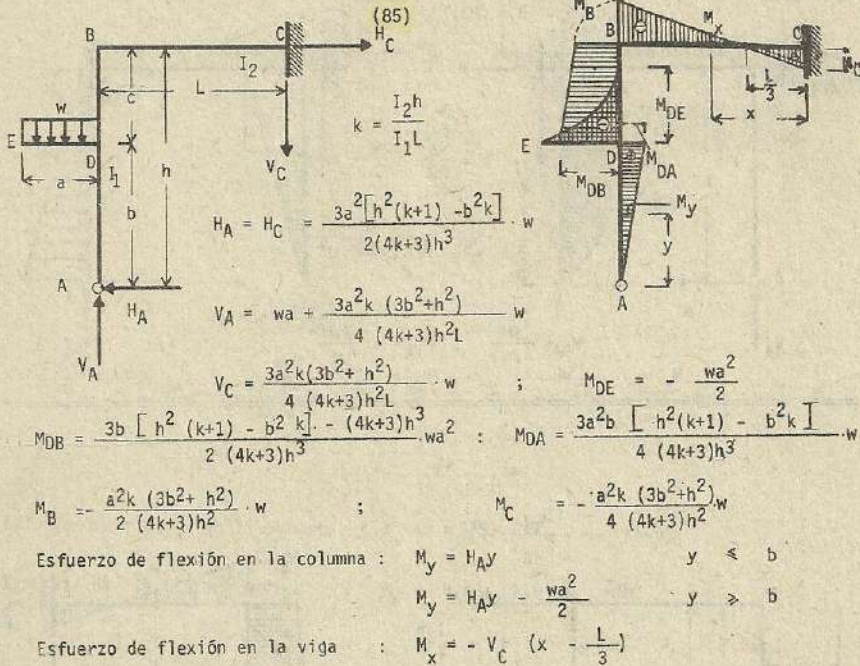
$V_C = \frac{3ak(3b^2+h^2)}{2h^2L(4k+3)} \cdot P$; $M_B = -\frac{ak(3b^2+h^2)}{(4k+3)h^2} \cdot P$

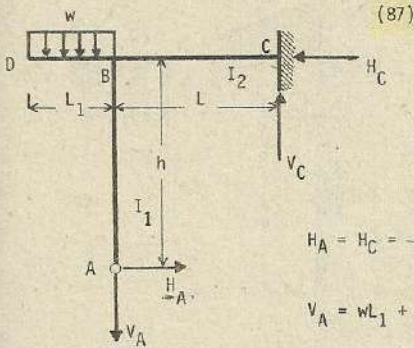
$M_C = \frac{ak(3b^2+h^2)}{2(4k+3)h^2} \cdot P$; $M_{DB} = \frac{3b[h^2(k+1) - b^2k] - (4k+3)h^3}{(4k+3)h^3} Pa$; $M_{DE} = -Pa$

$M_{DA} = \frac{3ab[h^2(k+1) - b^2k]}{(4k+3)h^3} \cdot P$

Esfuerzo de flexión en la columna : $M_y = H_A y$; $y < b$
 $M_y = H_A y - Pa$; $y \geq b$

Esfuerzo de flexión en la viga : $M_x = -V_C (x - \frac{L}{3})$





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3L_1^2}{2(4k+3)h} w$$

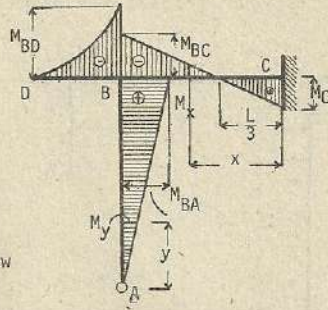
$$V_A = wL_1 + \frac{3L_1^2 k}{(4k+3)L} w$$

$$V_C = \frac{3L_1^2 k}{(4k+3)L} w ; \quad M_{BD} = -\frac{wL_1^2}{2} ; \quad M_{BA} = \frac{3L_1^2}{2(4k+3)} w$$

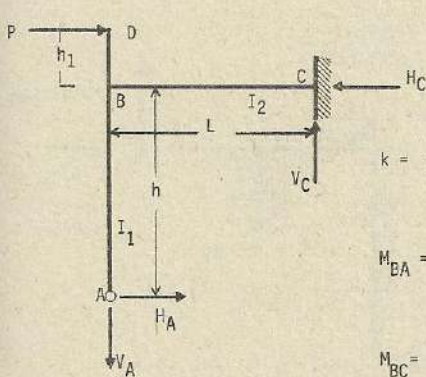
$$M_{BC} = -\frac{2L_1^2 k}{(4k+3)} w ; \quad M_C = \frac{L_1^2 k}{(4k+3)} w$$

Esfuerzo de flexión en la columna : $M_y = H_A y$

Esfuerzo de flexión en la viga : $M_x = -V_C \left(x - \frac{L}{3}\right)$



(88)

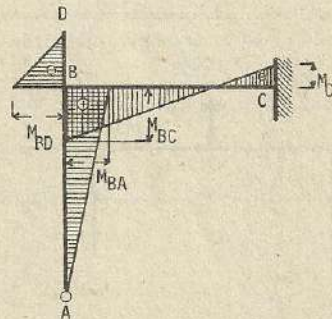


$$k = \frac{I_2 h}{I_1 L}$$

$$M_{BA} = -\frac{Ph_1}{4k+3}$$

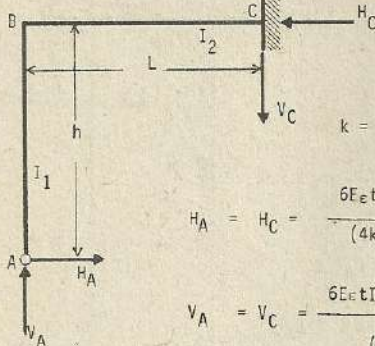
$$M_{BC} = \frac{Ph_1(2k+1)}{4k+3}$$

$$M_{BD} = -Ph_1 ; \quad H_C = -\frac{Ph_1(2k+1)}{4k+3}$$



Esfuerzos por cambio de temperatura

(89)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{6E\alpha t I_2 (2L^2 + 3h^2)}{(4k + 3) h^2 L^2}$$

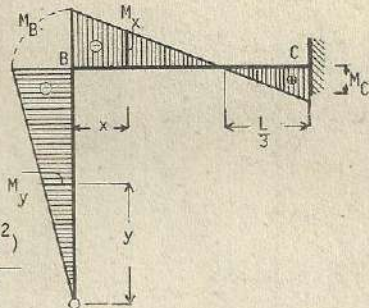
$$V_A = V_C = \frac{6E\alpha t I_2 [2(k+3)h^2 + 3L^2]^A}{(4k+3)hL^3}$$

$$M_B = -\frac{6E\alpha t I_2 (2L^2 + 3h^2)}{(4k + 3) h L^2}$$

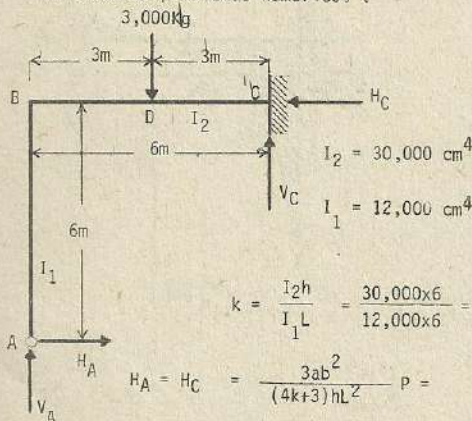
$$M_C = \frac{6E\alpha t I_2 [(2k+3)h^2 + L^2]}{(4k+3)hL^2}$$

Esfuerzo de flexión en la columna : $M_y = -H_A y$ Esfuerzo de flexión en la viga : $M_x = M_B + V_A x$

Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios



Solución de problemas numéricos :



$$I_2 = 30,000 \text{ cm}^4$$

$$I_1 = 12,000 \text{ cm}^4$$

$$k = \frac{I_2 h}{I_1 L} = \frac{30,000 \times 6}{12,000 \times 6} = 2.5$$

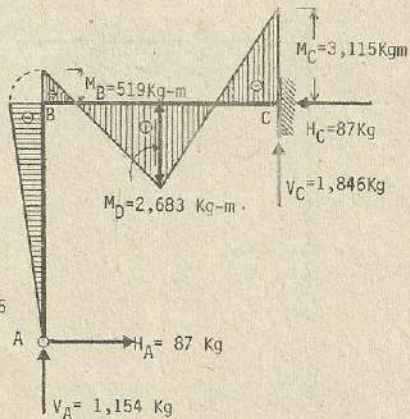
$$H_A = H_C = \frac{3ab^2}{(4k+3)hL^2} p =$$

$$= \frac{3 \times 3 \times 3 \times 3}{(4 \times 2.5 + 3) \times 6 \times 6 \times 6} \times 3,000$$

$$= 87 \text{ Kg.}$$

$$V_A = \frac{3(3a+L) + 2(a+2L)k}{(4k+3)L^3} p b^2 = \frac{3(3 \times 3 + 6) + 2(3 + 2 \times 6) \times 2.5}{(4 \times 2.5 + 3) \times 6 \times 6 \times 6} \times 3,000 \times 3 \times 3$$

$$= 1,154 \text{ Kg} ; V_C = P - V_A = 3,000 - 1,154 = 1,846 \text{ Kg.}$$



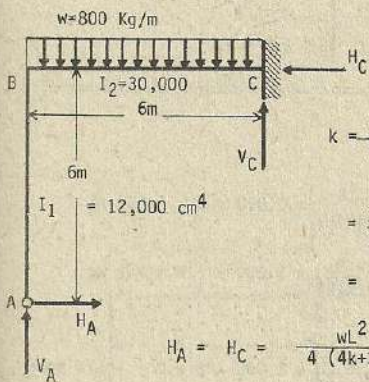
$$M_B = - \frac{3ab^2}{(4k+3)L^2} \cdot p = - \frac{3 \times 3 \times 3 \times 3}{(4 \times 2.5 + 3) \times 6 \times 6} \times 3,000 = -519 \text{ Kg-m}$$

$$M_C = - \frac{3 \cdot a + 2(a+L)k}{(4k+3)L^2} \times P_{ab} = - \frac{3 \times 3 + 2(3+6) \times 2.5}{(4 \times 2.5 + 3) \times 6 \times 6} \times 3,000 \times 3 \times 3$$

$$= - 3,115 \text{ Kg-m}$$

$$M_D = M_{\max.} = \frac{6a + 2(a+2L)k}{(4k+3)L^3} \cdot P_{ab}^2 = \frac{6 \times 3 + 2(3+2 \times 6) \cdot 2.5}{(4 \times 2.5 + 3) \times 6 \times 6 \times 6} \times 3,000 \times 3 \times 3 \times 3$$

$$\approx 2,683 \text{ Kg-m}$$



$$k = \frac{I_2 h}{I_1 L}$$

$$= \frac{30,000 \times 6}{12,000 \times 6}$$

$$= 2.5$$

$$H_A = H_C = \frac{wL^2}{4(4k+3)k}$$

$$= \frac{800 \times 6 \times 6}{4(4 \times 2.5 + 3) \times 6} \approx 92 \text{ Kg}$$

$$V_A = \frac{3(k+1)}{2(4k+3)} \cdot wL$$

$$= \frac{3(2.5+1)}{2(4 \times 2.5 + 3)} \times 800 \times 6 \approx 1,938 \text{ Kg}$$

$$V_C = \frac{5k+3}{2(4k+3)} \cdot wL = \frac{5 \times 2.5 + 3}{2(4 \times 2.5 + 3)} \times 800 \times 6 = 2,862 \text{ Kg}$$

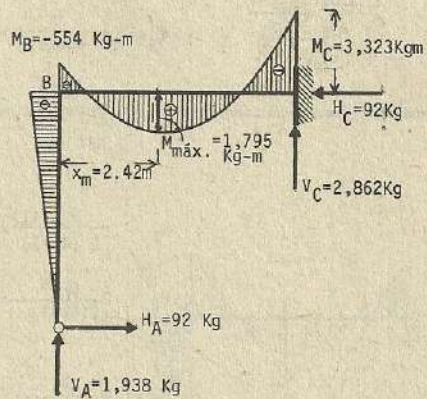
$$M_B = - \frac{wL^2}{4(4k+3)} = - \frac{800 \times 6 \times 6}{4(4 \times 2.5 + 3)} \approx - 554 \text{ Kg-m} ; M_C = - \frac{(2k+1)}{4(4k+3)} wL^2$$

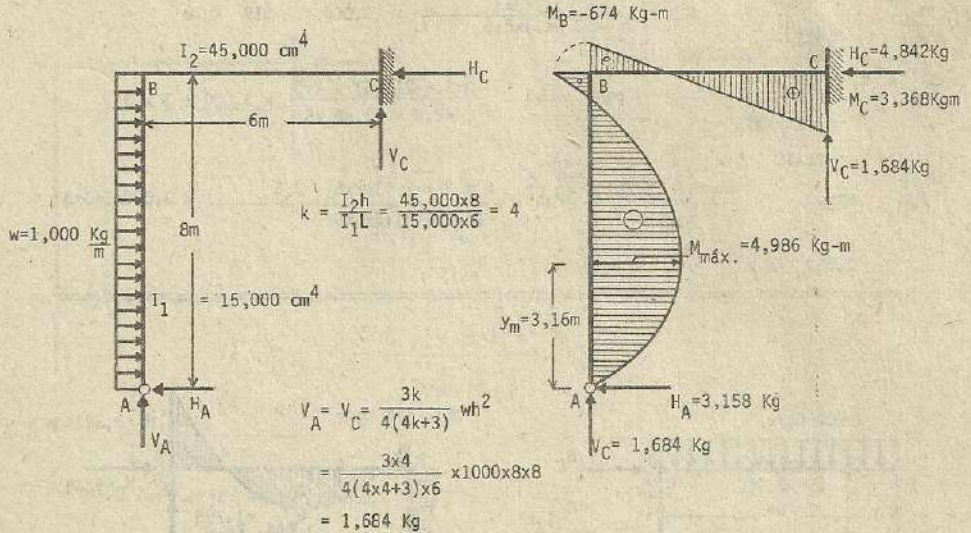
$$= - \frac{2 \times 2.5 + 1}{4(4 \times 2.5 + 3)} \times 800 \times 6 \times 6$$

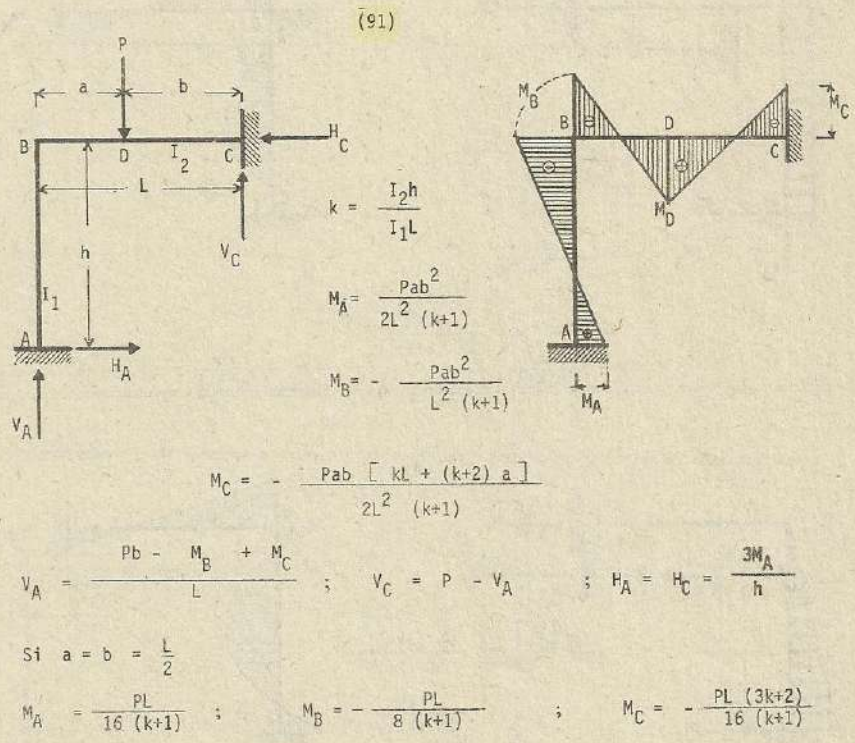
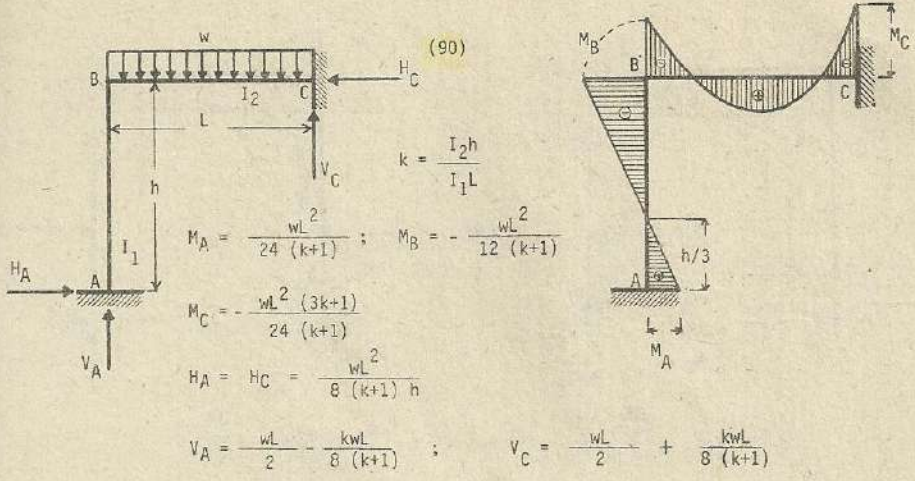
$$\approx - 3,323 \text{ Kg-m}$$

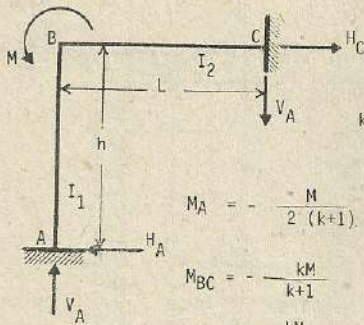
$$M_{\max.} = \frac{3+10k+9k^2}{8(4k+3)^2} \cdot wL^2 = \frac{3 + 10 \times 2.5 + 9 \times 2.5 \times 2.5}{8(4 \times 2.5 + 3)^2} \times 800 \times 6 \times 6 = 1,795 \text{ Kg-m}$$

$$x_m = \frac{3(k+1)L}{2(4k+3)} = \frac{3(2.5+1) \times 6}{2(4 \times 2.5 + 3)} = 2.42 \text{ m}$$









(92)

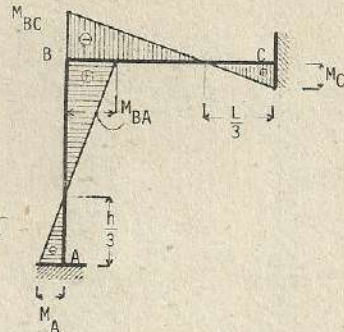
$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{M}{2(k+1)}; \quad M_{BA} = \frac{M}{k+1}$$

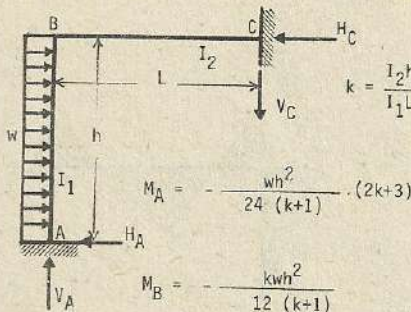
$$M_{BC} = -\frac{kM}{k+1}$$

$$M_C = \frac{kM}{2(k+1)}$$

$$V_A = V_C = \frac{3M_C}{L}; \quad H_A = H_C = -\frac{3M_A}{h}$$



(93)



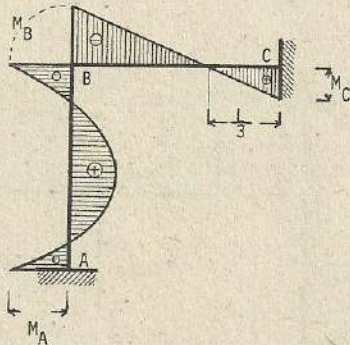
$$M_A = -\frac{wh^2}{24(k+1)} \cdot (2k+3)$$

$$M_B = -\frac{kwh^2}{12(k+1)}$$

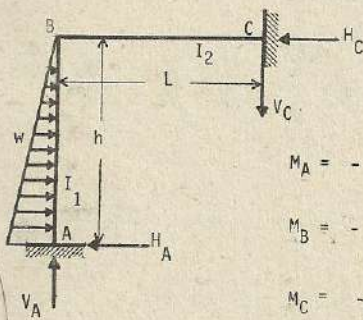
$$M_C = -\frac{M_B}{2} = +\frac{kwh^2}{24(k+1)}$$

$$V_A = V_C = \frac{kwh^2}{8L(k+1)}$$

$$H_A = \frac{wh}{2} + \frac{wh}{8(k+1)}; \quad H_C = \frac{wh}{2} - \frac{wh}{8(k+1)}$$



(94)



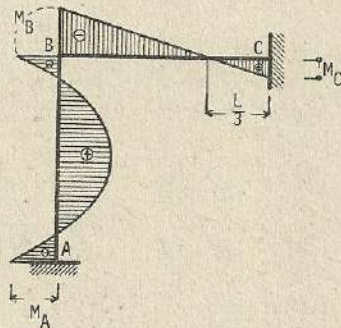
$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{Wh(3k+4)}{30(k+1)}$$

$$M_B = -\frac{kWh}{15(k+1)}$$

$$M_C = \frac{kWh}{30(k+1)}$$

$$H_A = \frac{1}{h} \cdot (W \times \frac{2}{3} h + M_B - M_A); \quad H_C = W - H_A; \quad V_A = V_C = \frac{3M_C}{L}$$



(95)

$k = \frac{I_2 h}{I_1 L}$
 $M_A = - \frac{P a b [h + (2k+1) b]}{2h^2 (k+1)}$
 $M_B = - \frac{k P a^2 b}{h^2 (k+1)}$
 $M_C = \frac{k P a^2 b}{2h^2 (k+1)}$; $H_A = \frac{P b + M_B - M_C}{h}$; $H_C = P - H_A$
 $V_A = V_C = \frac{3M_C}{L}$

Si $a = b = \frac{h}{2}$

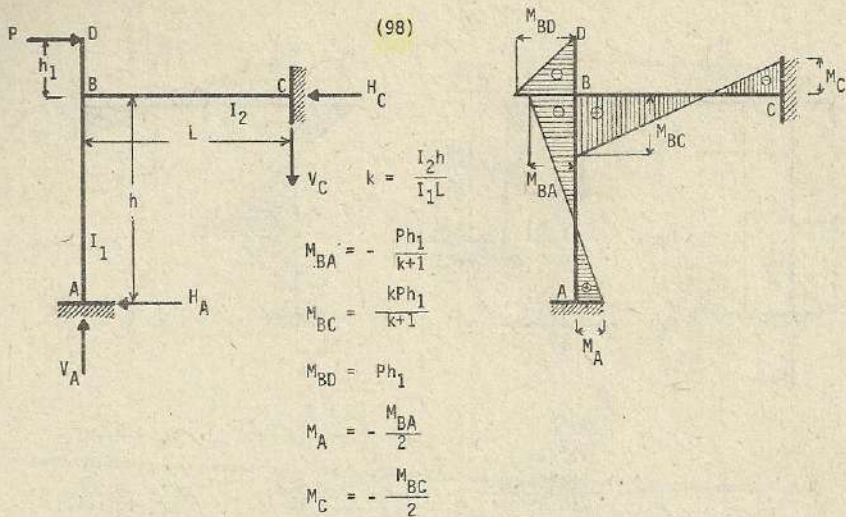
$M_A = - \frac{P h (2k+3)}{16 (k+1)}$; $M_B = - \frac{k P h}{8 (k+1)}$; $M_C = \frac{k P h}{16 (k+1)}$

(96)

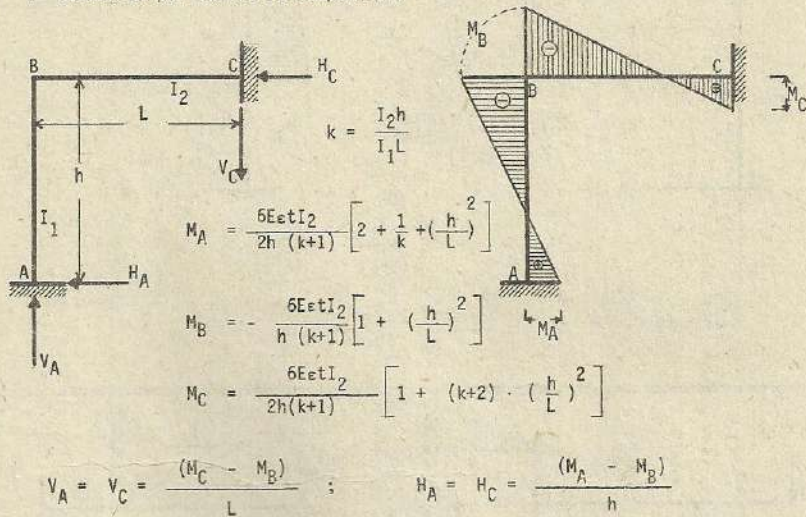
$k = \frac{I_2 h}{I_1 L}$
 $M_A = \frac{M (3b^2 - L^2) (2k+1)}{2L^2 (k+1)}$
 $M_B = - \frac{M (3b^2 - L^2) k}{L^2 (k+1)}$
 $M_C = \frac{M (3b^2 - L^2) (2k+1)}{6L^2 (k+1)}$

(97)

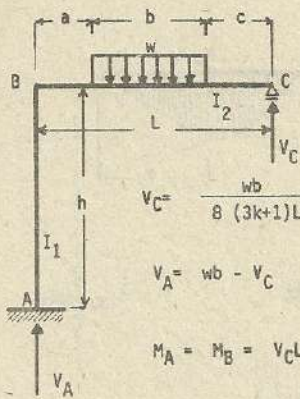
$k = \frac{I_2 h}{I_1 L}$
 $M_{BA} = \frac{P L_1}{k+1}$; $M_{BC} = - \frac{k P L_1}{k+1}$
 $M_{BD} = - P L_1$; $M_A = - \frac{M_{BA}}{2}$
 $M_C = - \frac{M_{BC}}{2}$



(99) : Esfuerzos por cambio de temperatura



Nota : El presente problema corresponde al caso de incremento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.



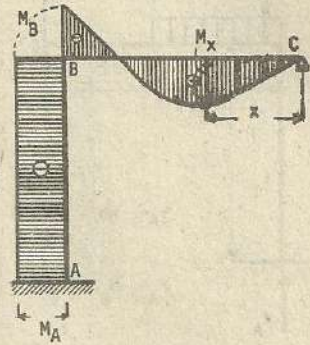
(100)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_C = \frac{wb}{8(3k+1)L^3} \left[\alpha + 12L^2 k(2a+b) \right]$$

$$V_A = wb - V_C$$

$$M_A = M_B = V_C L - \frac{wb}{2} (2a+b)$$



$$\alpha = 4b^2c + 3b^3 + 12abc + 12ab^2 + 12a^2c + 18a^2b + 8a^3$$

Esfuerzo de flexión en la viga : $M_x = V_C x$

$$x \leq c$$

$$M_x = V_C x - \frac{w(x-c)^2}{2}$$

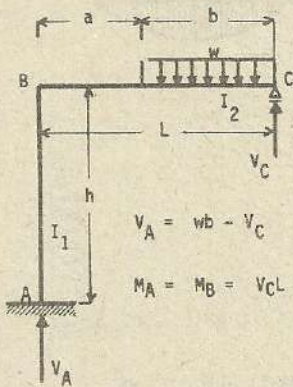
$$c \leq x \leq b+c$$

$$M_x = V_C x - \frac{wb}{2} (2x-b-2c)$$

$$x > b+c$$

Esfuerzo de flexión en la columna :

$$M_y = V_C L - \frac{wb}{2} (2a+b)$$

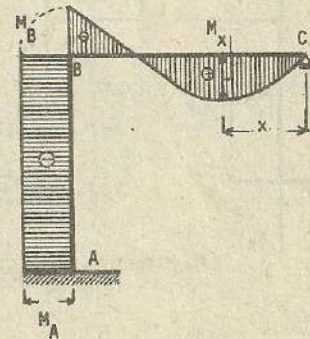


(101)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = wb - V_C$$

$$M_A = M_B = V_C L - \frac{wb}{2} (2a+b)$$



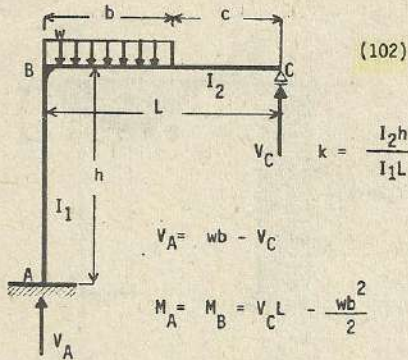
$$V_C = \frac{wb}{8(3k+1)L^2} \left[3b^3 + 12ab^2 + 18a^2b + 8a^3 + 12kL^2(2a+b) \right]$$

Esfuerzo de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2} \quad x \leq b$$

$$M_x = V_C x - \frac{wb}{2} (2x-b) \quad x \geq b$$

Esfuerzo de flexión en la columna : $M_y = V_C L - \frac{wb}{2} (2a+b)$

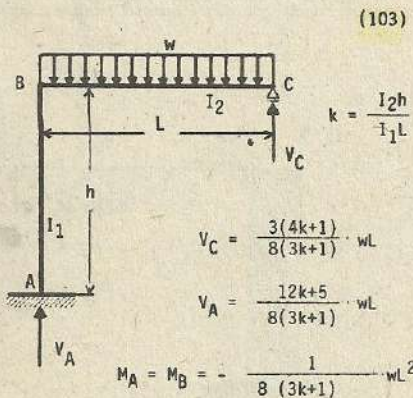
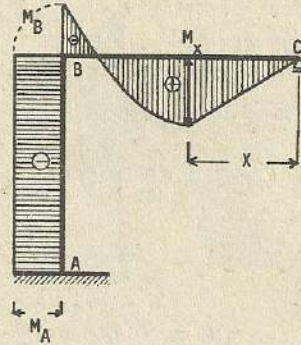


$$V_C = \frac{wb^2}{8(3k+1)L^2} [4bc + 3b^2 + 12kL^2]$$

Esfuerzo de flexión en la viga : $M_x = V_C x$ $x \leq c$

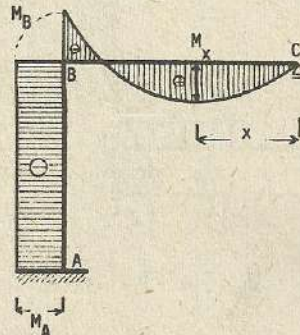
$$M_x = V_C x - \frac{w(x-c)^2}{2} \quad x > c$$

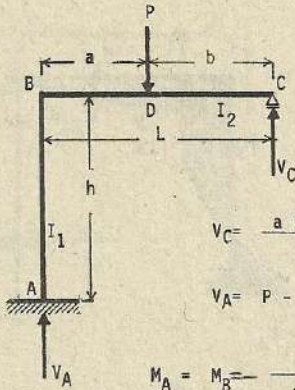
Esfuerzo de flexión en la columna : $M_y = V_C L - \frac{wb^2}{2}$



Esfuerzo de flexión en la viga : $M_x = - \frac{3(4k+1)}{8(3k+1)} \cdot wLx - \frac{wx^2}{2}$

Esfuerzo de flexión en la columna : $M_y = - \frac{1}{8(3k+1)} \cdot wL^2$





(104)

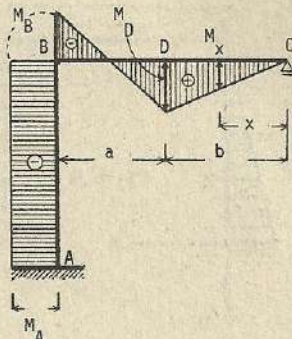
$$k = \frac{I_2 h}{I_1 L}$$

$$V_C = \frac{a(3L-a) + 6kL^2}{2(3k+1)L^3} \cdot Pa$$

$$V_A = P - V_C$$

$$M_A = M_B = -\frac{L+b}{2(3k+1)L^2} \cdot Pab$$

$$M_D = \frac{a(3L-a) + 6kL^2}{2(3k+1)L^3} \cdot Pab$$



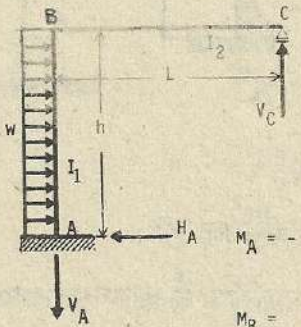
Esfuerzo de flexión en la viga : $M_x = V_C x$ $x \leq b$

$M_x = V_C x - P(x-b)$ $x > b$

Esfuerzo de flexión en la columna :

$$= -\frac{L+b}{2(3k+1)L^2} \cdot Pab$$

(105)



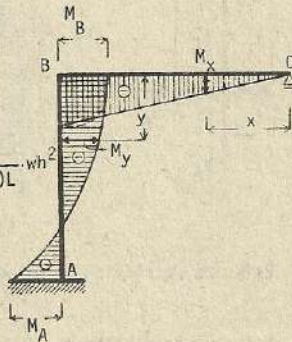
$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{k}{2(3k+1)L} \cdot wh^2$$

$$H_A = wh$$

$$M_A = -\frac{2k+1}{2(3k+1)} \cdot wh^2$$

$$M_B = \frac{k}{2(3k+1)} \cdot wh^2$$

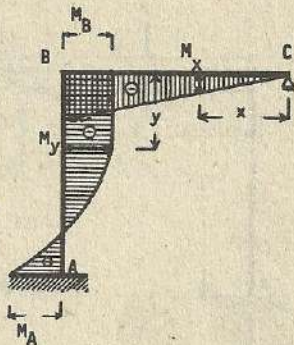
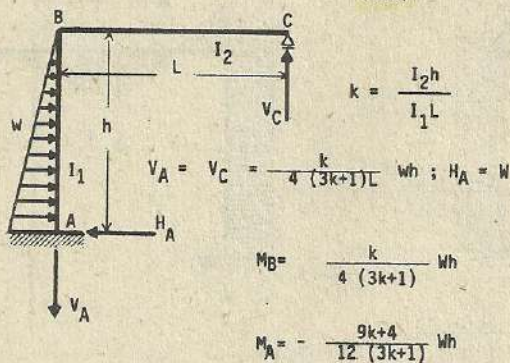


Esfuerzo de flexión en la viga : $\frac{k}{2(3k+1)L} \cdot wh^2 x$

Esfuerzo de flexión en la columna :

$$-\frac{2k+1}{2(3k+1)} \cdot wh^2 - \frac{wy^2}{2}$$

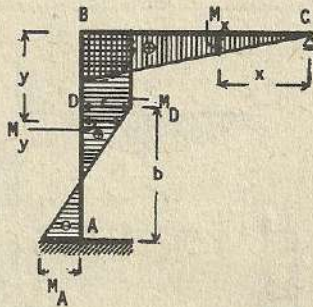
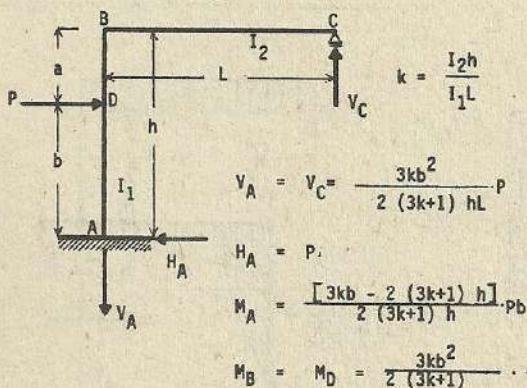
(106)



Esfuerzo de flexión en la viga : $M_x = -\frac{k}{4(3k+1)L} Whx$

Esfuerzo de flexión en la columna : $M_y = \frac{k}{4(3k+1)} wh - \frac{Wy^3}{3h^2}$

(107)



Esfuerzo de flexión en la viga : $M_x = \frac{3kb^2}{2(3k+1)hL} \cdot Px$

Esfuerzo de flexión en la columna : $M_y = \frac{3kb^2}{2(3k+1)} \cdot \frac{P}{h} ; (\text{Constante entre B-D})$

$M_y = \frac{3kb^2}{2(3k+1)} \cdot \frac{P}{h} - P(y-a) ; y \geq a$



(108)

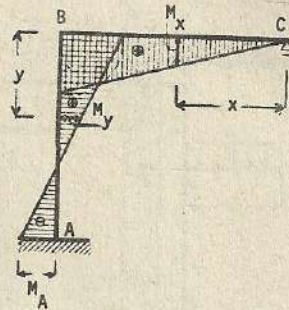
$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3k}{2(3k+1)} \cdot Ph$$

$$H_A = P$$

$$M_A = -\frac{3k+2}{2(3k+1)} \cdot Ph$$

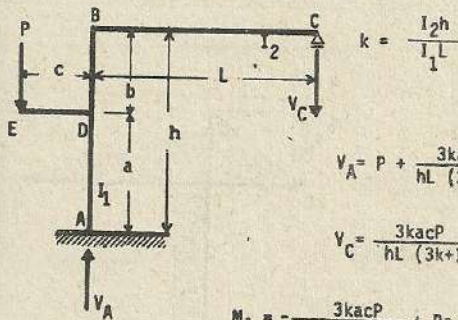
$$M_B = \frac{3k}{2(3k+1)} \cdot Ph$$



Esfuerzo de flexión en la viga : $M_x = \frac{3k}{2(3k+1)L} \cdot Phx$

Esfuerzo de flexión en la columna : $M_y = \frac{3k}{2(3k+1)} \cdot Ph - Py$

(109)



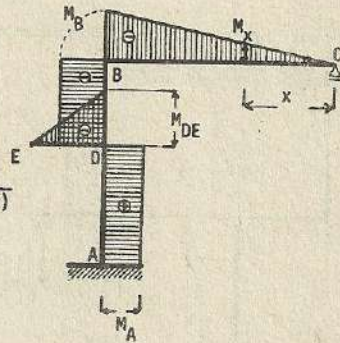
$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = P + \frac{3kacP}{hL(3k+1)}$$

$$V_C = \frac{3kacP}{hL(3k+1)}$$

$$M_A = -\frac{3kacP}{h(3k+1)} + Pc$$

$$M_B = -\frac{3kacP}{h(3k+1)}$$



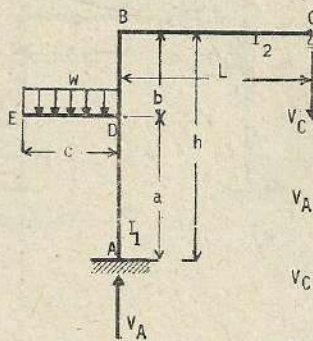
Esfuerzo de flexión en la viga: $M_x = -V_C x$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{3kacP}{h(3k+1)} + Pc \quad (A - D)$$

$$M_y = -\frac{3kacP}{h(3k+1)} \quad (D - B)$$

(110)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = wc + \frac{3ka}{2(3k+1)hL} \cdot wc^2$$

$$V_C = \frac{3ka}{2(3k+1)hL} \cdot wc^2$$

$$M_A = \frac{h + 3k(h-a)}{2(3k+1)h} \cdot wc^2$$

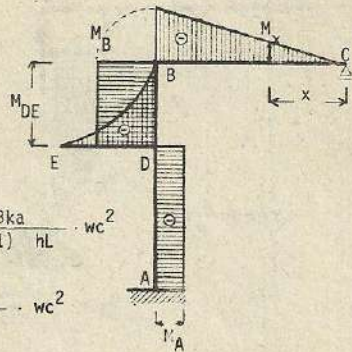
$$M_{DE} = -\frac{wc^2}{2}; \quad M_{DB} = -\frac{3ka}{2(3k+1)h} \cdot wc^2; \quad M_{DA} = \frac{h+3k(h-a)}{2(3k+1)h} \cdot wc^2$$

$$M_B = -\frac{3ka}{2(3k+1)h} \cdot wc^2$$

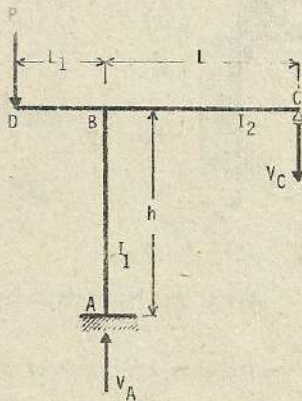
Esfuerzo de flexión en la viga : $M_x = -\frac{3ka}{2(3k+1)hL} \cdot wc^2 x$

Esfuerzo de flexión en la columna: $M_y = \frac{h + 3k(h-a)}{2(3k+1)h} \cdot wc^2$ porción (A-D)

$$M_y = -\frac{3ka}{2(3k+1)h} \cdot wc^2 \text{ porción (D-B)}$$



(111)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = P + \frac{3L_1 k P}{(3k+1)L}$$

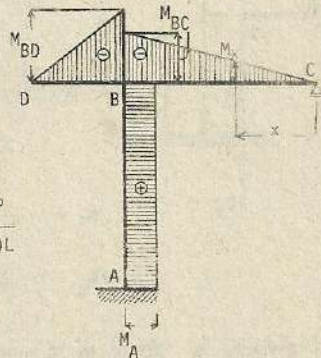
$$V_C = \frac{3L_1 k P}{(3k+1)L}$$

$$M_A = \frac{PL_1}{(3k+1)}$$

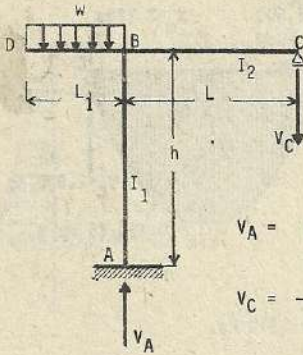
$$M_{BD} = -PL_1; \quad M_{BA} = \frac{PL_1}{(3k+1)}; \quad M_{BC} = -\frac{3PL_1 k}{(3k+1)}$$

Esfuerzo de flexión en la viga : $M_x = -\dots x$

Esfuerzo de flexión en la columna: $M_y = \dots$



(112)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = wL_1 + \frac{3k}{2(3k+1)} \frac{wL_1^2}{L}$$

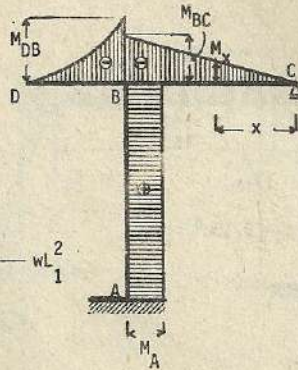
$$V_C = \frac{3k}{2(3k+1)} \frac{wL_1^2}{L}$$

$$M_A = M_{BA} = \frac{wL_1^2}{2(3k+1)}$$

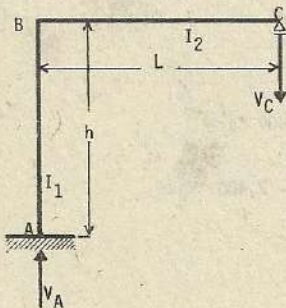
$$M_{BD} = -\frac{wL_1^2}{2} ; \quad M_{BC} = -\frac{3k}{2(3k+1)} \frac{wL_1^2}{L}$$

Esfuerzo de flexión en la viga : $M_x = -\frac{3k}{2(3k+1)} \frac{wL_1^2}{L} x$

Esfuerzo de flexión en la columna : $M_y = \frac{wL_1^2}{2(3k+1)}$



Esfuerzo por cambio de temperatura (113)



$$k = \frac{I_2 h}{I_1 L}$$

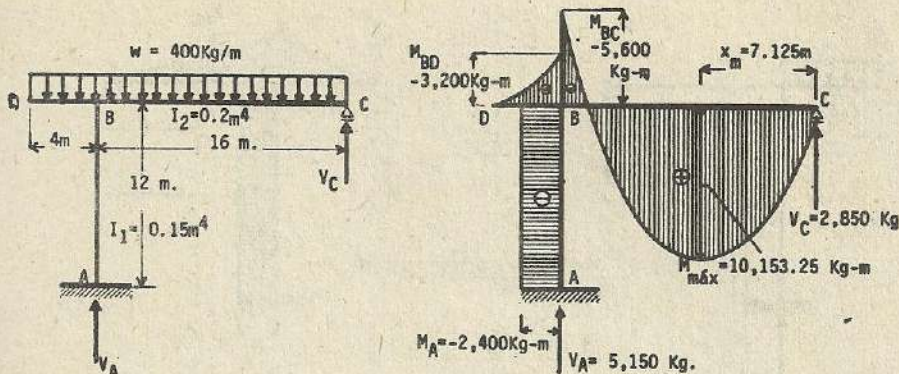
$$V_A = V_C = \frac{3E\alpha t I_1 k}{(3k+1) L^2}$$

$$M_A = M_B = -\frac{3E\alpha t I_1 k}{(3k+1) L}$$

Esfuerzo de flexión en la viga : $M_x = -\frac{3E\alpha t I_1 k}{(3k+1) L^2} \cdot x$

Esfuerzo de flexión en la columna : $M_y = -\frac{3E\alpha t I_1 k}{(3k+1) L}$ Constante

Nota : El presente pórtico trata del problema de aumento de temperatura, en caso contrario, los esfuerzos se anotan con los signos contrarios.



$$k = \frac{I_2 h}{I_1 L} = \frac{0.2 \times 12}{0.15 \times 16} = 1$$

Reacción debido a la carga en la viga BC :

$$V_C = \frac{3(4k+1)}{8(3k+1)} \cdot wL = \frac{3 \times 5}{8 \times 4} \times 400 \times 16 = 3,000 \text{ Kg.}$$

Debido al volado :

$$V_C'' = \frac{3k}{2(3k+1)L} \cdot wL^2 = \frac{3 \times 400 \times 4 \times 4}{2(3 \times 1 + 1) \times 16} = 150 \text{ Kg.} \quad \therefore V_C = 3,000 - 150 = 2,850 \text{ Kg.}$$

Nota: La reacción hacia arriba se considera como signo positivo

$$V_A = w(L + L_1) - V_C = 400(16 + 4) - 2,850 = 5,150 \text{ Kg.}$$

$$M_{BD} = -\frac{wL_1^2}{2} = -400 \times \frac{4 \times 4}{2} = -3,200 \text{ Kg-m}$$

$$M_{BC} = V_C L - w \frac{L^2}{2} = 2,850 \times 16 - \frac{400 \times 16 \times 16}{2} = -5,600 \text{ Kg-m}$$

$$M_{BA} = V_C L - w \frac{L^2}{2} + w \frac{L_1^2}{2} = -5,600 + 3,200 = -2,400 \text{ Kg-m}$$

$$M_A = M_{BA} = -2,400 \text{ Kg-m}$$

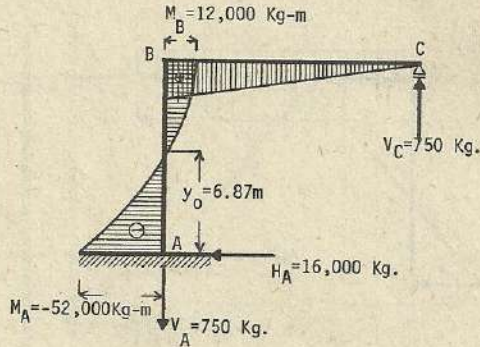
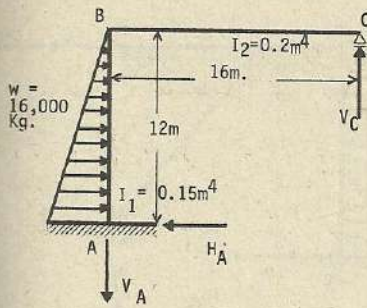
Para encontrar el punto de máximo esfuerzo de momento

$$V = V_C - wx = 0$$

$$2,850 - 400x = 0 \quad \therefore x = \frac{2,850}{400} = 7.125 \text{ m}$$

$$M_{\text{máx}} = V_C x - w \frac{x^2}{2} = 2,850 \times 7.125 - 400 \times \frac{7.125^2}{2} =$$

$$= 10,153.125 \text{ Kg -m.}$$



$$k = \frac{I_2 h}{I_1 L} = \frac{0.2 \times 12}{0.15 \times 16} = 1$$

$$V_A = V_C = \frac{k}{4(3k+1)} wh = \frac{1}{4 \times (3+1)} \times 16,000 \times 12 = 750 \text{ Kg.}$$

$$H_A = W = 16,000 \text{ Kg.}$$

$$M_B = \frac{k}{4(3k+1)} \cdot wh = \frac{1}{4 \times (3+1)} \times 16,000 \times 12 = 12,000 \text{ Kg-m}$$

$$M_A = - \frac{9k+4}{12(3k+1)} \cdot wh = - \frac{9+4}{12(3+1)} \times 16,000 \times 12 = -52,000 \text{ Kg-m}$$

Considerando como (y_0) la distancia que media entre el punto (A) y el punto donde el esfuerzo de momento es cero, tenemos :

$$M_y = V_C L - \frac{W y_0^3}{3h^2} = 0$$

$$750 \times 16 - \frac{16,000 y_0^3}{3 \times 12 \times 12} = 0$$

$$y_0^3 = \frac{750 \times 16 \times 3 \times 12 \times 12}{16,000} = 324$$

$$\therefore y_0 = \sqrt[3]{324} = 6.87 \text{ m.}$$

(114)

$$k = \frac{I_2 h}{I_1 L}$$

$$M_B = -\frac{3wL^2}{2(24k+21)}$$

$$M_D = -\frac{3(4k+3)}{4(24k+21)} \cdot wL^2$$

$$H_A = H_C = \frac{M_B}{h}$$

(115)

$$k = \frac{I_2 h}{I_1 L}$$

$$M_B = -\frac{3k}{24k+21} wh^2$$

$$M_D = -\frac{M_B}{4}$$

$$H_A = \frac{wh}{2} + \frac{M_B}{h} ; H_C = \frac{wh}{2} - \frac{M_B}{h}$$

(116)

$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = \frac{1}{4(6k+7)} \cdot wL^2$$

$$M_B = -\frac{1}{2(6k+7)} \cdot wL^2$$

$$M_D = -\frac{3(k+1)}{4(6k+7)} \cdot wL^2$$

$$H_A = H_C = -\frac{3M_B}{2h}$$

(117)

$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{2k+7}{8(6k+7)} \cdot wh^2$$

$$M_B = -\frac{k}{2(6k+7)} \cdot wh^2$$

$$M_D = -\frac{M_B}{4}$$

(118)

$$W = \frac{wL}{2} ; M_B = -\frac{wL}{64}$$

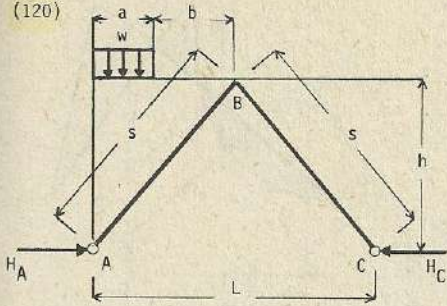
$$H_A = H_C = \frac{5wL^2}{64h} = 0.07813 w \cdot \frac{L^2}{h}$$

(119)

$$M_B = -\frac{wL}{32}$$

$$H_A = H_C = \frac{5wL^2}{32h} = 0.15625 w \cdot \frac{L^2}{h}$$

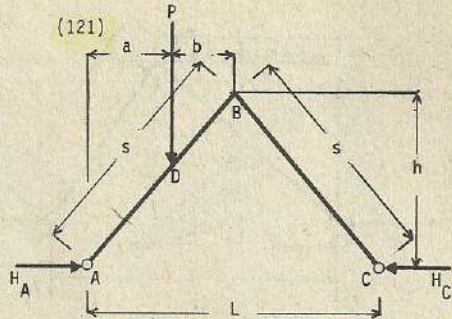
(120)



$$H_A = H_C = \frac{wL}{8} \cdot \frac{L}{h} \alpha^2 (3 - 2\alpha^2)$$

$$\alpha = \frac{a}{h}$$

(121)



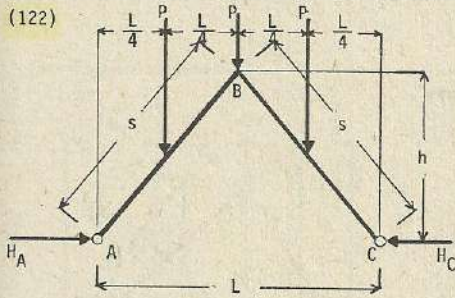
$$\alpha = \frac{a}{L}; \quad M_B = - \frac{Pab(L+2b)}{2L^2}$$

$$H_A = H_C = \frac{P}{4} \cdot \frac{L}{h} \alpha (3 - 4\alpha^2)$$

$$\text{Si } a = b = \frac{L}{2} \rightarrow H_A = H_C = \frac{PL}{4h}$$

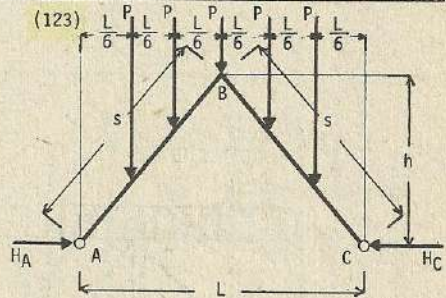
$$\text{Si } a = b = \frac{L}{4} \rightarrow M_B = - \frac{3PL}{64}$$

(122)



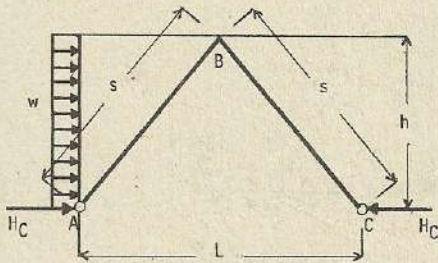
$$M_B = - \frac{3PL}{32}$$

(123)



$$M_B = - \frac{PL}{6}$$

(124)

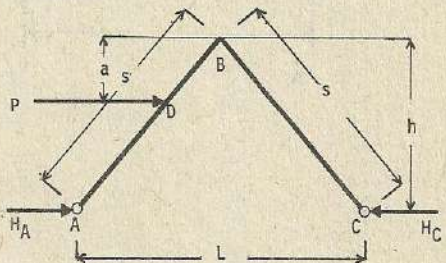


$$H_A = - \frac{11}{16} wh = - 0.6875 wh$$

$$H_C = \frac{5}{16} wh = - 0.3125 wh$$

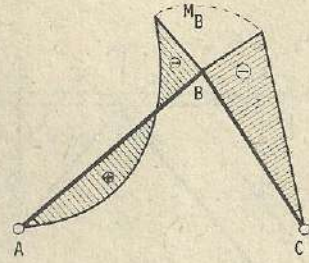
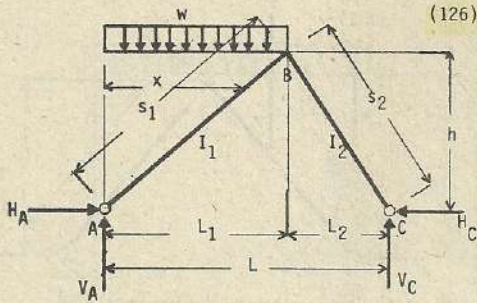
$$M_B = - \frac{wl^2}{16}$$

(125)



$$H_A = \frac{P}{4} \left[2 + 3 \cdot \left(\frac{a}{h}\right)^2 - \left(\frac{a}{h}\right)^3 \right]$$

$$H_C = \frac{P}{4} \left[-2 + 3 \cdot \left(\frac{a}{h}\right)^2 - \left(\frac{a}{h}\right)^3 \right]$$



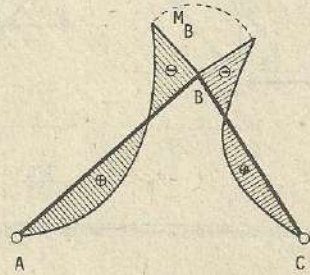
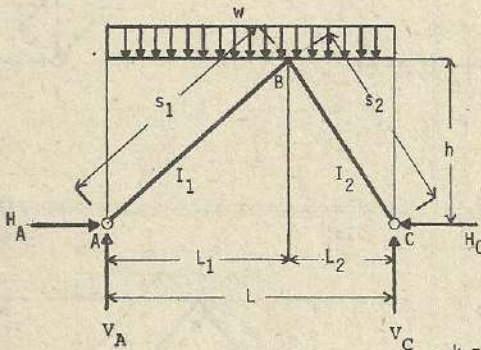
$$k = \frac{I_2 s_1}{I_1 s_2} ; \quad W = \frac{wL_1}{2}$$

$$M_B = - \frac{kNL_1}{8(k+1)} ; \quad V_A = \frac{wL_1}{2L} \cdot (L + L_2) ; \quad V_C = \frac{wL_1}{2L}$$

$$H_A = H_C = \frac{wL_1^2}{8hL} \cdot \frac{[4L_2 + k(4L_2 + L)]}{(k+1)} ; \quad M_x = V_A x - H_A \frac{hx}{L_1} - \frac{wx^2}{2}$$

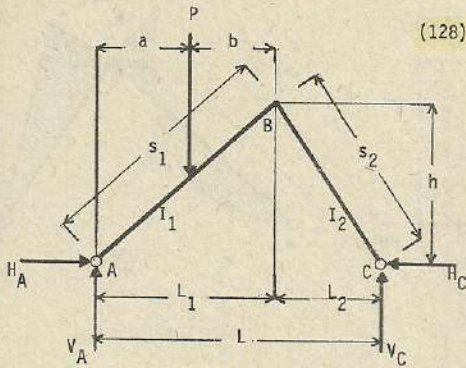
$$(M_B = V_C L_2 - H_C h)$$

(127)

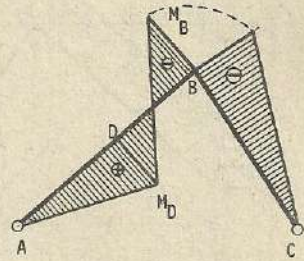


$$k = \frac{I_2 s_1}{I_1 s_2}$$

$$M_B = - \frac{w (kL_1^2 + L_2^2)}{8(k+1)}$$



(128)



$$k = \frac{I_2 s_1}{I_1 s_2} ; M_B = - \frac{k P a b (L_1 + a)}{2 L_1^2 (k+1)}$$

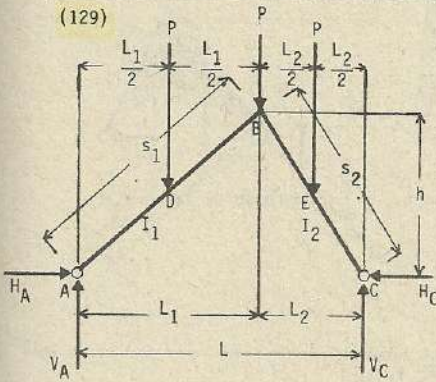
Si $a = b = \frac{L}{2} \rightarrow M_B = - \frac{3 k P L_1}{16 (k+1)}$

Considerando $\alpha = \frac{a}{L_1}$

$$V_A = \frac{P(L-a)}{L} ; V_C = \frac{Pa}{L}$$

$$H_A = H_C = \frac{Pa}{2hL} \left[\frac{2L_2 + (3L - L\alpha^2 - 2L_1 k)}{k+1} \right]$$

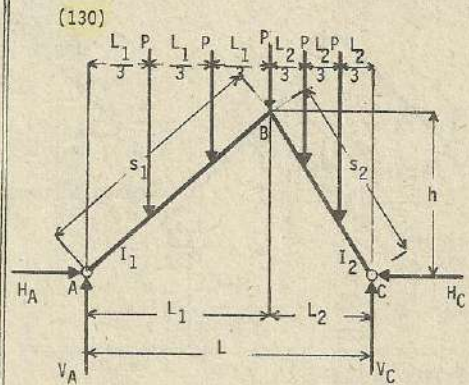
$$M_D = V_A a - H_A \frac{ha}{L_1} \quad M_B = V_C L_2 - H_C h$$



(129)

$$k = \frac{I_2 s_1}{I_1 s_2}$$

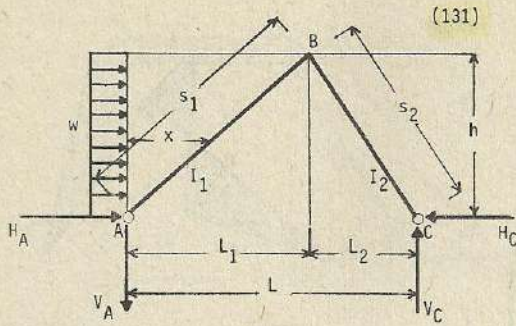
$$M_B = - \frac{3P(kL_1 + L_2)}{16(k+1)}$$



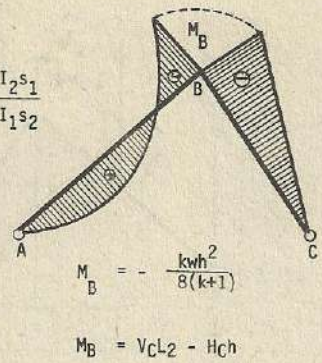
(130)

$$k = \frac{I_2 s_1}{I_1 s_2}$$

$$M_B = - \frac{P(kL_1 + L_2)}{3(k+1)}$$



$$k = \frac{I_2 s_1}{I_1 s_2}$$



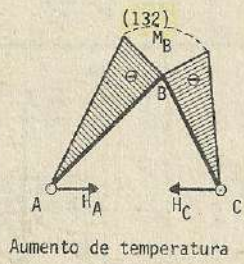
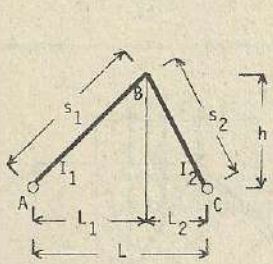
$$M_B = - \frac{kwh^2}{8(k+1)}$$

$$M_B = V_C L_2 - H_C h$$

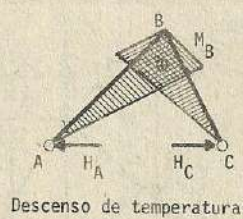
$$-V_A = V_C = \frac{wh^2}{2L} ; H_C = \frac{wh}{8L} \cdot \left[\frac{4L_2 + k(L + 4L_2)}{k+1} \right]$$

$$H_A = wh - H_C$$

$$H_x = H_A \frac{hx}{L_1} - V_A x - \frac{wh^2 x^2}{2L_1^2}$$



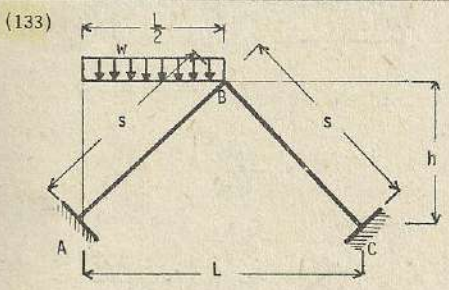
Aumento de temperatura



Descenso de temperatura

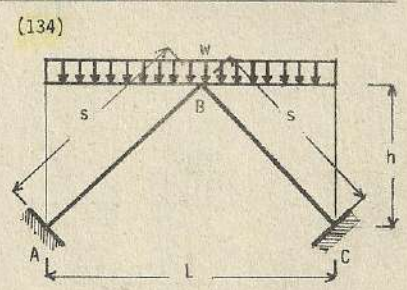
$$k = \frac{I_2 s_1}{I_1 s_2}$$

$$H_A = H_C = H = \frac{2E\epsilon t I_2 L}{h^2 s_2 (k+1)} ; M_B = Hh$$



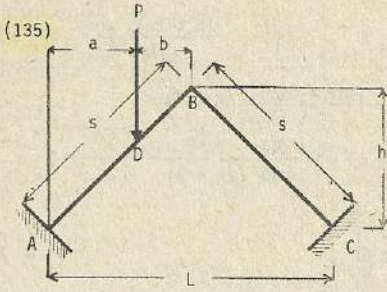
$$W = w \cdot \frac{L}{2} ; M_A = - \frac{5}{96} WL$$

$$M_B = - \frac{1}{48} WL ; M_C = - \frac{1}{96} WL$$



$$W = w \cdot \frac{L}{2} ; M_A = - \frac{1}{24} WL$$

$$M_B = - \frac{1}{24} WL ; M_C = - \frac{1}{24} WL$$



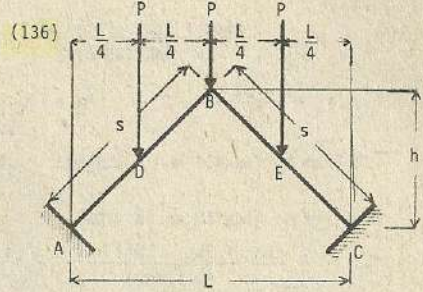
$$M_A = -\frac{Pab(L+6b)}{2L^2}$$

$$M_B = -\frac{2Pa^2b}{L^2} \quad ; \quad M_C = \frac{Pa^2b}{L^2}$$

Si $a = b = \frac{L}{4}$

$$M_A = -\frac{5}{64} PL \quad ; \quad M_B = -\frac{1}{32} PL$$

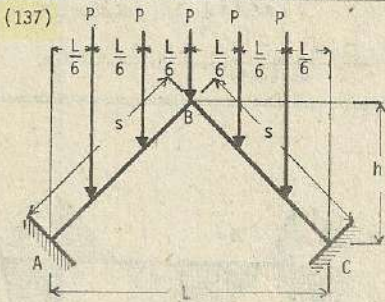
$$M_C = \frac{1}{64} PL$$



$$M_A = -\frac{1}{16} P$$

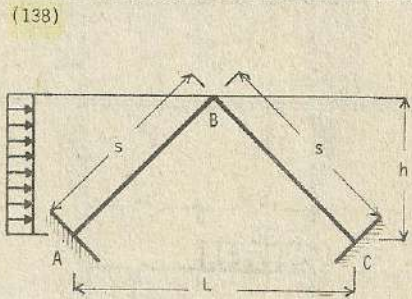
$$M_B = -\frac{1}{16} P$$

$$M_C = -\frac{1}{16} P$$



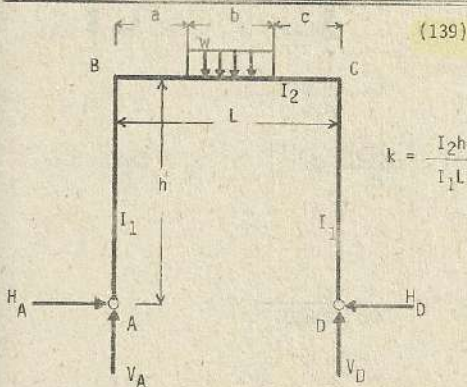
$$M_A = -\frac{1}{9} PL \quad ; \quad M_B = -\frac{1}{9} PL$$

$$M_C = -\frac{1}{9} PL$$

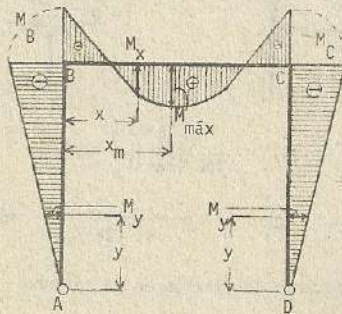


$$M_A = -\frac{5}{96} wh^2 \quad ; \quad M_B = -\frac{1}{48} wh^2$$

$$M_C = \frac{1}{96} wh^2$$



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{(6ac + 3bL - 2b^2) \cdot wb}{4hL(2k+3)}$$

$$V_A = \frac{(b+2c) \cdot wb}{2L}$$

$$V_D = \frac{(2a+b) \cdot wb}{2L} ; \quad M_B = M_C = - \frac{(6ac + 3bL - 2b^2) \cdot wb}{4L(2k+3)}$$

$$\text{Esfuerzo de flexión en la columna : } M_y = - \frac{(6ac + 3bL - 2b^2) \cdot wb}{4hL(2k+3)} y$$

Esfuerzo de flexión en la viga :

$$M_x = - \frac{(6ac + 3bL - 2b^2) \cdot wb}{4L(2k+3)} + \frac{(b+2c) \cdot wb}{2L} x \quad x \leq a$$

$$M_x = - \frac{(6ac + 3bL - 2b^2) \cdot wb}{4L(2k+3)} + \frac{(b+2c) \cdot wb}{2L} x - \frac{w(x-a)^2}{2} \quad a \leq x \leq a+b$$

$$M_x = - \frac{(6ac + 3bL - 2b^2) \cdot wb}{4L(2k+3)} + \frac{(b+2c) \cdot wb}{2L} x - \frac{wb(2x-2a-b)}{2} \quad x \geq a+b$$

$$x_m = a + \frac{b(b+2c)}{2L}$$

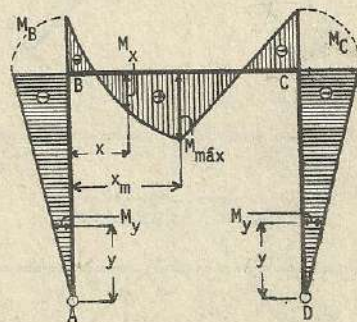
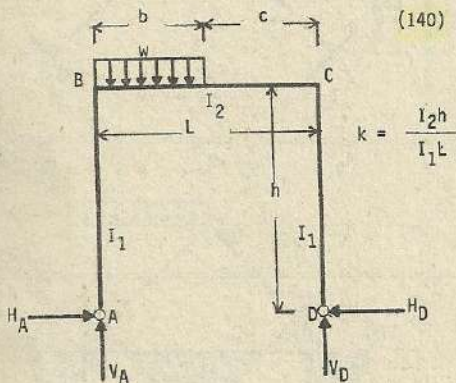
$$M_{\text{máx.}} = \frac{[(2k+3)(b+2c)(4aL+b^2+2bc) - (12acL+6bL^2-4b^2L)] \cdot wb}{8L^2(2k+3)}$$

Si $a = c$:

$$H_A = H_D = \frac{(3L^2 - b^2)}{8hL(2k+3)} \cdot wb ; \quad V_A = V_D = \frac{wb}{2}$$

$$M_B = M_C = \frac{(3L^2 - b^2)}{8L(2k+3)} \cdot wb$$

$$x_m = \frac{L}{2} ; \quad M_{\text{máx}} = \frac{(2k+3)(4a+b)L - [6aL + 2b(L+a)]}{8L(2k+3)} \cdot wb$$



$$H_A = H_D = \frac{(L+2c)}{4hL(2k+3)} \cdot wb^2 ; \quad V_A = \frac{(L+c)}{2L} \cdot wb ; \quad V_D = \frac{wb^2}{2L}$$

$$M_B = M_C = - \frac{(L+2c)}{4L(2k+3)} \cdot wb^2$$

$$\text{Esfuerzo de flexión en la columna : } M_y = - \frac{(L+2c)}{4hL(2k+3)} \cdot wb^2 y$$

Esfuerzo de flexión en la viga :

$$M_x = -\frac{wb^2(L+2c)}{4L(2k+3)} + \frac{wb(L+c)}{2L}x - \frac{wx^2}{2} \quad x \leq b$$

$$M_x = -\frac{wb^2(L+2c)}{4L(2k+3)} + \frac{wb(L+c)}{2L}x - \frac{wb}{2}(2x-b) \quad x > b$$

$$x_m = \frac{b(L+c)}{2L} \quad M_{m\acute{a}x.} = \frac{wb^2[(L+c)^2(2k+1) + 2c^2]}{8L^2(2k+3)}$$

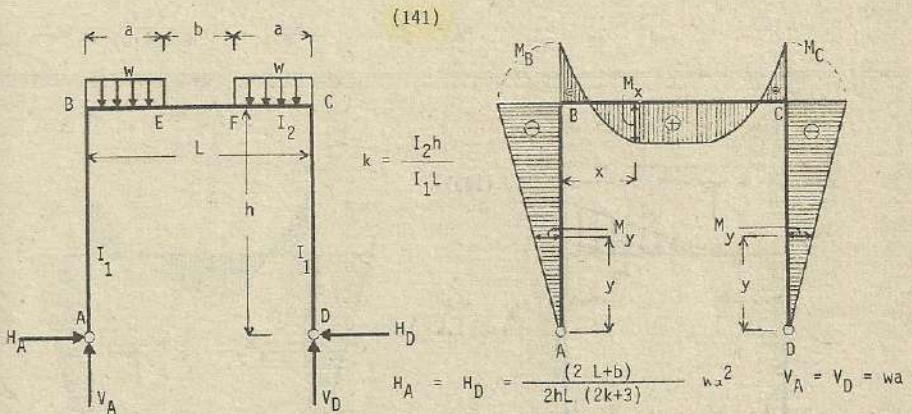
$$\text{Si } b = c = \frac{L}{2}$$

$$H_A = H_D = \frac{wL^2}{8h(2k+3)} \quad ; \quad V_A = \frac{3wL}{8} \quad ; \quad V_D = \frac{wL}{8}$$

$$M_B = M_C = -\frac{wL^2}{8(2k+3)}$$

$$x_m = \frac{3L}{8}$$

$$M_{m\acute{a}x.} = \frac{(18k+11)}{128(2k+3)} wL^2$$



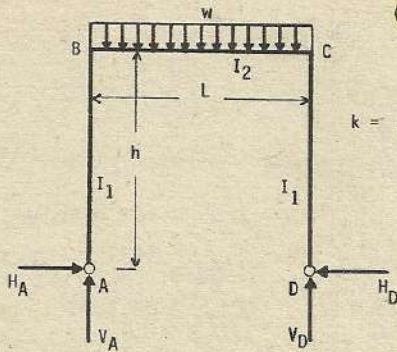
$$M_B = M_C = -\frac{(2L+b)}{2L(2k+3)} wa^2$$

Esfuerzos de flexión en la columna : $M_y = -\frac{(2L+b)}{2hL(2k+3)} wa^2 y$

Esfuerzo de flexión en la viga : $M_x = -\frac{(2L+b)}{2L(2k+3)} wa^2 + wax - \frac{wx^2}{2} \quad x \leq a$

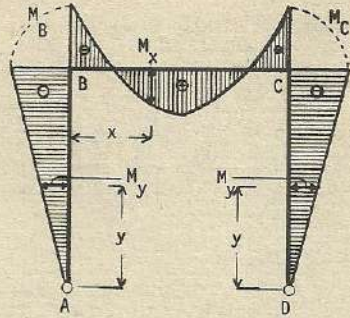
$$M_E = M_F = \frac{(a+Lk)}{L(2k+3)} wa^2$$

Entre los puntos (E) y (F) el momento de doblado o flexión es constante.



(142)

$$k = \frac{I_2 h}{I_1 L}$$



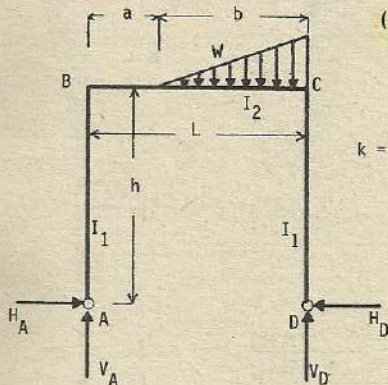
$$H_A = H_D = \frac{wL^2}{4h(2k+3)} \quad V_A = V_D = \frac{wL}{2}$$

$$M_B = M_C = -\frac{wL^2}{4(2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = \frac{wL^2 y}{4h(2k+3)}$

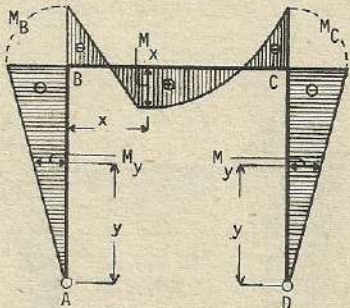
Esfuerzo de flexión en la viga : $M_x = -\frac{wL^2}{4(2k+3)} + \frac{wL}{2}x - \frac{wx^2}{2}$

$$x_m = \frac{L}{2} \quad M_{\text{máx.}} = \frac{2k+1}{8(2k+3)} \cdot wL^2$$



(143)

$$k = \frac{I_2 h}{I_1 L}$$



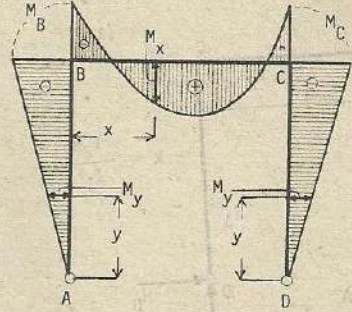
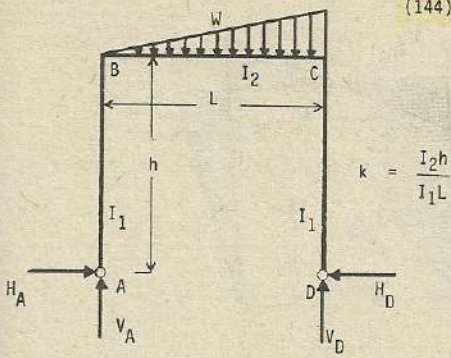
$$H_A = H_D = \frac{Wb(L+a)}{4hL(2k+3)}; \quad V_A = \frac{Wb}{3L}$$

$$V_D = \frac{W(2L+a)}{3L}; \quad M_B = M_C = -\frac{Wb(L+a)}{4L(2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = -\frac{Wb(L+a)}{4hL(2k+3)} \cdot y$

Esfuerzo de flexión en la viga : $M_x = -\frac{Wb(L+a)}{4L(2k+3)} + \frac{Wb}{3L}x \quad x \leq a$

$$M_x = -\frac{Wb(L+a)}{4L(2k+3)} + \frac{Wb}{3L}x - \frac{W(x-a)^3}{3b^2} \quad x \geq a$$



$$V_D = \frac{2}{3} W \quad M_B = M_C = - \frac{WL}{4(2k+3)}$$

$$H_A = H_D = \frac{WL}{4h(2k+3)} ; V_A = \frac{W}{3}$$

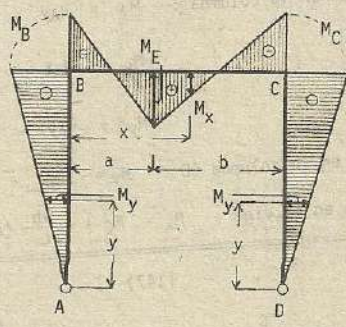
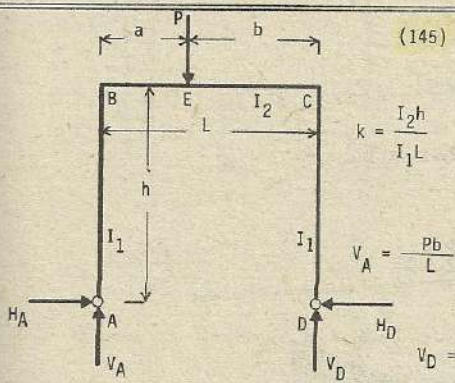
Esfuerzo de flexión en la columna :

$$M_y = - \frac{WLy}{4h(2k+3)}$$

Esfuerzo de flexión en la viga :

$$M_x = - \frac{WL}{4(2k+3)} - \frac{Wx^3}{3L^2} + \frac{W}{3}x$$

$$x_m = \frac{L}{\sqrt{3}} ; M_{\text{máx.}} = \frac{8\sqrt{3}}{108(2k+3)} \cdot WL$$



$$M_B = M_C = - \frac{3ab}{2L(2k+3)} \cdot P$$

$$H_A = H_D = \frac{3ab}{2hL(2k+3)} \cdot P$$

$$M_E = \frac{ab(4k+3)}{2L(2k+3)} \cdot P$$

Esfuerzo de flexión en la columna :

$$M_y = - \frac{3ab}{2hL(2k+3)} \cdot Py$$

Esfuerzo de flexión en la viga :

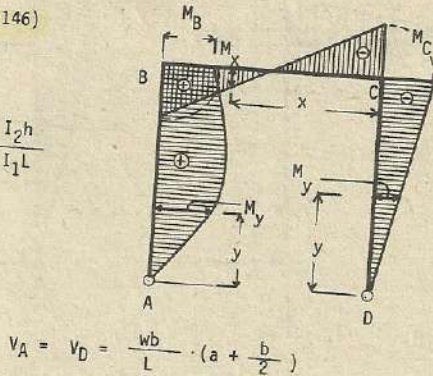
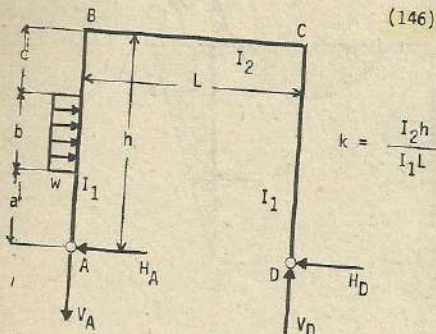
$$M_x = - \frac{3ab}{2L(2k+3)} \cdot P + \frac{Pb}{L} \cdot x \quad x \leq a$$

$$M_x = - \frac{3ab}{2L(2k+3)} \cdot P + \frac{Pb}{L} \cdot x - P(x-a) \quad x > a$$

Si $a = b = \frac{L}{2}$

$$H_A = H_D = \frac{3L}{8h(2k+3)} \cdot P \quad V_A = V_D = \frac{P}{2}$$

$$M_B = M_C = - \frac{3}{8(2k+3)} \cdot PL \quad M_E = \frac{(4k+3)}{8(2k+3)} \cdot PL$$



$$V_A = V_D = \frac{wb}{L} \cdot \left(a + \frac{b}{2}\right)$$

$$H_A = wb - \frac{(2a+b)}{8h^3(2k+3)} \left[6h^2(k+1) - k(2a^2 + 2ab + b^2) \right] \cdot wb$$

$$H_D = \frac{(2a+b)}{8h^3(2k+3)} \left[6h^2(k+1) - k(2a^2 + 2ab + b^2) \right] \cdot wb ; M_B = H_A h - \frac{wb}{2}(b+2c)$$

$$M_C = - \frac{(2a+b)}{8h^2(2k+3)} \left[6h^2(k+1) - k(2a^2 + 2ab + b^2) \right] wb$$

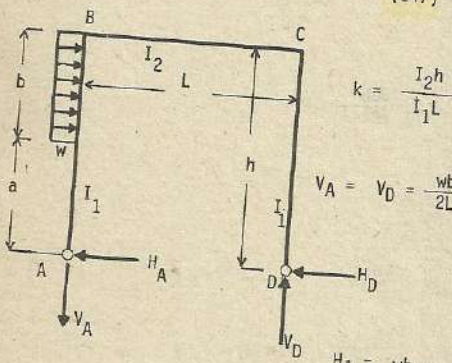
Esfuerzo de flexión en la columna : $M_y = H_A y \rightarrow y \leq a$

$$M_y = H_A y - \frac{w(y-a)^2}{2} \rightarrow a \leq y \leq a+b$$

$$M_y = H_A y - \frac{wb}{2}(2y-2a-b) \rightarrow y \geq a+b$$

Momento de flexión en la columna CD : $M_y = -H_D y$

Esfuerzo de momento en la viga : $M_x = M_C + \frac{wb}{L} \left(a + \frac{b}{2}\right) x$

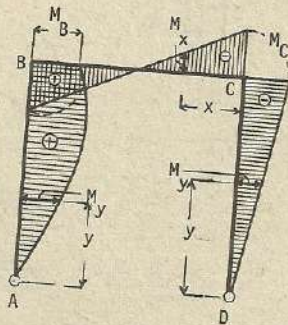


$$V_A = V_D = \frac{wb}{2L} (a+h)$$

$$H_A = wb - \frac{(h+a)}{8h^3(2k+3)} \left[6h^2(k+1) - k(h^2 + a^2) \right] \cdot wb$$

$$H_D = \frac{(h+a)}{8h^3(2k+3)} \left[6h^2(k+1) - k(h^2 + a^2) \right] \cdot wb ; M_B = H_A h - \frac{wb^2}{2}$$

$$M_C = - \frac{(h+a)}{8h^3(2k+3)} \left[6h^2(k+1) - k(h^2 + a^2) \right] \cdot wb$$



Esfuerzo de momento por flexión en la columna (AB) :

$$M_y = H_A y \quad y \leq a$$

$$M_y = H_A y - \frac{w(y-a)^2}{2} \quad y \geq a$$

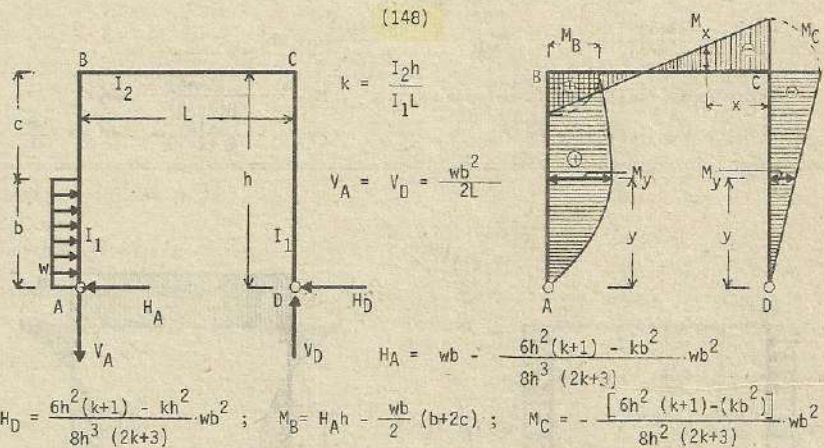
Esfuerzo de momento por flexión en la columna (CD) : $M_y = -H_D y$

Esfuerzo de flexión en la viga :

$$M_x = M_C + \frac{wb}{2L} (a+h) x$$

Si $a = b = \frac{h}{2}$: $V_A = V_D = \frac{3h^2 w}{8L}$; $H_A = \frac{71k+120}{128(2k+3)} wh$; $H_D = \frac{3(19k+24)}{128(2k+3)} wh$

$$M_B = \frac{39k+72}{128(2k+3)} wh^2 ; M_C = -\frac{3(19k+24)}{128(2k+3)} wh^2$$



Esfuerzo de flexión en la columna AB : $M_y = H_A y - \frac{wy^2}{2} \quad y \leq b$

$$M_y = H_A y - \frac{wb}{2} (2y-b) \quad y \geq b$$

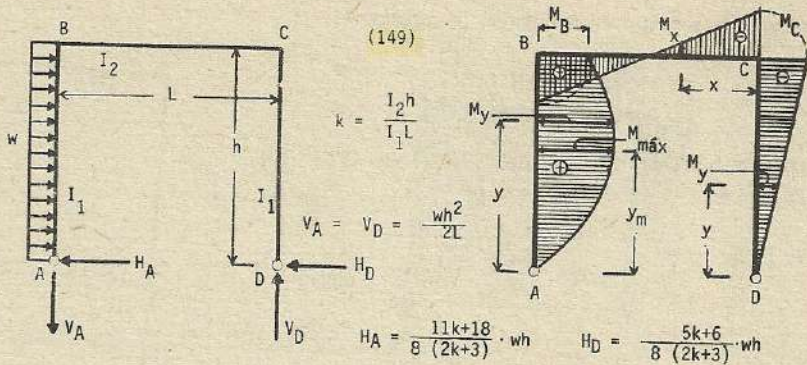
Esfuerzo de flexión en la columna CD : $M_y = -H_D y$

Esfuerzo de flexión en la viga : $M_x = M_C + \frac{wb}{2L} x$

Si $b = c = \frac{h}{2}$: $V_A = V_D = \frac{wh^2}{8L}$

$$H_A = \frac{105k+168}{128(2k+3)} wh \quad ; \quad H_D = \frac{23k+24}{128(2k+3)} wh$$

$$M_B = \frac{9k+24}{128(2k+3)} wh^2 \quad ; \quad M_C = -\frac{(23k+24)}{128(2k+3)} wh^2$$



$$M_B = \frac{3(k+2)}{8(2k+3)} \cdot wh^2 \quad M_C = - \frac{(5k+6)}{8(2k+3)} \cdot wh^2$$

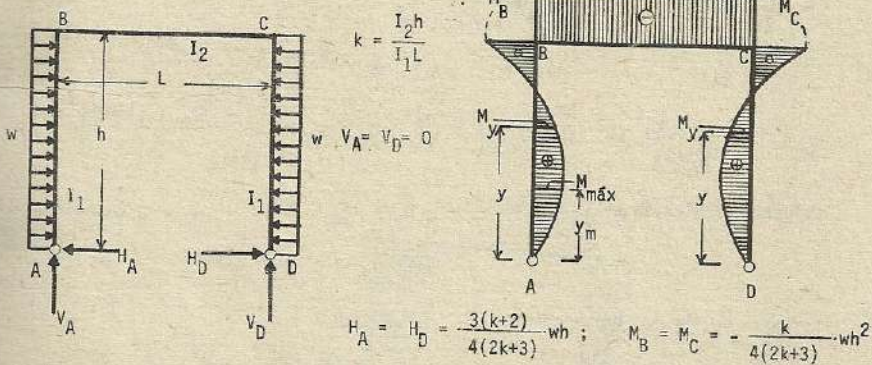
Esfuerzo de flexión en la columna AB : $M_y = \frac{11k+18}{8(2k+3)} \cdot why - \frac{wy^2}{2}$

$$y_m = \frac{11k+18}{8(2k+3)} h \quad ; \quad M_{m\acute{a}x.} = \frac{(11k+18)^2}{128(2k+3)^2} wh^2$$

Esfuerzo de flexión en la columna CD : $M_y = - \frac{(5k+6)}{8(2k+3)} \cdot why$

Esfuerzo de flexión en la viga : $M_x = - \frac{5k+6}{8(2k+3)} \cdot wh^2 + \frac{wh^2}{2L} x$

(150)

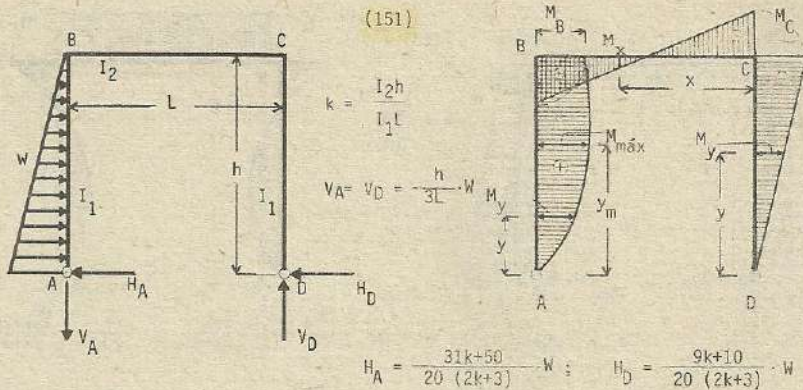


Esfuerzo de flexión en la columna :

$$M_y = \frac{3(k+2)}{4(2k+3)} \cdot why - \frac{wy^2}{2}$$

$$y_m = \frac{3(k+2)}{4(2k+3)} h \quad \longrightarrow \quad M_{m\acute{a}x.} = \frac{9(k+2)^2}{32(2k+3)^2} wh^2$$

Esfuerzo de flexión en la viga : $M_x = - \frac{k}{4(2k+3)} \cdot wh^2$ (constante)



$$M_B = \frac{13k+30}{60(2k+3)} \cdot Wh$$
 ; $M_C = -\frac{(9k+10)}{20(2k+3)} \cdot Wh$

Esfuerzo de flexión en la columna AB : $M_y = \frac{(31k+50)}{20(2k+3)} W y - \frac{W y^2}{3h^2} (3h-y)$

$$y_m = h \left[1 - \sqrt{\frac{9k+10}{20(2k+3)}} \right]$$
 ; $M_{m\acute{a}x.} = 2Wh \left[1 - \sqrt{\frac{9k+10}{20(2k+3)}} \right] \times \left[\frac{(31k+50)}{40(2k+3)} \right]$

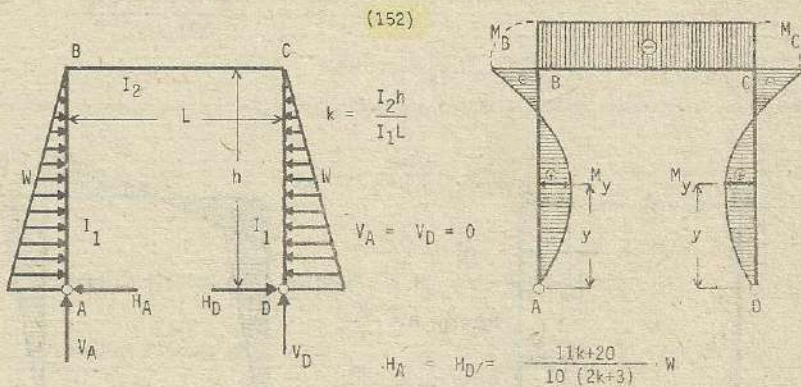
$$- \frac{1}{6} \left[2 + \sqrt{\frac{9k+10}{20(2k+3)}} \right] \cdot \left[1 - \sqrt{\frac{9k+10}{20(2k+3)}} \right]$$

Esfuerzo de flexión en la columna DC :

$$M_y = -\frac{(9k+10)}{20(2k+3)} W y$$

Esfuerzo de flexión en la viga :

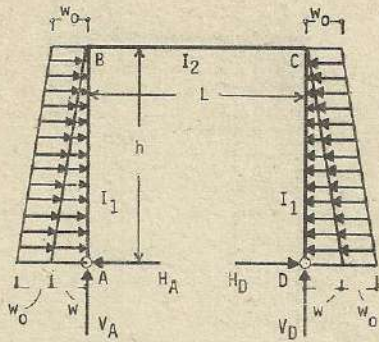
$$M_x = -\frac{(9k+10)}{20(2k+3)} \cdot Wh + \frac{h}{3L} \cdot W x$$



$$M_B = M_C = -\frac{7k}{30(2k+3)} Wh$$

Esfuerzo de flexión en la columna : $M_y = \frac{11k+20}{10(2k+3)} W y - \frac{W y^2}{3h^2} (3h-y)$

Esfuerzo de flexión en la viga : $M_x = -\frac{7k}{30(2k+3)} Wh$ (constante)



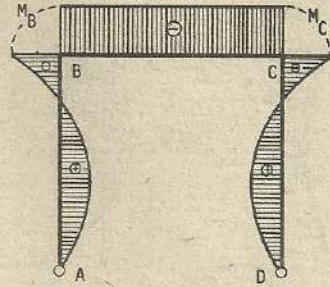
(153)

$$k = \frac{I_2 h}{I_1 L}$$

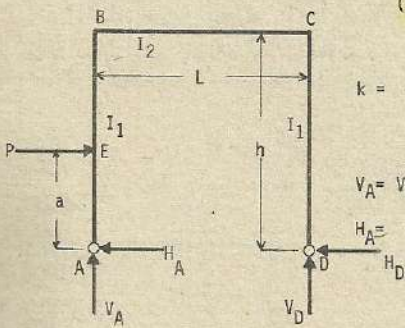
$$V_A = V_D = 0$$

$$H_A = H_D = \frac{3w_0 h}{4} \cdot \frac{k+2}{2k+3} + \frac{wh}{20} \cdot \frac{11k+20}{2k+3}$$

$$M_B = M_C = -\frac{w_0 h^2}{4} \cdot \frac{k}{2k+3} - \frac{wh^2}{60} \cdot \frac{7k}{2k+3}$$



Nota : para hallar (M_{\max}) analizar el problema (12)



(154)

$$k = \frac{I_2 h}{I_1 L}$$

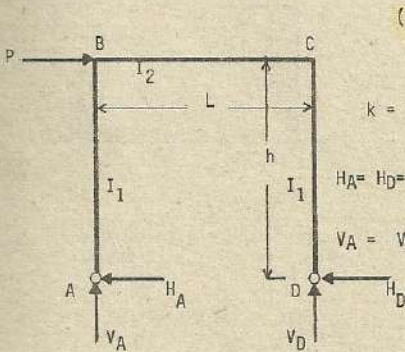
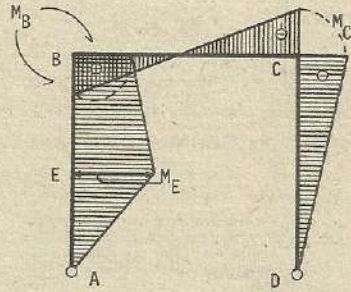
$$V_A = V_D = \frac{Pa}{L}$$

$$H_A = P - H_D$$

$$H_D = \frac{Pa}{2} \cdot \frac{3h^2 + k(3h^2 - a^2)}{h^3(2k+3)}$$

$$M_B = Pa - H_D h$$

$$M_C = -H_D h$$



(155)

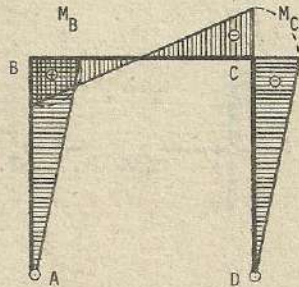
$$k = \frac{I_2 h}{I_1 L}$$

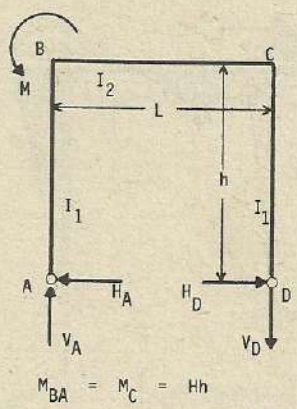
$$H_A = H_D = H = \frac{P}{2}$$

$$V_A = V_D = \frac{Ph}{L}$$

$$M_B = \frac{1}{2} \cdot Ph$$

$$M_C = -\frac{1}{2} \cdot Ph$$





(156)

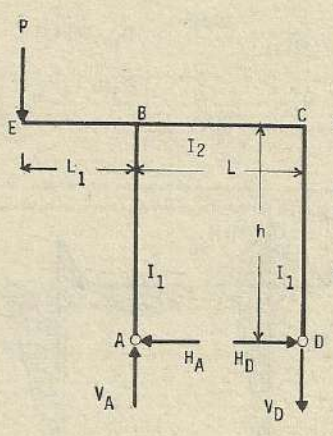
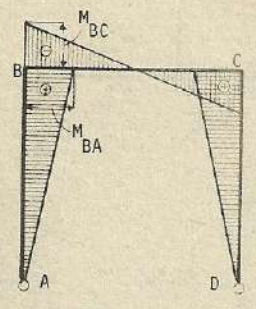
$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{M}{L}$$

$$H_A = H_D = H = \frac{3M}{2(2k+3) + h}$$

$$M_{BA} = M_C = Hh$$

$$M_{BC} = M_{BA} - M$$



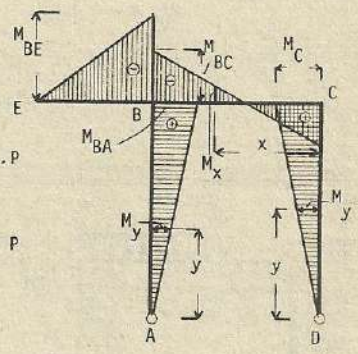
(157)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{L_1 + L}{L} \cdot P$$

$$V_D = \frac{L_1}{L} \cdot P$$

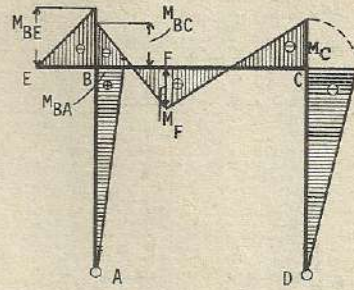
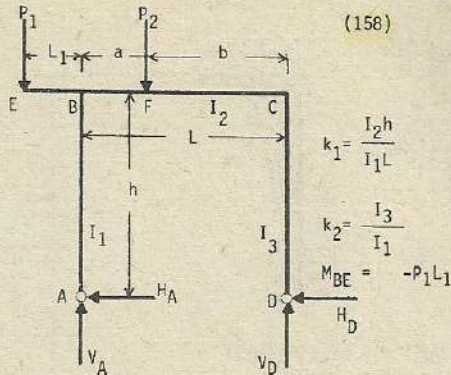
$$H_A = H_D = \frac{3PL_1}{2(2k+3)h}$$



$$M_{BA} = \frac{3PL_1}{2(2k+3)} ; \quad M_{BC} = -\frac{4k+3}{2(2k+3)} \cdot PL_1 ; \quad M_C = \frac{3PL_1}{2(2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = \frac{3PL_1 y}{2(2k+3)h}$

Esfuerzo de flexión en la viga : $M_x = \frac{3PL_1}{2(2k+3)} - P \cdot \frac{L_1}{L} \cdot x$



$$M_{BC} = -\frac{1}{2(3k_2 + k_1 + k_1 k_2)} \left[\frac{2k_2 P_2 a b}{L} + P_1 L_1 (3k_2 + 2k_1 k_2 + 2k_1) \right]$$

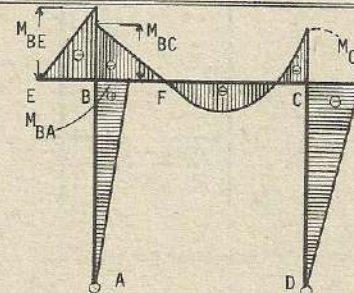
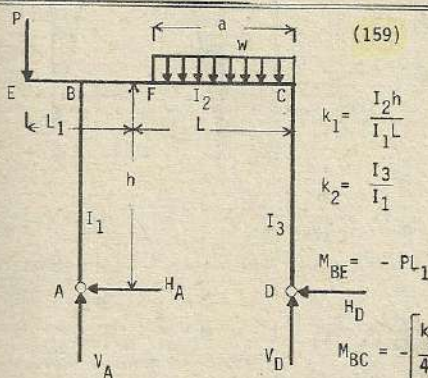
$$M_{BA} = -\frac{3k_2}{2(3k_2 + k_1 + k_1 k_2)} \left[\frac{P_2 a b}{L} - P_1 L_1 \right]; \quad M_C = -\frac{3k_2}{2(3k_2 + k_1 + k_1 k_2)} \left[\frac{P_2 a b}{L} - P_1 L_1 \right]$$

Si $I_1 = I_3$

$$M_{BE} = -P_1 L_1; \quad M_{BC} = -\frac{1}{2(2k+3)} \left[\frac{3P_2 a b}{L} + P_1 L_1 (4k+3) \right]$$

$$k = \frac{I_2 h}{I_1 L}$$

$$M_{BA} = -\frac{3}{2(2k+3)} \left[\frac{P_2 a b}{L} - P_1 L_1 \right]; \quad M_C = -\frac{3}{2(2k+3)} \left[\frac{P_2 a b}{L} - P_1 L_1 \right]$$



$$M_{BC} = -\left[\frac{k_2 w a^2 (3L-2a)}{4L(3k_2 + k_1 + k_1 k_2)} + \frac{P_1 L_1 (3k_2 + 2k_1 k_2 + 2k_1)}{2(3k_2 + k_1 + k_1 k_2)} \right]$$

$$M_{BA} = -\left[\frac{k_2 w a^2 (3L-2a)}{4L(3k_2 + k_1 + k_1 k_2)} - \frac{3k_2 P_1 L_1}{2(3k_2 + k_1 + k_1 k_2)} \right]$$

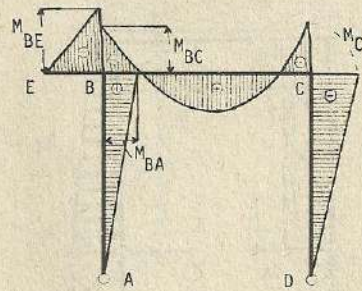
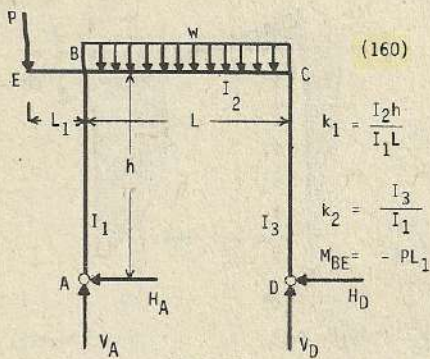
$$M_C = -\left[\frac{k_2 w a^2 (3L-2a)}{4L(3k_1 + k_1 + k_1 k_2)} - \frac{3k_2 P_1 L_1}{2(3k_2 + k_1 + k_1 k_2)} \right]$$

Si $k_3 = k_1$ $k = \frac{I_2 h}{I_1 L}$

$$M_{BE} = -PL_1$$

$$M_{BC} = -\left[\frac{w a^2 (3L-2a)}{4L(2k+3)} + \frac{P_1 L_1 (4k+3)}{2(2k+3)} \right]$$

$$M_{BA} = -\left[\frac{w a^2 (3L-2a)}{4L(2k+3)} - \frac{3P_1 L_1}{2(2k+3)} \right]; \quad M_C = -\left[\frac{w a^2 (3L-2a)}{4L(2k+3)} - \frac{3P_1 L_1}{2(2k+3)} \right]$$



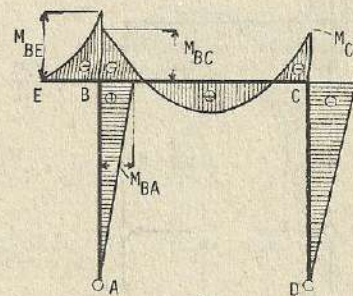
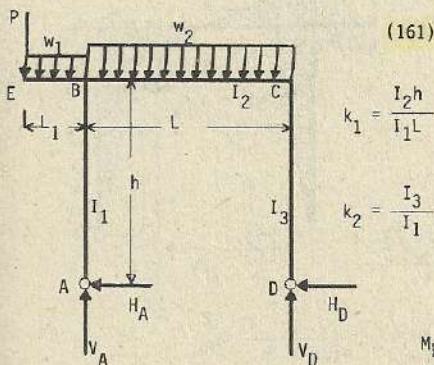
$$M_{BC} = -\frac{1}{4(3k_2 + k_1 + k_1 + k_2)} \left[k_2 w L^2 + 2PL_1(3k_2 + 2k_1 k_2 + 2k_1) \right]$$

$$M_{BA} = +\frac{3k_2}{4(3k_2 + k_1 + k_1 + k_2)} \left[\frac{wL^2}{3} - 2PL_1 \right]; \quad M_C = -\frac{3k_2}{4(3k_2 + k_1 + k_1 + k_2)} \left[\frac{wL^2}{3} - 2PL_1 \right]$$

$$\text{Si } I_1 = I_3 \quad k = \frac{I_2 h}{I_1 L} \quad M_{BE} = -PL_1$$

$$M_{BC} = -\frac{1}{4(2k+3)} \left[wL^2 + 2PL_1(4k+3) \right]; \quad M_{BA} = -\frac{3}{4(2k+3)} \left[\frac{wL^2}{3} - 2PL_1 \right]$$

$$M_C = -\frac{3}{4(2k+3)} \left[\frac{wL^2}{3} - 2PL_1 \right]$$



$$M_{BE} = -\left(PL_1 + \frac{w_1 L_1^2}{2} \right)$$

$$M_{BC} = -\frac{1}{4(3k_2 + k_1 + k_1 k_2)} \left[k_2 w_2 L^2 + (2PL_1 + w_1 L_1^2) \cdot (3k_2 + 2k_1 k_2 + 2k_1) \right]$$

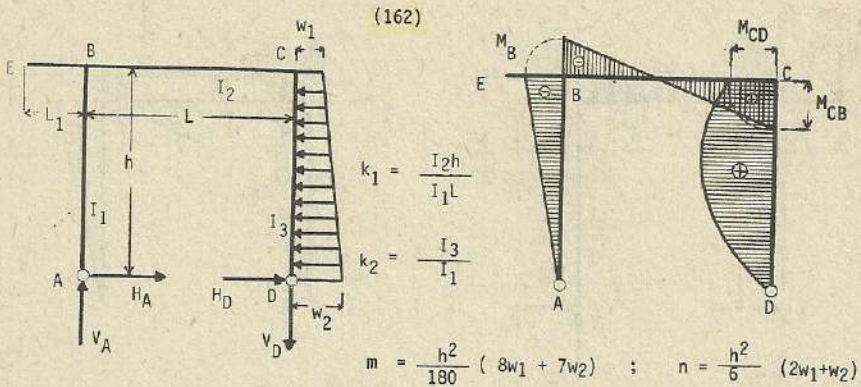
$$M_{BA} = -\frac{3k_2}{4(3k_2 + k_1 + k_1 k_2)} \left[\frac{w_2 L^2}{3} - 2PL_1 - w_1 L_1^2 \right]$$

$$M_C = -\frac{3k_2}{4(3k_2 + k_1 + k_1 k_2)} \left[\frac{w_2 L^2}{3} - 2PL_1 + w_1 L_1^2 \right]$$

$$\text{Si } I_3 = I_1 \quad k = \frac{I_2 h}{I_1 L} \quad M_{BE} = -\left(PL_1 + \frac{w_1 L_1^2}{2} \right)$$

$$M_{BC} = -\frac{1}{4(2k+3)} \left[w_2 L^2 + (2PL_1 + w_1 L_1^2) (4k+3) \right]$$

$$M_{BA} = +\frac{3}{4(2k+3)} \left[\frac{w_2 L^2}{3} - 2PL_1 - w_1 L_1^2 \right] = -M_C$$



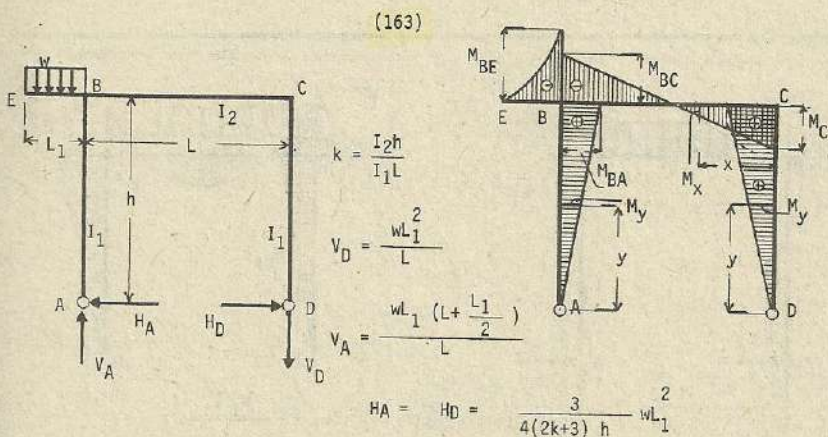
$$M_B = - \frac{2k_1 m + n (3k_2 + 2k_1)}{2 (3k_2 + k_1 + k_1 k_2)}$$

$$M_C = \frac{(3k_2 + 2k_1 k_2) n - 2k_1 m}{2 (3k_2 + k_1 + k_1 k_2)}$$

Si $I_3 = I_1$ $k = \frac{I_2 h}{I_1 L}$

$$M_B = - \frac{(2k m + n (2k+3))}{2 (2k+3)}$$

$$M_C = \frac{(2k+3) n - 2k m}{2 (2k+3)}$$

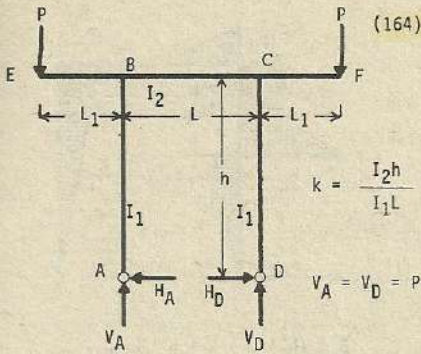


$$M_C = \frac{3}{4(2k+3)} \cdot w L_1^2 \quad ; \quad M_{BE} = - \frac{w L_1^2}{2} \quad ; \quad M_{BC} = - \frac{4k+3}{4(2k+3)} \cdot w L_1^2$$

$$M_{BA} = \frac{3}{4(2k+3)} \cdot w L_1^2$$

Esfuerzo de flexión en la columna : $M_y = \frac{3}{4(2k+3)} \cdot w L_1^2 \cdot y$

Esfuerzo de flexión en la viga : $M_x = \frac{3}{4(2k+3)} \cdot w L_1^2 - \frac{w L_1}{L} \cdot x$

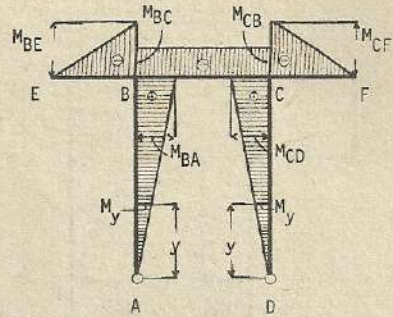


$$H_A = H_D = \frac{3PL_1}{(2k+3)h}$$

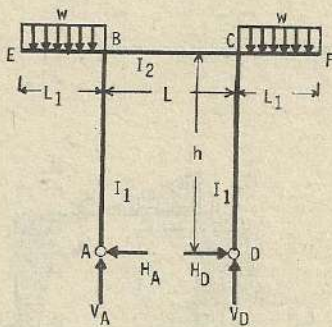
$$M_{BE} = M_{CF} = -PL_1 \quad ; \quad M_{BA} = M_{CD} = \frac{3PL_1}{(2k+3)} \quad ; \quad M_{BC} = M_{CB} = -\frac{2k}{(2k+3)} \cdot PL_1$$

Esfuerzo de flexión en la columna : $M_y = \frac{3PL_1 y}{(2k+3)h}$

Esfuerzo de flexión en la viga : $M_x = -\frac{2k}{(2k+3)} \cdot PL_1$ (constante)



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$$k = \frac{I_2 h}{I_1 L}$$

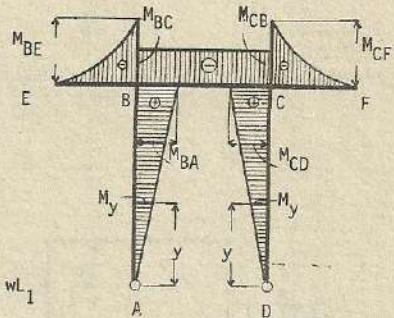
$$V_A = V_D = wL_1$$

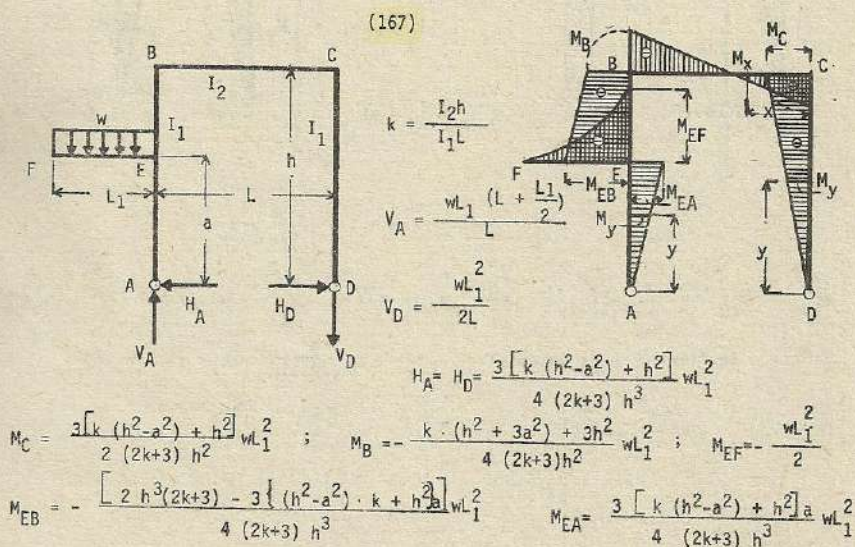
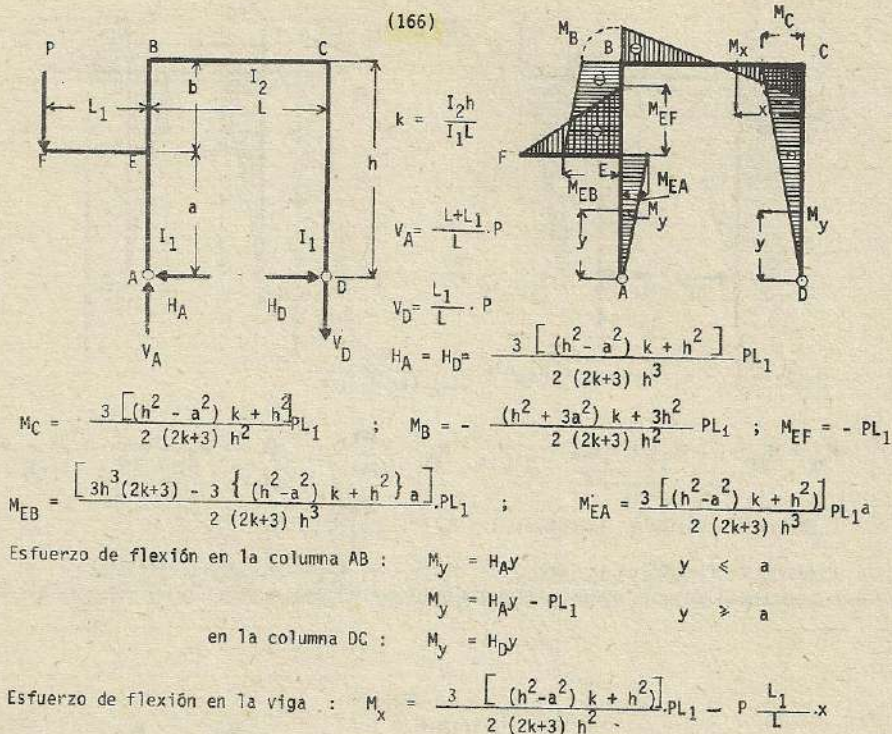
$$H_A = H_D = \frac{3}{2(2k+3)h} \cdot wL_1^2$$

$$M_{BA} = M_{CD} = \frac{3}{2(2k+3)} \cdot wL_1^2 \quad ; \quad M_{BE} = M_{CF} = -\frac{wL_1^2}{2} \quad ; \quad M_{BC} = M_{CB} = -\frac{k}{(2k+3)} \cdot wL_1^2$$

Esfuerzo de flexión en la columna : $M_y = \frac{3}{2(2k+3)h} \cdot wL_1^2 y$

Esfuerzo de flexión en viga : $M_x = -\frac{k}{(2k+3)} \cdot wL_1^2$ (constante)





Esfuerzo de flexión en la columna AB :

$$M_y = \frac{3 [k (h^2 - a^2) + h^2]}{4 (2k+3) h^3} wL_1^2 y \quad y \leq a$$

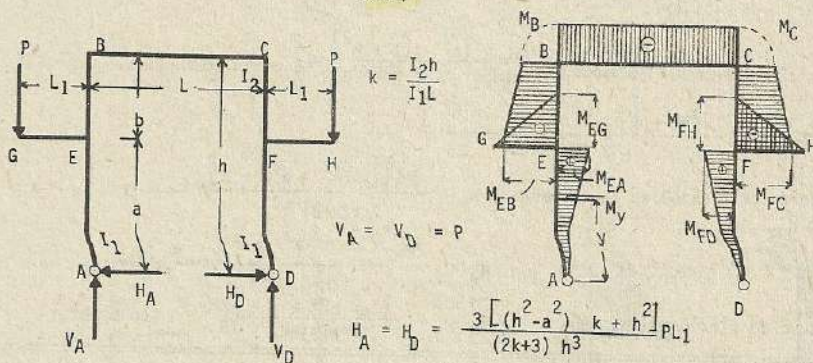
$$M_y = \frac{3 [k (h^2 - a^2) + h^2]}{4 (2k+3) h^3} wL_1^2 y - \frac{wL_1^2}{2} \quad y \geq a$$

Esfuerzo de flexión en la columna CD :

$$M_y = \frac{3 [k (h^2 - a^2) + h^2]}{4 (2k+3) h^3} wL_1^2 y$$

Esfuerzo de flexión en la viga : $M_x = \frac{3 [(h^2 - a^2) k + h^2]}{4 (2k+3) h^2} wL_1^2 - \frac{wL_1^2}{2L} x$

(168)



$$M_B = M_C = - \frac{(3a^2 - h^2) k}{(2k+3) h^2} PL_1 \quad ; \quad M_{EG} = M_{FH} = - PL_1$$

$$M_{EB} = M_{FC} = - \frac{(2k+3) h^3 - 3a [(h^2 - a^2) k + h^2]}{(2k+3) h^3} PL_1$$

$$M_{EA} = M_{ED} = \frac{3 [(h^2 - a^2) k + h^2]}{(2k+3) h^3} PL_1 a$$

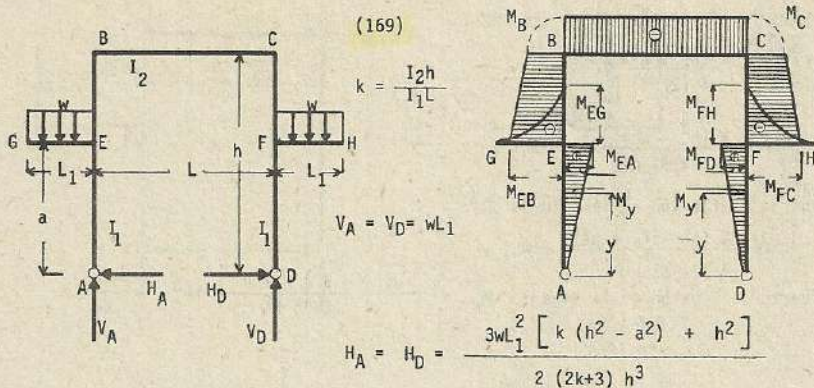
Momento de flexión en la columna

$$M_y = H_A y \quad y \leq a$$

$$M_y = H_A y - PL_1 \quad y \geq a$$

Esfuerzo de flexión en la viga :

$$M_x = - \frac{(3a^2 - h^2) k}{(2k+3) h^2} PL_1 \quad (\text{constante})$$



$$M_B = M_C = -\frac{(2a^2 - h^2)k}{2(2k+3)h^2} wL_1^2$$

$$M_{EG} = M_{FH} = -\frac{wL_1^2}{2}$$

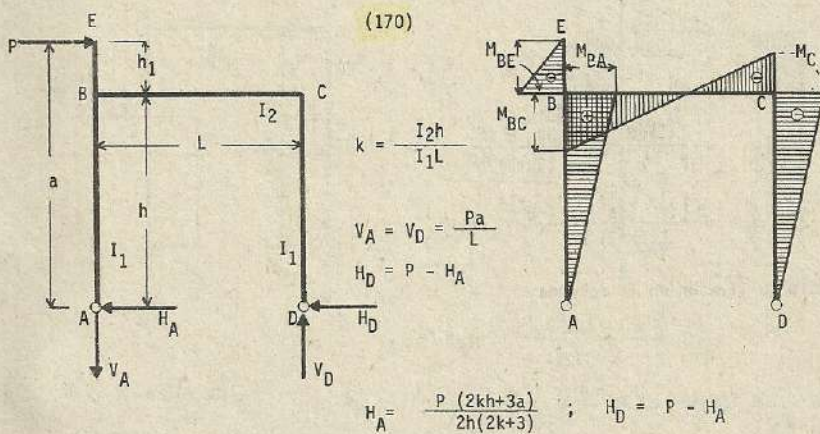
$$M_{EB} = M_{FC} = -\frac{(2k+3)h^3 - 3a[k(h^2 - a^2) + h^2]}{2(2k+3)h^3} wL_1^2$$

$$M_{EA} = M_{FD} = \frac{3waL_1^2 [k(h^2 - a^2) + h^2]}{2(2k+3)h^3}$$

Esfuerzo de flexión en la columna: $M_y = \frac{3wL_1^2 [k(h^2 - a^2) + h^2]}{2(2k+3)h^3} y \quad |y| \leq a$

$$M_y = \frac{3wL_1^2 [k(h^2 - a^2) + h^2]}{2(2k+3)h^3} y - \frac{wL_1^2}{2} \quad |y| \geq a$$

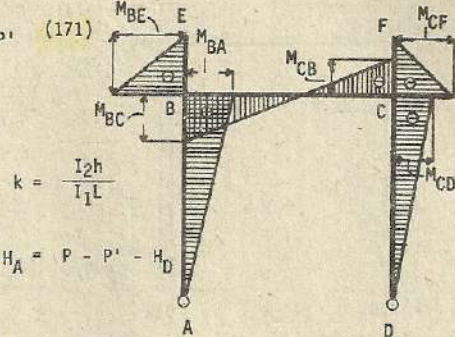
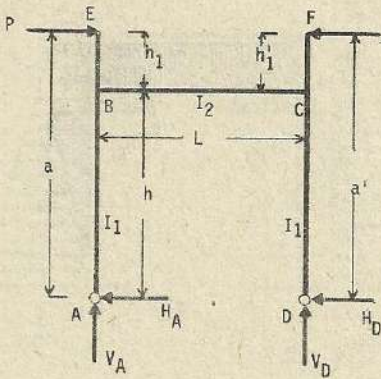
Esfuerzo de flexión en la viga: $M_x = -\frac{(3a^2 - h^2)k}{2(2k+3)h^2} wL_1^2 \quad (\text{constante})$



$$M_{BA} = H_A h \quad ; \quad M_{BC} = M_{BA} + Ph_1 = H_A h + Ph_1$$

$$M_{CB} = M_{CD} = -(P - H_A) h$$

$$M_{BE} = -Ph_1$$



$$k = \frac{I_2 h}{I_1 L}$$

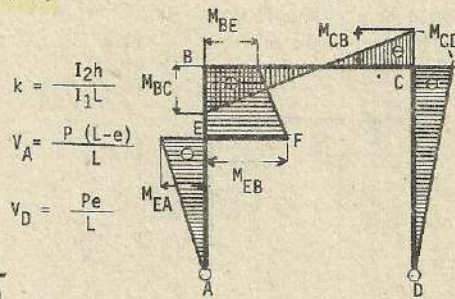
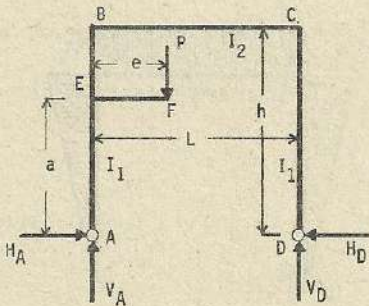
$$H_A = P - P' - H_D$$

$$H_D = \frac{(P - P') 2kh + 3(Pa + P'a') - 6P'h}{2h(2k+3)}$$

$$V_A = V_D = \frac{Pa - P'a'}{L}$$

$$M_{BA} = H_A h \quad ; \quad M_{BC} = M_{BA} + Ph_1 \quad ; \quad M_{CB} = M_{CD} + P'h_1 \quad ; \quad M_{CD} = -H_D h$$

(172)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{P(L-e)}{L}$$

$$V_D = \frac{Pe}{L}$$

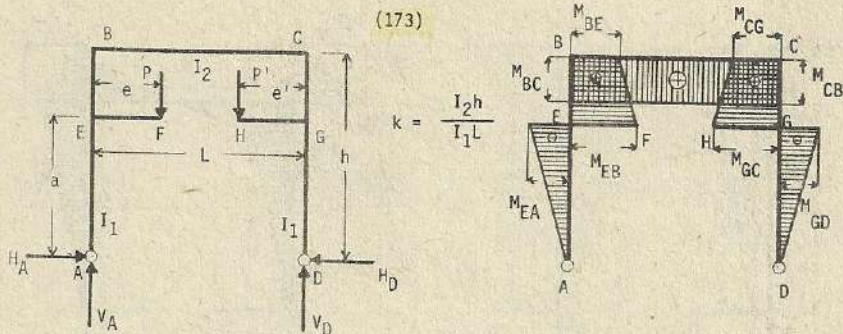
$$H_A = H_D = H = 3Pe \frac{k(h^2 - a^2) + h^2}{2h^3(2k+3)}$$

$$M_{BE} = Pe - Hh = M_{BC}$$

$$M_{EA} = -H_A a$$

$$M_{CB} = -Hh = M_{CD}$$

$$M_{FB} = -H_A a + Pe$$



$$V_A = \frac{P' e' + P(L-e)}{L}$$

$$V_D = \frac{Pe + P'(1-e')}{L}$$

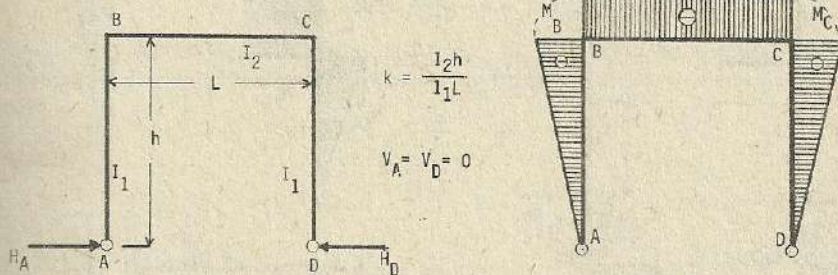
$$H_A = H_D = H = 3(Pe + P'e') \cdot \frac{k(h^2 - a^2) + h^2}{2h^3(2k+3)} ; M_{BE} = Pe - Hh = M_{BC}$$

$$M_{CB} = P' e' - Hh = M_{CG} ; M_{EA} = -H_A a ; M_{GD} = -H_D a$$

$$M_{EB} = -H_A a + pe ; M_{GC} = -H_D a + P'e'$$

Esfuerzos por cambio de Temperatura

(174)

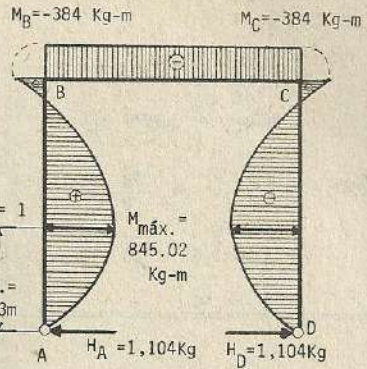
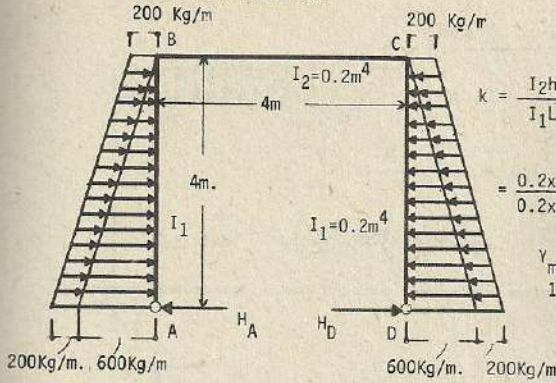


$$H_A = H_D = \frac{3EtI_2}{(3+2k)h^2}$$

$$M_B = M_C = -\frac{3EtI_2}{(2k+3)h^2}$$

Nota : El presente problema corresponde al caso de aumento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.

Problemas Numéricos



De acuerdo a (153) :

$$H_A = H_D = \frac{3w_0 h}{4} \cdot \frac{k+2}{2k+3} + \frac{wh}{20} \cdot \frac{11k+20}{2k+3}$$

$$= \frac{3}{4} \times 200 \times 4 \times \frac{1+2}{2+3} + \frac{600 \times 4}{20} \times \frac{11+20}{2+3} = 1,104 \text{ Kg.}$$

$$V_A = V_D = 0$$

$$M_B = M_C = -\frac{w_0 h^2}{4} \frac{k}{2k+3} - \frac{wh^2}{60} \frac{7k}{2k+3} = -\frac{200 \times 4 \times 4}{4} \cdot \frac{1}{2+3} - \frac{600 \times 4 \times 4}{60} \cdot \frac{7}{2+3} = -384 \text{ Kg-m}$$

Similares resultados se obtienen descomponiendo las cargas como uniformemente distribuida y de forma triangular :

$$w_0 = 200 \text{ Kg/m} ; \quad W = \frac{600 \times 4}{2} = 1,200 \text{ Kg.}$$

Carga uniformemente distribuida :

$$H'_A = H'_D = \frac{3(k+2)}{4(2k+3)} \cdot wh$$

$$= \frac{3 \times (1+2)}{4 \times (2+3)} \times 200 \times 4 = 360 \text{ Kg}$$

Carga triangular :

$$H''_D = H''_D = \frac{11k+20}{10(2k+3)} \cdot W = \frac{11+20}{10(2+3)} \times 1,200 = 744 \text{ Kg.}$$

$$H_A = H_D = 360 + 744 = 1,104 \text{ Kg.}$$

Carga uniformemente distribuida :

$$M'_B = M'_C = -\frac{k}{4(2k+3)} \cdot w_0 h^2$$

$$= -\frac{1}{4(2+3)} \times 200 \times 4 \times 4 = -160 \text{ Kg-m}$$

Carga triangular :

$$M''_B = M''_C = -\frac{7k}{30(2k+3)} \cdot Wh = -\frac{7}{30(2+3)} \times 1200 \times 4 = -224 \text{ Kg-m}$$

$$M_B = M_C = -160 - 224 = -384 \text{ Kg-m}$$

Si consideramos como (y_m) la distancia que media entre (A) y el punto donde el esfuerzo de flexión es máximo ($M_{m\acute{a}x.}$) :

$$V = H_A - w_0 y_m - W \cdot \frac{(2hy_m - y_m^2)}{h^2} = 0$$

$$1,104 - 200y_m - 1,200x \frac{(2.4 \cdot y_m - y_m^2)}{4 \times 4} = 0$$

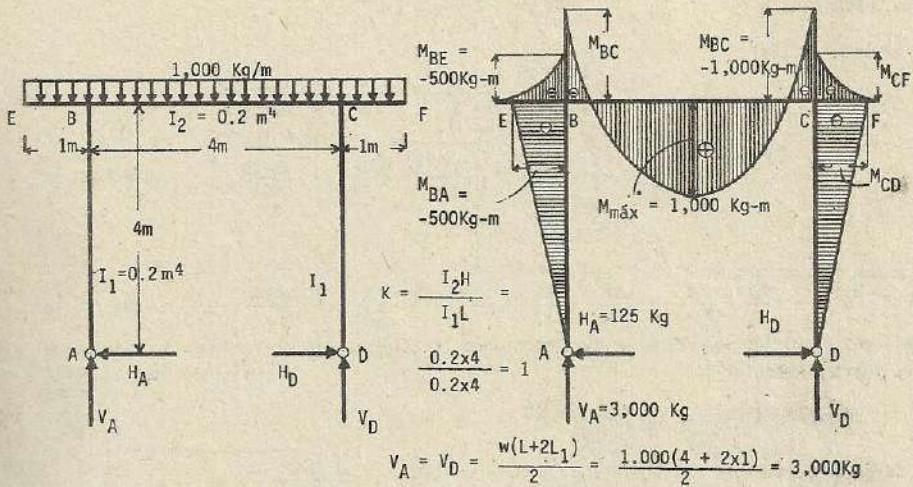
$$75y_m^2 - 800y_m + 1,104 = 0$$

$$y_m = 1.63 \text{ m.}$$

$$M_{\max.} = H_A y_m - w_o \frac{y_m^2}{2} - \frac{W y_m^2}{3h^2} \cdot (3h - y_m)$$

$$= 1,104 \times 1.63 - 200 \times \frac{1.63 \times 1.63}{2} - \frac{1,200 \times 1.63 \times 1.63}{3 \times 4 \times 4} \times (3 \times 4 - 1.63)$$

$$= 845.02 \text{ Kg.}$$



Carga en (B-C) - 142 -

$$H'_A = H'_D = \frac{wL^2}{4(3+2K)h} = \frac{1,000 \times 4 \times 4}{4(3+2) \times 4} = 200 \text{ Kg}$$

Carga por volado - 165 -

$$H''_A = H''_D = \frac{3wL_1^2}{2(2K+3)h} = \frac{3 \times 1,000 \times 1 \times 1}{2(2+3) \times 4} = 75 \text{ Kg}$$

$\therefore H_A = H_D = 200 - 75 = 125 \text{ Kg}$ Nota: tener cuidado con los signos.

$$M_{BE} = M_{CF} = -\frac{wL_1^2}{2} = -\frac{1,000 \times 1 \times 1}{2} = -500 \text{ Kg-m}$$

$$M_{BA} = M_{CD} = -H_A h = -125 \times 4 = -500 \text{ Kg-m}$$

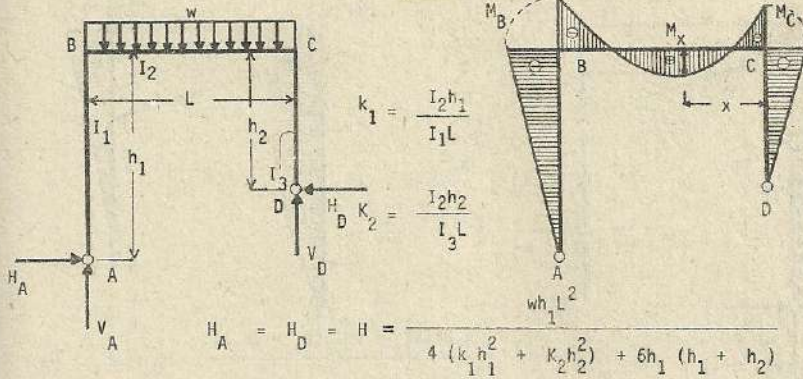
$$M_{BC} = M_{CB} = -500 - 500 = -1,000 \text{ Kg-m}$$

Máximo esfuerzo de flexión o doblado ($M_{\max.}$)

$$M_{\max.} = V_A \cdot \frac{L}{2} - H_A h - \frac{w}{2} (L_1 + \frac{L}{2})^2 = 3,000 \times \frac{4}{2} - 125 \times 4 - \frac{1,000}{2} (1 + \frac{4}{2})^2$$

$$= 1,000 \text{ Kg-m}$$

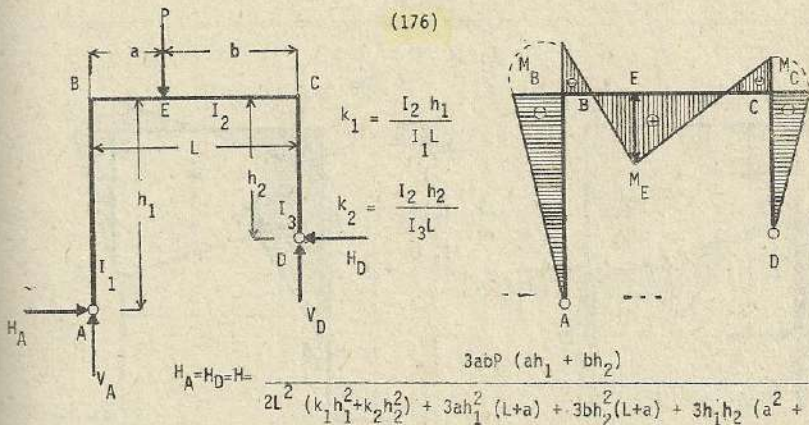
(175)



$$V_A = \frac{wL}{2} + H \cdot \frac{h_1 - h_2}{L}; \quad V_D = \frac{wL}{2} - H \cdot \frac{h_1 - h_2}{L}; \quad M_B = -H h_1; \quad M_C = -H h_2$$

$$M_x = M_o - \left[M_C + \frac{x}{L} \cdot (M_B - M_C) \right] \quad \text{siendo} \quad M_o = \frac{w x}{2} \cdot (L - x)$$

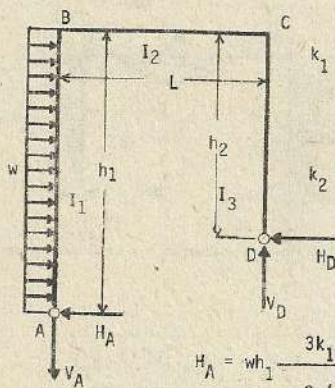
(176)



$$V_A = P \cdot \frac{b}{L} + H \cdot \frac{h_1 - h_2}{L}; \quad V_D = P \cdot \frac{a}{L} - H \cdot \frac{h_1 - h_2}{L}; \quad M_B = -H h_1; \quad M_C = -H h_2$$

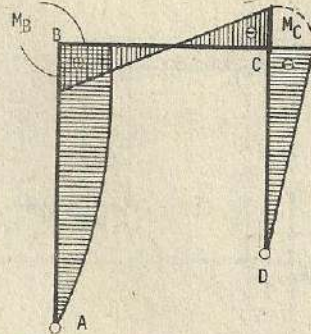
$$M_E = V_A a - H h_1 = V_D b - H h_2$$

(177)



$$k_1 = \frac{I_2 h_1}{I_1 L}$$

$$k_2 = \frac{I_2 h_2}{I_3 L}$$

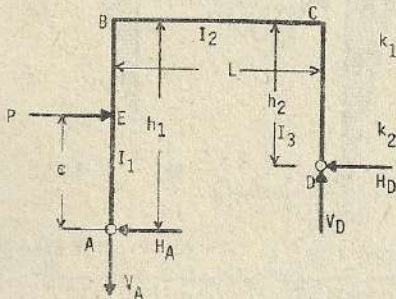


$$H_A = wh_1 \frac{3k_1 h_1^2 + 8k_2 h_2^2 + 6h_1 (h_1 + 2h_2)}{8(k_1 h_1^2 + k_2 h_2^2) + 12h_1 (h_1 + h_2)} ; H_D = wh_1 - H_A$$

$$-V_A = V_D = \frac{wh_1 (h_2 - \frac{h_1}{2}) + H_A (h_1 - h_2)}{L} ; M_B = -H_A h_1 + \frac{wh_1^2}{2}$$

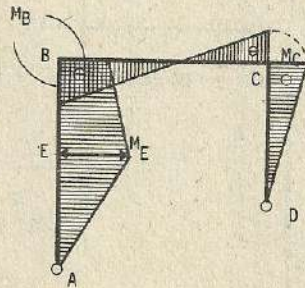
$$M_C = -H_D h_2$$

(178)



$$k_1 = \frac{I_2 h_1}{I_1 L}$$

$$k_2 = \frac{I_2 h_2}{I_3 L}$$

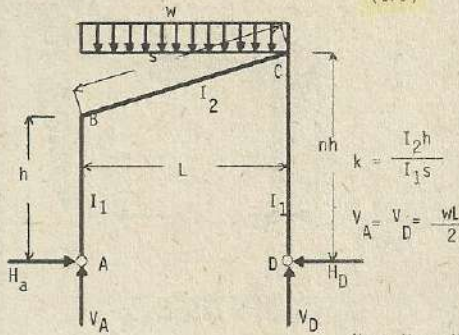


$$H_A = P \frac{k_1 (2h_1^3 + c^3 - 3ch_1^2) + 2k_2 h_1 h_2^2 + 3k_1^2 (h_1 - c + h_2)}{h_1^2 (2k_1 + 3) + h_2 (3k_1^2 + 2k_2 h_1 h_2)} ; H_D = P - H_A$$

$$-V_A = V_D = \frac{P(h_2 + c - h_1) + H_A (h_1 - h_2)}{L}$$

$$M_B = -H_A h_1 + P(h_1 - c) ; M_C = -H_D h_2 ; M_E = -H_A c$$

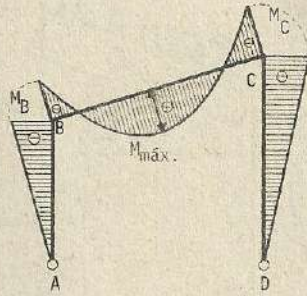
(179)



$$V_A = V_D = \frac{wL}{2}$$

$$H_A = H_D = H = \frac{wL^2}{8h} \cdot \frac{1+n}{(1+n+n^2+k(1+n^3))}$$

$$M_B = -Hh \quad ; \quad M = -nHh \quad ; \quad M_{\max.} = \frac{wL^2}{8} - Hh \left(\frac{1+n}{2} \right)$$



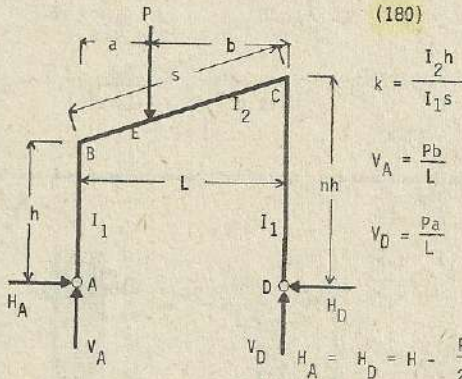
Cuando la carga uniformemente distribuida acciona perpendicularmente en la cara de proyección de la viga :

$$- V_A = V_D = \frac{wh^2(n^2-1)}{2L} \quad ; \quad H_A = \frac{wh}{8} \cdot \frac{\alpha}{\beta} \quad ; \quad H_D = wh(n-1) - H_A \quad ; \quad M_B = H_A h$$

$$M_C = -nh \{ wh(n-1) - H_A \} \quad ; \quad \alpha = 8n^3 k(n-1) + 7n^3 - 2n^2 - 3n - 1$$

$$\beta = 1 + n + n^2 + k(1+n^3)$$

(180)



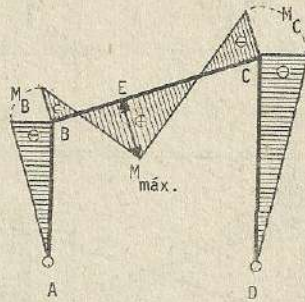
$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = \frac{Pb}{L}$$

$$V_D = \frac{Pa}{L}$$

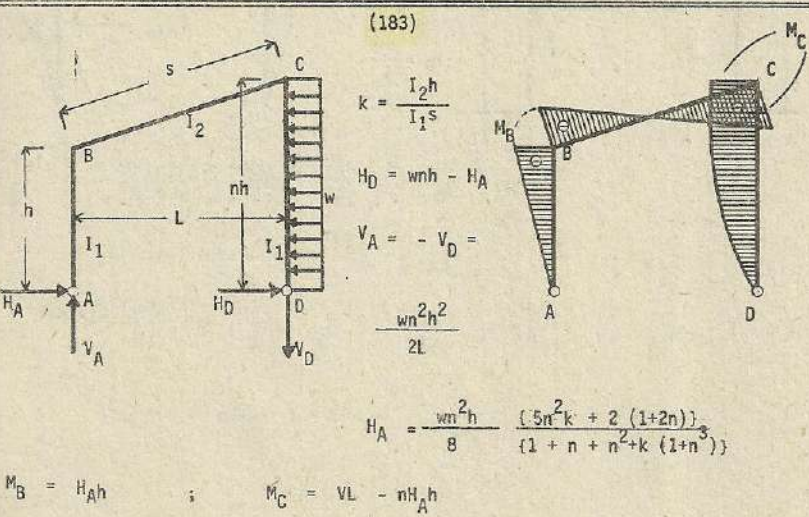
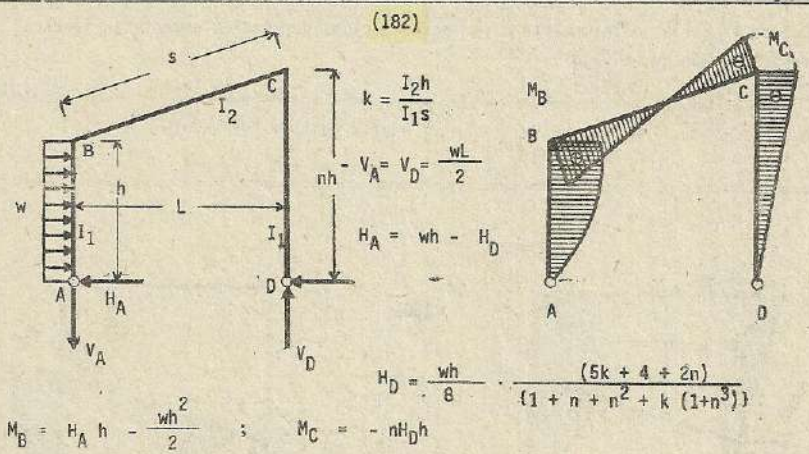
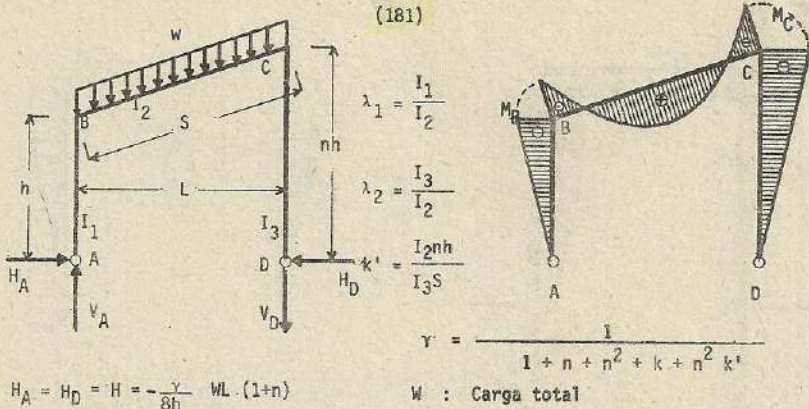
$$H_A = H_D = H = \frac{Pab \{ L(2+n) + a(n-1) \}}{2hL^2 \{ 1 + n + n^2 + k(1+n^3) \}}$$

$$M_{\max} = \frac{Pab}{L} - H \left(h + \frac{a}{L} (nh - h) \right)$$

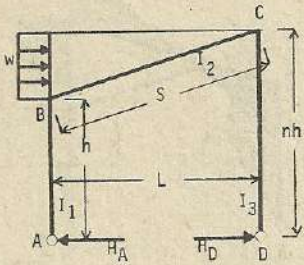


$$\text{Si } a = b = \frac{L}{2} \quad ; \quad V_A = V_D = \frac{P}{2} \quad ; \quad H_A = H_D = H = \frac{3PL}{16h} \cdot \frac{(1+n)}{\beta}$$

$$\beta = 1 + n + n^2 + k(1+n^3)$$



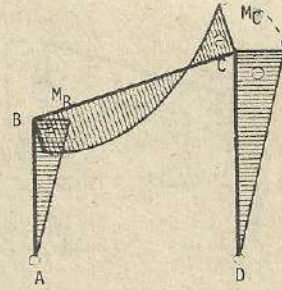
(184)



$$\lambda_1 = \frac{I_1}{I_2}$$

$$\lambda_2 = \frac{I_3}{I_2}$$

$$k' = \frac{I_2 nh}{I_3 S}$$

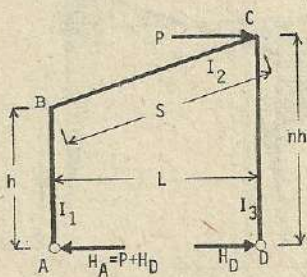


$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H = -\gamma W \left(\lambda_1 + \frac{7 + 4n + n^2}{8} \right)$$

$$H_A = W + H_D ; M_B = (W + H_D) h ; M_C = H_D nh \quad W : \text{Carga total}$$

(185)



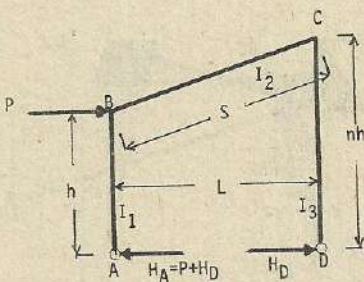
$$\lambda_1 = \frac{I_1}{I_2} ; \lambda_2 = \frac{I_3}{I_2}$$

$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H_D = -\frac{\gamma}{2} P (2 + n + 2k') ; H_A = P + H_D$$

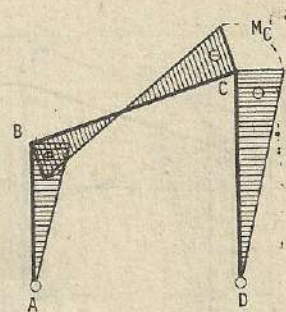
$$M_B = (P + H_D) h \quad M_C = H_D nh$$

(186)



$$\lambda_1 = \frac{I_1}{I_2}$$

$$\lambda_2 = \frac{I_3}{I_2}$$

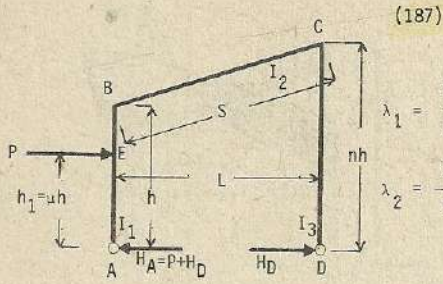


$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H_D = -\frac{\gamma}{2} P (2 + n + 2k')$$

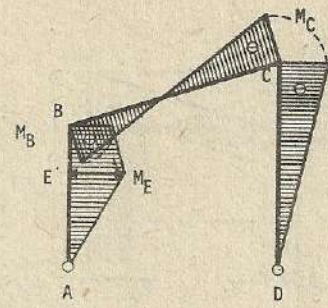
$$H_A = P + H_D$$

$$M_B = (P + H_D) h \quad M_C = H_D nh$$



$$\lambda_1 = \frac{I_1}{I_2}$$

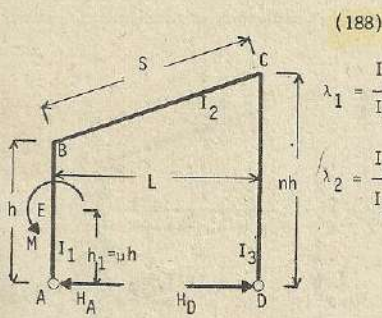
$$\lambda_2 = \frac{I_3}{I_2}$$



$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

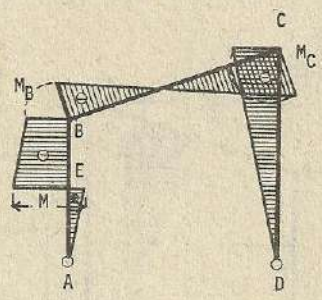
$$H_D = -\frac{\gamma}{2} P u (k(u^2 - 3u + 4) + n + 2) ; H_A = P + H_D$$

$$M_B = (P u + H_D) h ; M_C = H_D \cdot nh$$



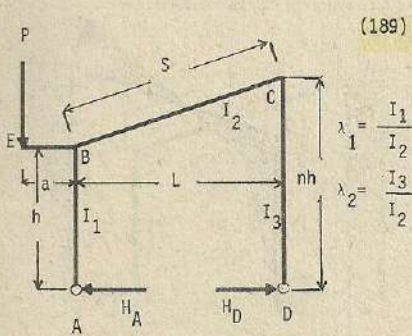
$$\lambda_1 = \frac{I_1}{I_2}$$

$$\lambda_2 = \frac{I_3}{I_2}$$



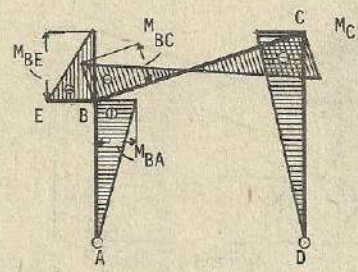
$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H_A = H_D = H = \frac{\gamma}{h} M (2 + n + 3k - 3k'u^2) ; M_B = Hh - M ; M_C = Hnh$$



$$\lambda_1 = \frac{I_1}{I_2}$$

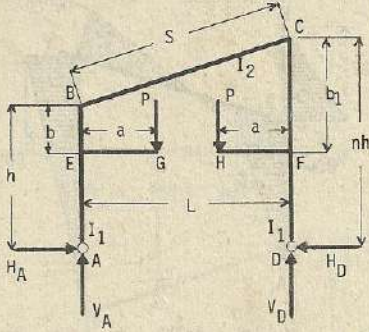
$$\lambda_2 = \frac{I_3}{I_2}$$



$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H_A = H_D = H = \frac{\gamma}{2h} Pa (2 + n) ; M_{BA} = Hh ; M_{BC} = Hh - Pa ; M_C = Hnh$$

(190)

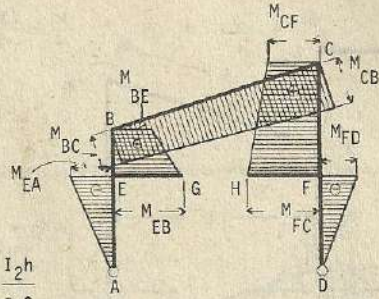


$$k = \frac{I_2 h}{I_1 S}$$

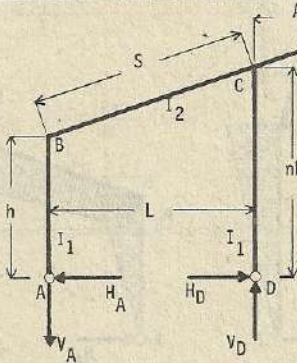
$$H_A = H_D = H = \frac{3Pa}{2h^3} \frac{V_A = V_D = \frac{P}{k(2bh - b^2 + 2b_1 nh - b_1^2)} + h^2(1+n)}{1 + n + n^2 + (1+n^3)k}$$

$$M_{BC} = M_{BA} = -Hh + Pa ; M_{CB} = M_{CD} = -Hnh + Pa ; M_{EA} = M_{FD} = -H(h-b)$$

$$M_{EB} = M_{FC} = -H(h-b) + Pa$$



(191)



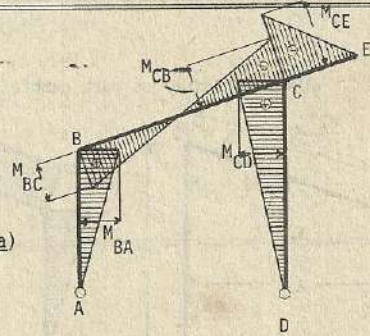
$$k = \frac{I_2 h}{I_1 S}$$

$$V_A = \frac{Pa}{L}$$

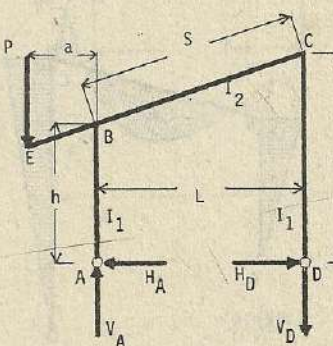
$$V_D = \frac{P(L+a)}{L}$$

$$H_A = H_D = H = \frac{Pa}{2h} \frac{1 + 2n}{1 + n + n^2 + (1+n^3)k}$$

$$M_{CD} = Hnh ; M_{CB} = Hnh - Pa ; M_{BA} = M_{BC} = +Hh$$



(192)



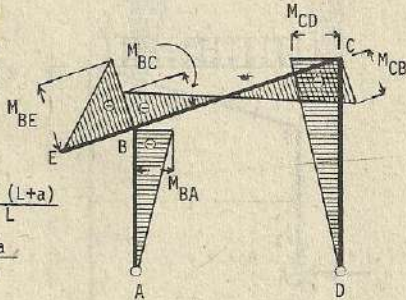
$$k = \frac{I_2 h}{I_1 S}$$

$$V_A = \frac{P(L+a)}{L}$$

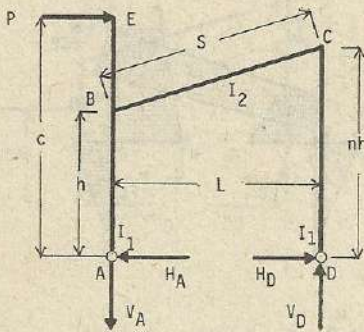
$$V_D = \frac{Pa}{L}$$

$$H_A = H_D = H = \frac{Pa}{2h} \frac{2 + n}{1 + n + n^2 + (1+n^3)k}$$

$$M_{BA} = Hh ; M_{BC} = Hh - Pa ; M_{CB} = M_{CD} = Hnh$$

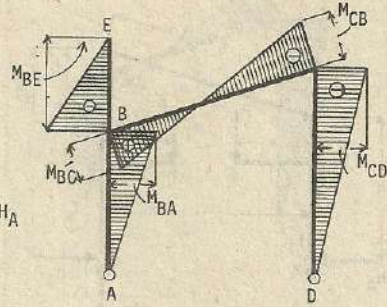


(193)



$$k = \frac{I_2 h}{I_1 s}$$

$$H_D = P - H_A$$

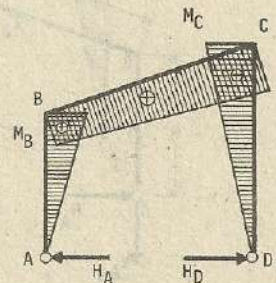
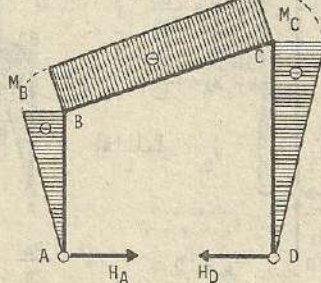
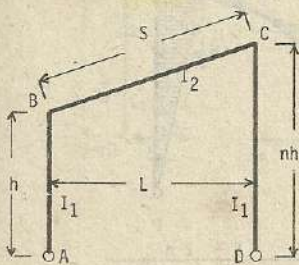


$$H_A = \frac{Pc}{2h} \cdot \frac{n(2nh + 2n^2hk + c) + 2c - 2h}{1 + n + n^2 + k(1+n^3)}$$

$$V_A = V_D = V = \frac{Pc}{L} ; \quad M_{BA} = H_A h ; \quad M_{BC} = VL - (P - H_A) h$$

$$M_{CB} = M_{CD} = -(P - H_A)nh ; \quad M_{BE} = -(c-h)P$$

(194) : Esfuerzos para cambio de temperatura



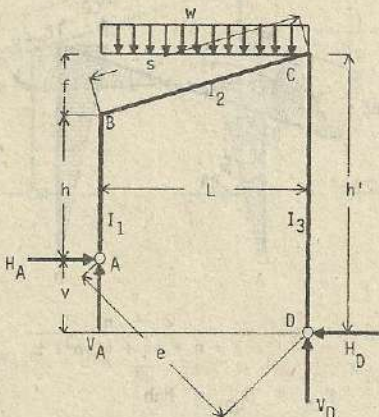
$$k = \frac{I_2 h}{I_1 L}$$

Por aumento de temperatura

Por descenso

$$H_A = H_D = H = \frac{3E\epsilon I_2 tL}{h^2 s (k(n^2+1) + (n^2+n+1))} ; \quad M_B = Hh ; \quad M_C = Hnh$$

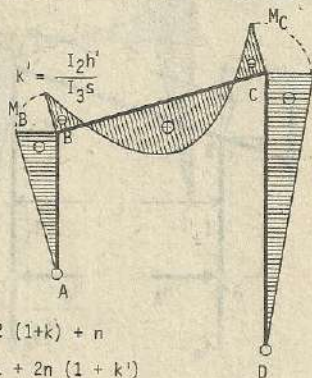
(195)



$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



$$\beta = 2(1+k) + n$$

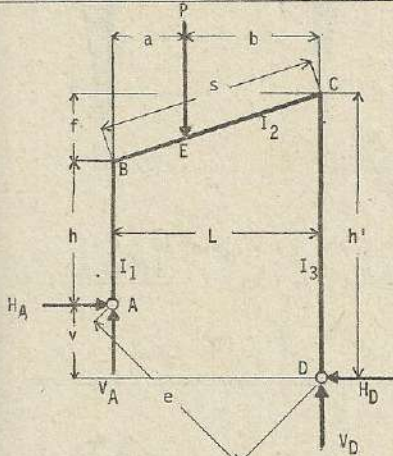
$$\gamma = 1 + 2n(1+k)$$

$$N = \beta + n\gamma = 2(1+k+n+n^2(1+k'))$$



$$H_A = H_D = H = \frac{(1+n)wL^2}{4Nh} ; \quad V_A = \frac{wL}{2} - \frac{Hv}{L} ; \quad V_D = \frac{wL}{2} + \frac{Hv}{L}$$

$$M_B = -Hh ; \quad M_C = -Hh'$$

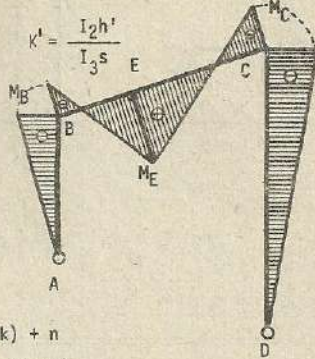


(196)

$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



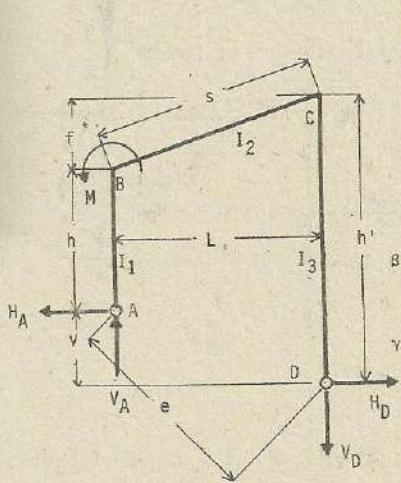
$$\beta = 2(1+k) + n$$

$$\gamma = 1 + 2n(1+k')$$

$$N = \beta + n\gamma = 2(1+k+n+n^2 \frac{(1+k')}{h})$$

$$H_{BC} = -\frac{Pab}{2L^2}(L+b) ; \quad H_{CB} = -\frac{Pab}{2L^2}(L+a) ; \quad H_A - H_D - H = -2 \frac{H_{BC} + H_{CB}}{Nh}$$

$$V_A = \frac{Pb - Hv}{L} ; \quad V_D = P - V_A ; \quad M_B = -Hh ; \quad M_C = -Hh'$$



(197)

$$k = \frac{I_2 h}{I_1 s} ; \quad k' = \frac{I_2 h'}{I_3 s}$$

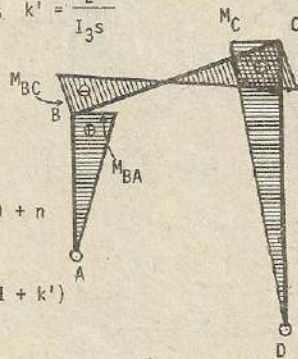
$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2(1+k) + n$$

$$\gamma = 1 + 2n(1+k')$$

$$N' = \beta + n\gamma = 2(1+k+n+n^2(1+k'))$$



$$H_A = H_D = H = \frac{(2+n)M}{Nh}$$

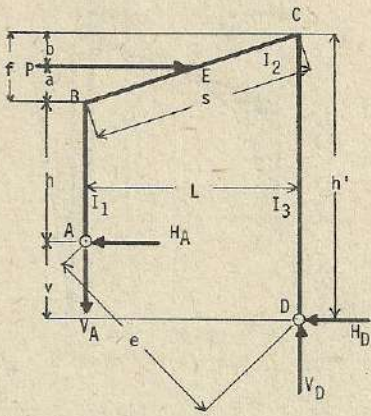
$$V_A = V_D = V = \frac{M + Hv}{L}$$

$$M_{BA} = Hh$$

$$M_{BC} = M_{BA} - M$$

$$M_C = Hh'$$

(200)



$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2(1+k) + n$$

$$\gamma = 1 + 2n(1+k')$$

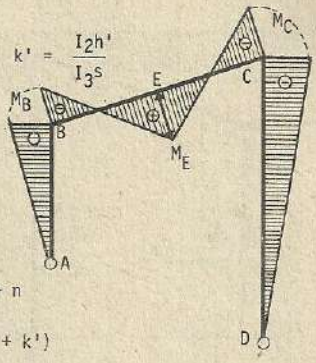
$$N = \beta + n\gamma = 2(1+k+n+n^2(1+k'))$$

$$H_{BC} = -\frac{Pab}{2f^2}(f+b); \quad H_{CB} = -\frac{Pab}{2f^2}(f+a)$$

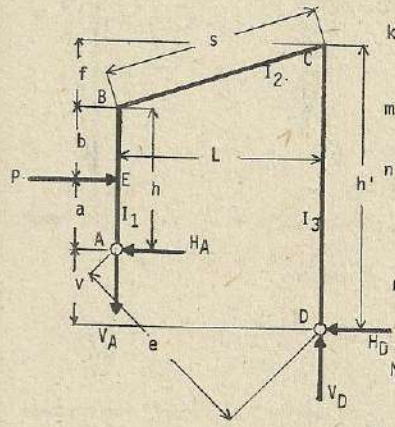
$$H_A = P - H_D; \quad H_D = \frac{\beta P}{N} - 2 \frac{H_{BC} + nH_{CB}}{Nh}; \quad V_A = V_D = \frac{P(h+a) + H_D v}{L}$$

$$M_B = (P - H_D)h$$

$$M_C = -H_D h'$$



(201)



$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

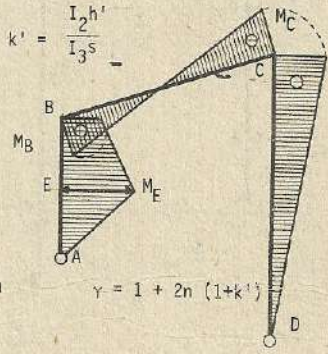
$$n = \frac{h'}{h}$$

$$\beta = 2(1+k) + n$$

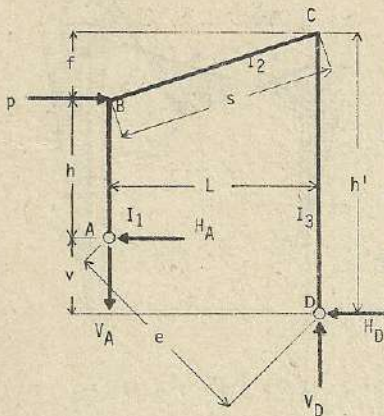
$$N = \beta + n\gamma = 2 \times [1+k+n+n^2(1+k')]$$

$$H_{BA} = -\frac{Pab}{2h^2}(h+b); \quad H_D = \frac{\beta Pa - 2k H_{BA}}{Nh}$$

$$H_A = P - H_D; \quad V_A = V_D = V = \frac{Pa + H_D v}{L}; \quad M_B = Pa - H_D h; \quad M_C = -H_D h'$$



(202)



$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

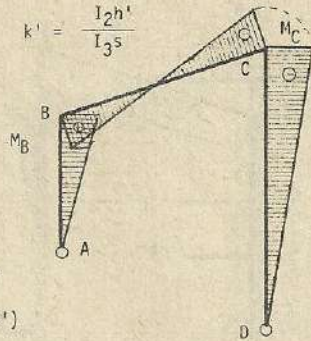
$$\beta = 2(1+k) + n$$

$$\gamma = 1 + 2n(1+k')$$

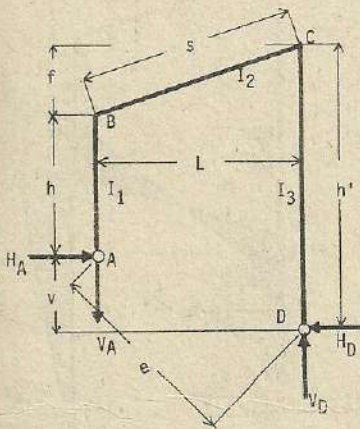
$$N = \beta + n\gamma = 2(1+k+n+n^2(1+k'))$$

$$H_D = \frac{\beta P}{N}; \quad H_A = P - H_D; \quad V_A = V_D = \frac{Ph + H_D v}{L}$$

$$M_B = (P - H_D)h; \quad M_C = -H_D h'$$



Esfuerzo por cambio de temperatura (203)



$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2(1+k) + n$$

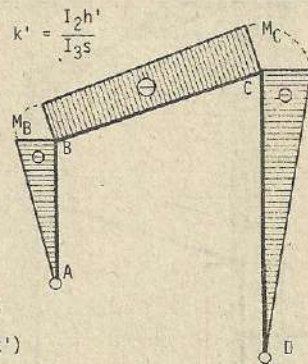
$$\gamma = 1 + 2n(1+k')$$

$$N = \beta + n\gamma = 2(1+k+n+n^2(1+k'))$$

$$H_A = H_D = H = \frac{T}{N} \frac{L^2 + v^2}{L^2}; \quad T = \frac{6E\epsilon I_2 t L}{sh^2}$$

$$V_A = V_D = \frac{Hv}{L}$$

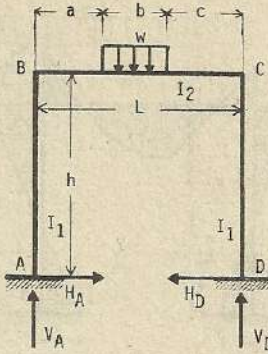
$$M_B = -Hh; \quad M_C = -Hh'$$



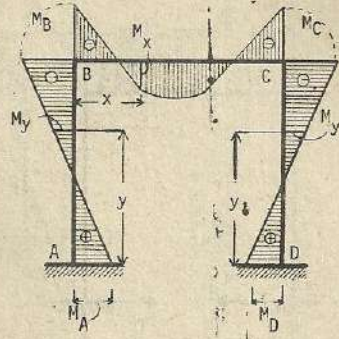
Nota :

El presente problema corresponde al caso de aumento de temperatura.
En caso de descenso, Los esfuerzos se anotan con los signos contrarios.

(204)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = \frac{\left[3abc + (3L-2c)c^2 + \left(L+c - \frac{b}{2}\right)b^2 + 3kL^2(b+2c) \right]}{(6k+1)L^3} wb$$

$$V_D = \frac{\left[3abc + (3L-2a)a^2 + \left(L+a - \frac{b}{2}\right)b^2 + 3kL^2(b+2a) \right]}{(6k+1)L^3} wb$$

$$H_A = H_D = \frac{(6ac + 3bL - 2b^2)}{4(k+2)hL} wb$$

$$M_A = \frac{wb}{12L^2(k+2)(6k+1)} \left[6a(3cL+2ab-4c^2) + b(10bL-3L^2-6b^2) + 6ka(7cL+ab-2c^2) + 3bk(5L^2 - 2bL-b^2) \right]$$

$$M_D = \frac{wb}{12L^2(k+2)(6k+1)} \left[6c(3aL+2cb-4a^2) + b(10bL-3L^2-6b^2) + 6kc(7aL+bc-2a^2) + 3bk(5L^2 - 2bL-b^2) \right]$$

$$M_B = M_A - H_A h \quad ; \quad M_C = M_D - H_D h$$

Momento por flexión en la columna AB : $M_y = M_A - H_A y$

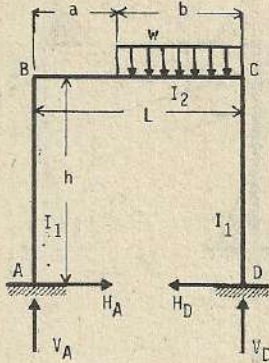
columna CD : $M_y = M_D - H_D y$

Momento por flexión en la viga :

$$M_x = M_A - H_A h + V_A x \quad x \leq a$$

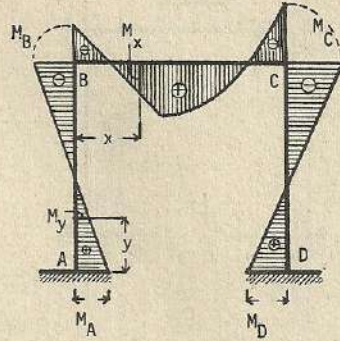
$$M_x = M_A - H_A h + V_A x - \frac{w(x-a)^2}{2} \quad a \leq x \leq a+b$$

$$M_x = M_A - H_A h + V_A x - \frac{wb(2x-2a-b)}{2} \quad x \geq a+b$$



(205)

$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{3L-2b}{4(k+2)hL} \cdot wb^2 \quad V_A = \frac{(L - \frac{b}{2})b + 3kL^2}{(6k+1)L^3} \cdot wb^2$$

$$V_D = \frac{(3L-2a)a^2 + (L+a - \frac{b}{2})b^2 + 3kL^2(b+2a)}{(6k+1)L^3} \cdot wb$$

$$M_A = \frac{(9a^2 + 4ab + b^2) + 3k(7a^2 + 2b^2 + 8ab)}{12L^2(k+2)(6k+1)} \cdot wb^2$$

$$M_B = M_A - H_A h$$

$$M_D = \frac{(10bL - 3L^2 - 6b^2) + 3k(5a^2 + 2b^2 + 8ab)}{12L^2(k+2)(6k+1)} \cdot wb^2$$

$$M_C = M_D - H_D h$$

Momento de flexión en la columna AB : $M_y = M_A - H_A y$

columna CD : $M_y = M_D - H_D y$

Momento de flexión en la viga :

$$M_x = M_A - H_A h + V_A x \quad x \leq a$$

$$M_x = M_A - H_A h + V_A x - \frac{w(x-a)^2}{2} \quad x \geq a$$

Si $a = b = \frac{L}{2}$ $V_D = \frac{(72k+13)}{32(6k+1)} \cdot wL$ $V_A = \frac{3(8k+1)}{32(6k+1)} \cdot wL$

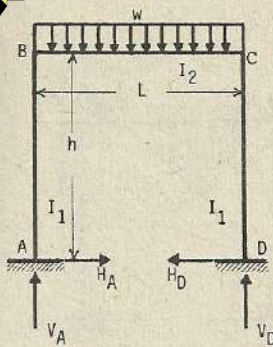
$$H_A = H_D = \frac{1}{8(k+2)h} \cdot wL^2$$

$$M_A = \frac{51k+14}{192(k+2)(6k+1)} \cdot wL^2$$

$$M_D = \frac{45k+2}{192(k+2)(6k+1)} \cdot wL^2$$

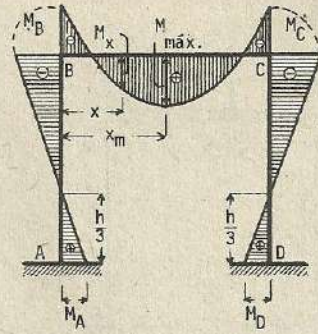
$$M_B = -\frac{(93k+10)}{192(k+2)(6k+1)} \cdot wL^2$$

$$M_C = -\frac{(99k+22)}{192(k+2)(6k+1)} \cdot wL^2$$



(206)

$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{wL^2}{4(k+2)h} ; \quad V_A = V_D = \frac{wL}{2} ; \quad M_A = M_D = \frac{wL^2}{12(k+2)} ; \quad M_B = M_C = -\frac{wL^2}{6(k+2)}$$

Momento por flexión en la columna :

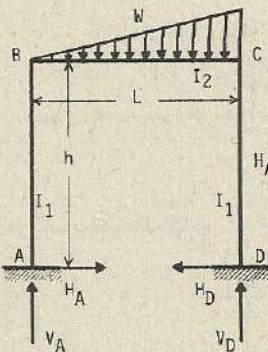
$$M_y = \frac{wL^2}{12(k+2)} - \frac{wL^2 y}{4(k+2)h}$$

Momento por flexión en la viga :

$$M_x = -\frac{wL^2}{6(k+2)} + \frac{wL}{2}x - \frac{wx^2}{2}$$

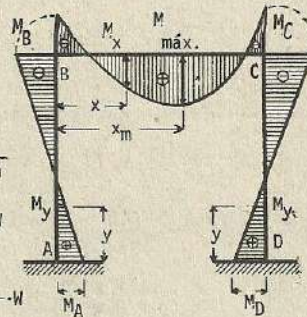
$$x_m = \frac{L}{2}$$

$$M_{m\acute{a}x.} = \frac{(3k+2)}{24(k+2)} \cdot wL^2$$



(207)

$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{WL}{4(k+2)h}$$

$$V_A = \frac{(20k+3)}{10(6k+1)} \cdot W$$

$$V_D = \frac{(40k+7)}{10(6k+1)} \cdot W$$

$$M_A = \frac{(31k+7)}{60(k+2)(6k+1)} \cdot WL$$

$$M_D = \frac{(29k+3)}{60(k+2)(6k+1)} \cdot WL$$

$$M_B = -\frac{(59k+8)}{60(k+2)(6k+1)} \cdot WL$$

$$M_C = -\frac{(61k+12)}{60(k+2)(6k+1)} \cdot WL$$

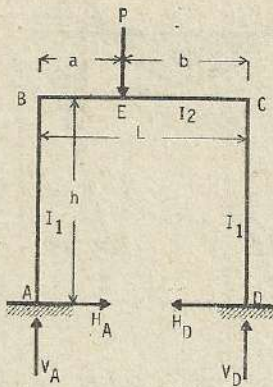
Esfuerzo de momento por flexión en la columna AB :

$$M_y = \frac{(31k+7)WL}{60(k+2)(6k+1)} - \frac{WLy}{4(k+2)h}$$

$$\text{Columna DC : } M_y = \frac{(29k+3)WL}{60(k+2)(6k+1)} - \frac{WLy}{4(k+2)h}$$

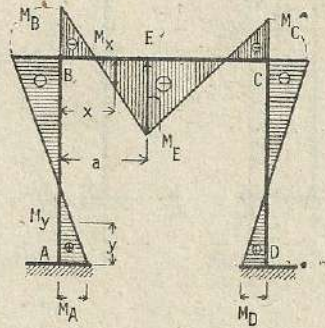
$$\text{Viga BC : } M_x = -\frac{(59k+8)}{60(k+2)(6k+1)} WL + \frac{(20k+3)}{10(6k+1)} Wx - \frac{Wx^3}{3L^2}$$

$$x_m = L \cdot \sqrt{\frac{2k+0.3}{6k+1}}$$



(208)

$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{3Pab}{2(k+2)hL} \quad ; \quad V_A = \frac{Pb}{L} + \frac{Pab(L-2a)}{(6k+1)L^3} \quad ; \quad V_D = \frac{Pa(6kL^2 + 3aL - 2a^2)}{L^3(6k+1)}$$

$$M_A = \frac{(5k-1)L + 2a(k+2)}{2(k+2)(6k+1)L^2} Pab \quad ; \quad M_D = \frac{(7k+3)L - 2a(k+2)}{2(k+2)(6k+1)L^2} Pab$$

$$M_B = -\frac{(13k+4)L - 2a(k+2)}{2(k+2)(6k+1)L^2} Pab \quad ; \quad M_C = -\frac{11kL + 2a(k+2)}{2(k+2)(6k+1)L^2} Pab$$

Momento por flexión en la columna AB ;

$$M_y = \frac{(5k-1)L - 2a(k+2)}{2(k+2)(6k+1)^2} Pab - \frac{3Paby}{2(k+2)hL}$$

Columna CD :

$$M_y = \frac{(7k+3)L - 2a(k+2)}{2(k+2)(6k+1)L^2} Pab - \frac{3Paby}{2(k+2)hL}$$

Viga BC :

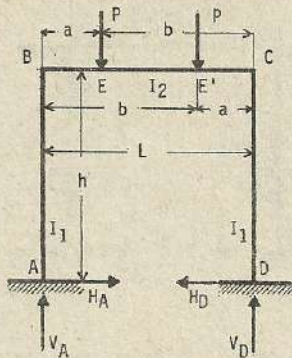
$$M_x = -\frac{(13k+4)L + 2a(k+2)}{2(k+2)(6k+1)L^2} Pab + \frac{Pb}{L}x + \frac{Pab(L-2a)x}{(6k+1)L^3} \quad x \leq a$$

$$M_x = -\frac{(13k+4)L + 2a(k+2)}{2(k+2)(6k+1)L^2} Pab + \frac{Pb}{L}x + \frac{Pab(L-2a)x}{(6k+1)L^3} - P(x-a) \quad x \geq a$$

$$M_E = -\frac{(13k+4)L + 2a(k+2)}{2(k+2)(6k+1)L^2} Pab + \frac{Pab}{L} + \frac{Pa^2b(L-2a)}{(6k+1)L^3}$$

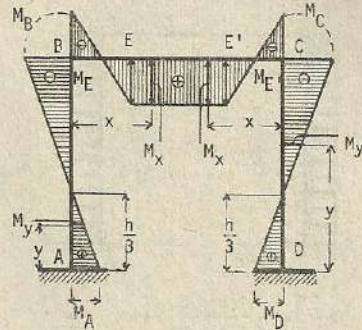
$$\text{Si } a = b = \frac{L}{2} \quad ; \quad H_A = H_D = \frac{3PL}{8(k+2)h} \quad ; \quad V_A = V_D = \frac{P}{2}$$

$$M_A = M_D = -\frac{PL}{8(k+2)} \quad ; \quad M_B = M_C = -\frac{PL}{4(k+2)} \quad ; \quad M_E = \frac{(k+1)}{4(k+2)} PL$$



(209)

$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{3Pab}{(k+2)hL}$$

$$V_A = V_D = P$$

$$M_A = M_D = \frac{Pab}{(k+2)L}; \quad M_B = M_C = -\frac{2Pab}{(k+2)L}$$

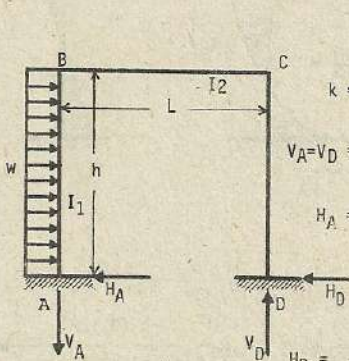
$$M_E = Pa - \frac{2Pab}{(k+2)L}$$

Momento por flexión en la columna :

$$M_y = \frac{Pab}{(k+2)L} - \frac{3Paby}{(k+2)hL}$$

En la viga : $M_x = -\frac{2Pab}{(k+2)L} + Px$ $x \leq a$

Esfuerzo de momento por flexión entre las cargas : $M_x = Pa - \frac{2Pab}{(k+2)L}$ (constante)



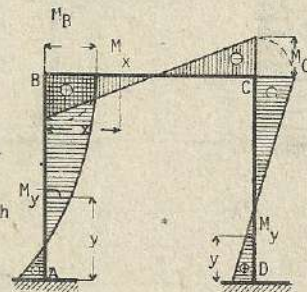
(210)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{kwh^2}{(6k+1)L}$$

$$H_A = \frac{(2k+3)}{8} \cdot wh$$

$$H_D = \frac{(6k+13)}{8(k+2)} \cdot wh$$



$$M_A = -\frac{(30k^2+73k+15)}{24(k+2)(6k+1)} \cdot wh^2$$

$$M_D = \frac{(18k^2+35k+9)}{24(k+2)(6k+1)} \cdot wh^2$$

$$M_B = \frac{k(6k+23)}{24(k+2)(6k+1)} \cdot wh^2$$

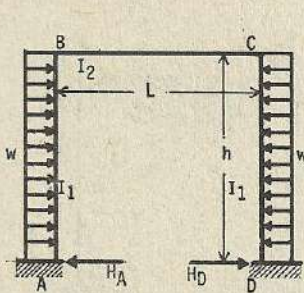
$$M_C = -\frac{k(18k+25)}{24(k+2)(6k+1)} \cdot wh^2$$

Esfuerzo de momento por flexión en la columna AB :

$$M_y = -\frac{(30k^2+73k+15)}{24(k+2)(6k+1)} \cdot wh^2 + \frac{6k+13}{8(k+2)} \cdot why - \frac{wy^2}{2}$$

Columna CD : $M_y = \frac{(18k^2+35k+9)}{24(k+2)(6k+1)} \cdot wh^2 - \frac{(2k+3)}{8(k+2)} \cdot why$

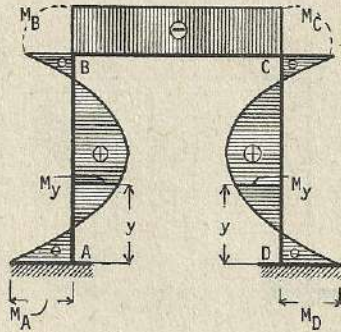
Viga BC : $M_x = \frac{k(6k+23)}{24(k+2)(6k+1)} \cdot wh^2 - \frac{k}{(6k+1)L} \cdot wh^2 x$



(211)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = 0$$

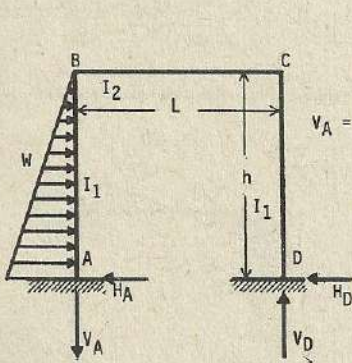


$$H_A = H_D = \frac{(2k+5)}{4(k+2)} \cdot wh$$

$$M_A = M_D = -\frac{(k+3)}{12(k+2)} \cdot wh^2 ; \quad M_B = M_C = -\frac{k}{12(k+2)} \cdot wh^2$$

$$\text{Esfuerzo de flexión en la columna : } M_y = -\frac{(k+3)}{12(k+2)} \cdot wh^2 + \frac{(2k+5)}{4(k+2)} \cdot why - \frac{wy^2}{2}$$

$$\text{Esfuerzo de flexión en la viga : } M_x = -\frac{k}{12(k+2)} \cdot wh^2$$

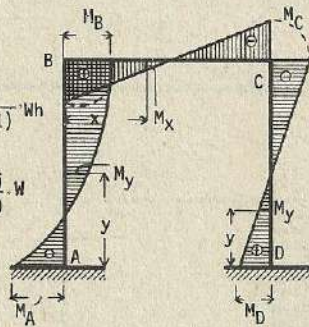


(212)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{k}{2(6k+1)} \cdot Wh$$

$$H_A = \frac{(17k+36)}{20(k+2)} \cdot W$$



$$H_D = \frac{(3k+4)}{20(k+2)} \cdot W$$

$$M_A = -\frac{(63k^2+151k+28)}{60(k+2)(6k+1)} \cdot Wh ; \quad M_D = \frac{(27k^2+49k+12)}{60(k+2)(6k+1)} \cdot Wh ; \quad M_B = \frac{k(3k+28)}{60(k+2)(6k+1)} \cdot Wh$$

$$M_C = -\frac{k(27k+32)}{60(k+2)(6k+1)} \cdot Wh$$

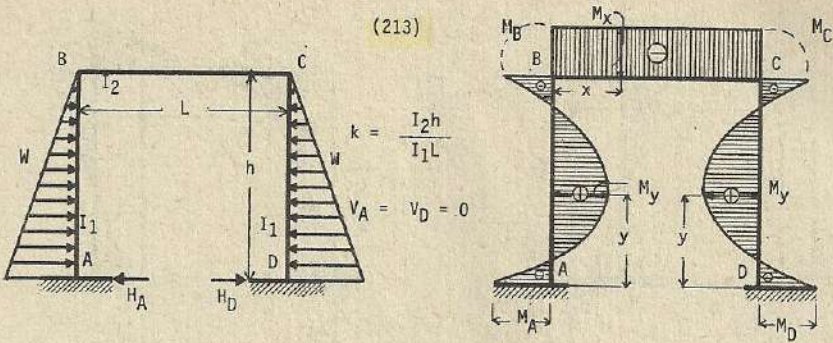
Momento por flexión en la columna AB :

$$M_y = -\frac{(63k^2+151k+28)}{60(k+2)(6k+1)} \cdot Wh + \frac{(17k+36)}{20(k+2)} \cdot Wy - \frac{Wy^2}{3h^2} \cdot (3h-y)$$

$$\text{Columna CD : } M_y = \frac{(27k^2+49k+12)}{60(k+2)(6k+1)} \cdot Wh - \frac{(3k+4)}{20(k+2)} \cdot Wy$$

$$\text{Viga BC : } M_x = \frac{k(3k+28)}{60(k+2)(6k+1)} \cdot Wh - \frac{k}{2(6k+1)} \cdot Whx$$

(213)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = 0$$

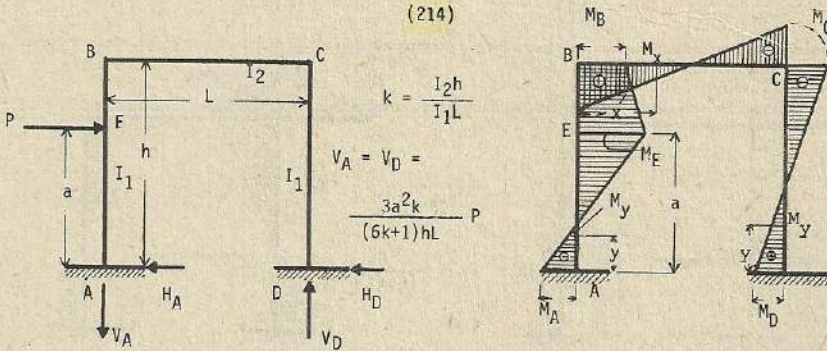
$$H_A = H_D = \frac{(7k+16)}{10(k+2)} \cdot w ; \quad M_A = M_D = - \frac{(3k+9)}{30(k+2)} \cdot wh ; \quad M_B = M_C = - \frac{k}{15(k+2)} \cdot wh$$

Esfuerzo de momento por flexión en la columna :

$$M_y = - \frac{(3k+8)}{30(k+2)} \cdot wh + \frac{(7k+16)}{10(k+2)} \cdot wy - \frac{wy^2}{3h^2} \cdot (3h-y)$$

En la viga : $M_x = - \frac{k}{15(k+2)} \cdot wh$

(214)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D =$$

$$\frac{3a^2 k}{(6k+1)hL} P$$

$$H_A = \frac{2(k+2)h^3 - a^2[3h(k+1) - a(2k+1)]}{2(k+2)h^3} \cdot P ; \quad H_D = \frac{a^2[3k(k+1) - a(2k+1)]}{2(k+2)h^3} \cdot P$$

$$M_A = - Pa + \frac{[(15k^2+26k+3)h - a(k+1)(6k+1)] \cdot Pa^2}{2(k+2)(6k+1)h^2}$$

$$M_D = \frac{[(9k^2+14k+3)h - a(k+1)(6k+1)]}{2(k+2)(6k+1)h^2} Pa^2 ; \quad M_B = \frac{k[a(6k+1) - h(3k-5)]}{2(k+2)(6k+1)h^2} \cdot Pa^2$$

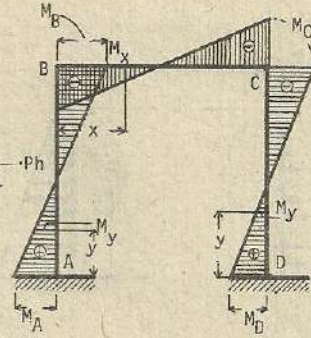
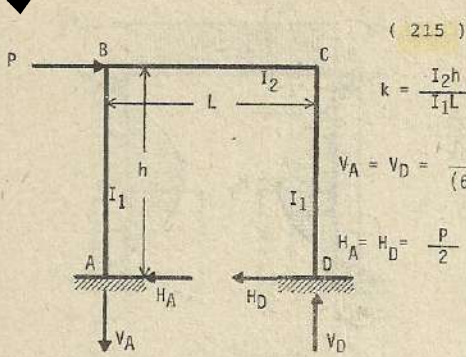
$$M_C = - \frac{k[(9k+7)h - (6k+1)a]}{2(k+2)(6k+1)h^2} Pa^2 ; \quad M_E = M_A + H_A a$$

Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y$ $y < a$

$M_y = M_A + H_A y - P(y-a)$ $y > a$

columna CD : $M_y = M_D - H_D y$

viga BC : $M_x = M_B - V_A x$



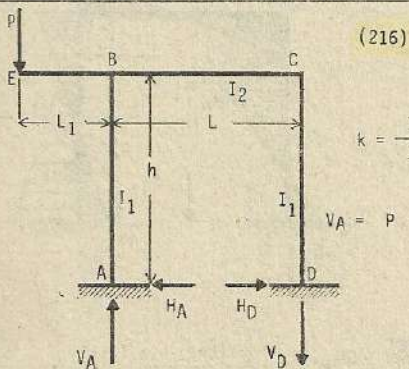
$$M_A = -\frac{(3k+1)}{2(6k+1)} \cdot Ph ; \quad M_D = \frac{(3k+1)}{2(6k+1)} \cdot Ph ; \quad M_B = \frac{3k}{2(6k+1)} \cdot Ph ; \quad M_C = -\frac{3k}{2(6k+1)} \cdot Ph$$

Esfuerzo de momento por flexión en la columna AB :

$$M_y = -\frac{(3k+1)}{2(6k+1)} \cdot Ph + \frac{P}{2} y$$

columna CD : $M_y = \frac{(3k+1)}{2(6k+1)} \cdot Ph - \frac{P}{2} y$

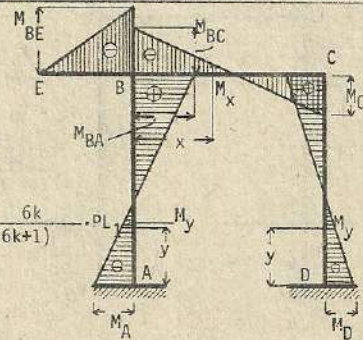
Viga BC : $M_x = \frac{3k}{2(6k+1)} \cdot Ph - \frac{3k}{(6k+1)L} \cdot Phx$



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = P + \frac{6k}{(6k+1)} \cdot PL_1$$

$$V_D = \frac{6k}{(6k+1)} \cdot PL_1 ; \quad H_A = H_D = \frac{3}{2(k+2)h} \cdot PL_1$$



$$M_A = -\frac{(5k-1)}{2(k+2)(6k+1)} \cdot PL_1 ; \quad M_D = \frac{7k+3}{2(k+2)(6k+1)} \cdot PL_1 ; \quad M_{BE} = -PL_1$$

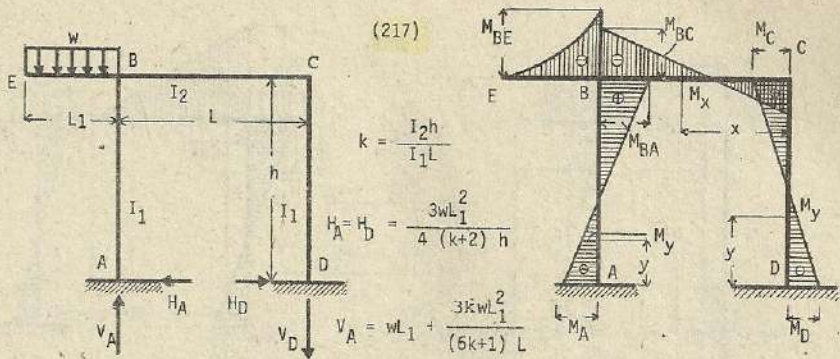
$$M_{BA} = \frac{(13k+4)}{2(k+2)(6k+1)} \cdot PL_1 ; \quad M_{BC} = \frac{k(12k+13)}{2(k+2)(6k+1)} \cdot PL_1 ; \quad M_C = \frac{11k}{2(k+2)(6k+1)} \cdot PL_1$$

Esfuerzo de flexión en la columna AB :

$$M_y = -\frac{5k-1}{2(k+2)(6k+1)} \cdot PL_1 + \frac{3}{2(k+2)h} \cdot PL_1 y$$

columna DC : $M_y = -\frac{7k+3}{2(k+2)(6k+1)} \cdot PL_1 + \frac{3}{2(k+2)h} \cdot PL_1 y$

Viga BC : $M_x = -\frac{k(12k+13)}{2(k+2)(6k+1)} \cdot PL_1 - Px - \frac{6k}{(6k+1)L} \cdot PL_1 x$



$$V_D = \frac{3kwL_1^2}{(6k+1)L} ; M_A = -\frac{(5k-1)wL_1^2}{4(6k+1)(k+2)}$$

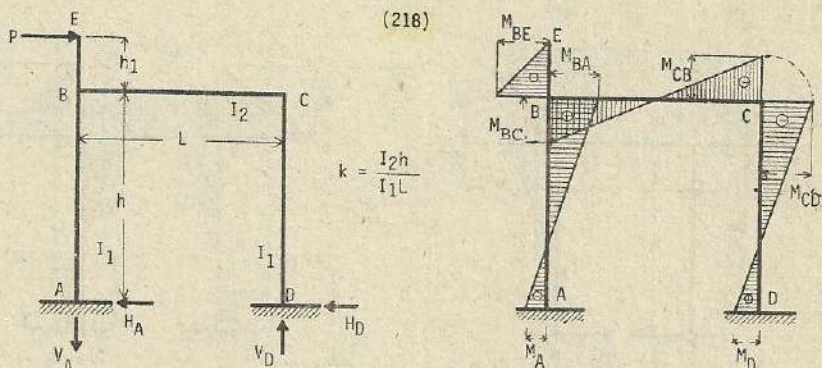
$$M_D = -\frac{(7k+3)wL_1^2}{4(6k+1)(k+2)} ; M_{BE} = -\frac{wL_1}{2} ; M_{BA} = \frac{(13k+4)wL_1^2}{4(6k+1)(k+2)}$$

$$M_{BC} = -\frac{k(12k+13)wL_1^2}{4(6k+1)(k+2)} ; M_C = \frac{11kwL_1^2}{4(6k+1)(k+2)}$$

Esfuerzo de flexión en la columna AB : $M_y = -\frac{(5k-1)wL_1^2}{4(6k+1)(k+2)} + \frac{3wL_1^2}{4(k+2)h} \cdot y$

Columna CD : $M_y = -\frac{(7k+3)wL_1^2}{4(6k+1)(k+2)} + \frac{3wL_1^2}{4(k+2)h} \cdot y$

Viga BC : $M_x = \frac{11kwL_1^2}{4(6k+1)(k+2)} - \frac{3kwL_1^2}{(6k+1)L} \cdot x$

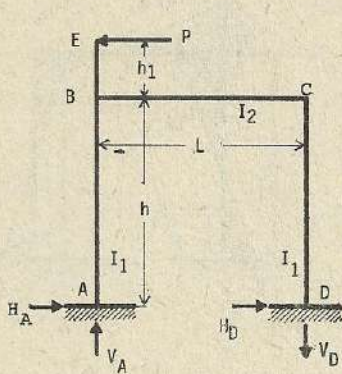


$$V_A = V_D = \frac{P(h+h_1)}{L} + \frac{M_{AB} - M_{DC}}{L} ; H_A = \frac{M_{BA} - M_{BC}}{h} ; H_D = \frac{M_{DC} - M_{CD}}{h}$$

$$M_{AB} = -\frac{P}{2} \cdot \frac{h_1(1-5k) + h(3k^2+7k+2)}{(k+2)(6k+1)} ; M_{BC} = \frac{Pk}{2} \cdot \frac{h_1(12k+13) + 3h(k+2)}{(k+2)(6k+1)}$$

$$M_{BA} = M_{BC} - Ph_1 ; M_{CD} = M_{CB} = -\frac{Pk}{2} \cdot \frac{11h_1 + 3h(k+2)}{(k+2)(6k+1)}$$

$$M_{DC} = \frac{P}{2} \cdot \frac{h_1(7k+3) + h(3k^2+7k+2)}{(k+2)(6k+1)}$$

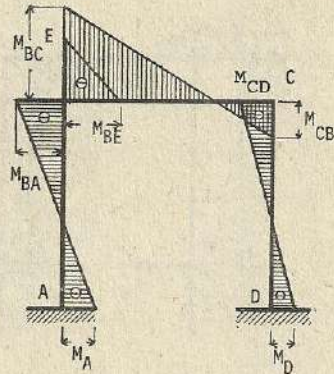


(219)

$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = \frac{M_{BA} - M_{AB}}{h}$$

$$H_D = \frac{M_{CD} - M_{DC}}{h}$$



$$V_A = V_D = \frac{P(h+h_1)}{L} + \frac{M_{AB} - M_{DC}}{L}$$

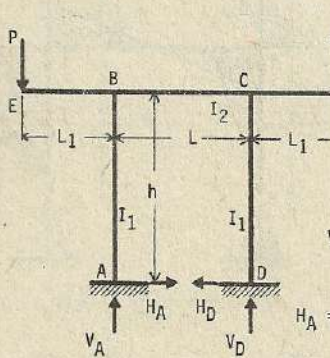
$$M_{AB} = \frac{P}{2} \cdot \frac{h_1(1-5k) + h(3k^2+7k+2)}{(k+2)(6k+1)}$$

$$; M_{BA} = M_{BC} - Ph_1$$

$$M_{BC} = -\frac{Pk}{2} \cdot \frac{h_1(12k+13) + 3h(k+2)}{(k+2)(6k+1)}$$

$$; M_{CD} = M_{CB} = \frac{Pk}{2} \cdot \frac{11h_1 + 3h(k+2)}{(k+2)(6k+1)}$$

$$M_{DC} = -\frac{P}{2} \cdot \frac{h_1(7k+3) + h(3k^2+7k+2)}{(k+2)(6k+1)}$$

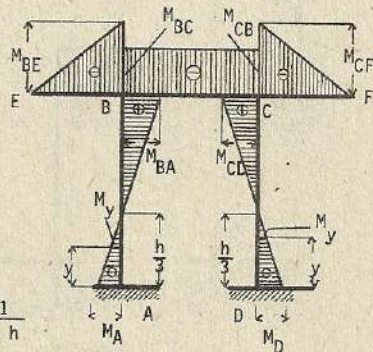


(220)

$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = P$$

$$H_A = H_D = \frac{3PL_1}{(k+2)h}$$



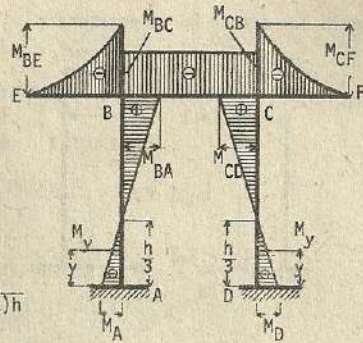
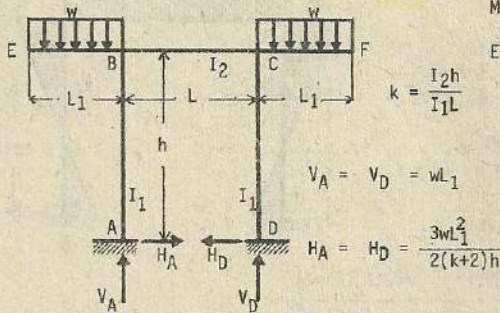
$$M_A = M_D = -\frac{PL_1}{(k+2)} ; M_{BE} = M_{CF} = PL_1 ; M_{BA} = M_{CD} = \frac{2PL_1}{(k+2)}$$

$$M_{BC} = M_{CB} = -\frac{kPL_1}{(k+2)}$$

Esfuerzo de flexión en la columna : $M_y = -\frac{PL_1}{(k+2)} + \frac{3PL_1}{(k+2)h} \cdot y$

En la viga : $M_x = -\frac{kPL_1}{(k+2)}$ (constante)

(221)



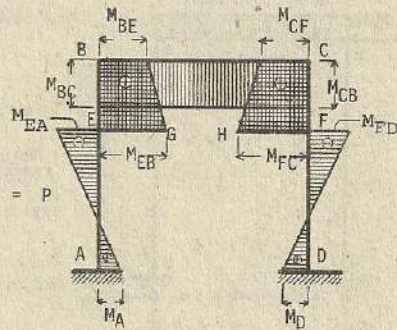
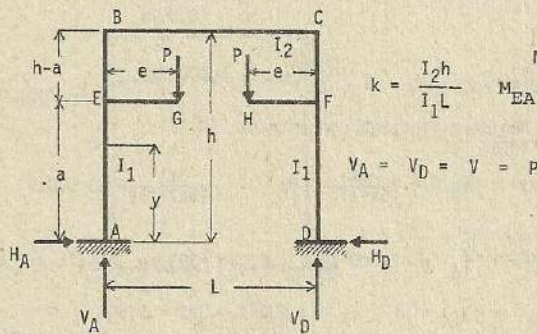
$$M_A = M_D = -\frac{wL_1^2}{2(k+2)} ; M_{BE} = M_{CF} = -\frac{wL_1^2}{2} ; M_{BA} = M_{CD} = \frac{wL_1^2}{k+2}$$

$$M_{BC} = M_{CB} = -\frac{kwL_1^2}{2(k+2)}$$

Esfuerzo de flexión en la columna : $M_y = -\frac{wL_1^2}{2(k+2)} + \frac{3wL_1^2}{2(k+2)h} \cdot y$

en la viga : $M_x = -\frac{kwL_1^2}{2(k+2)}$ (constante)

(222)



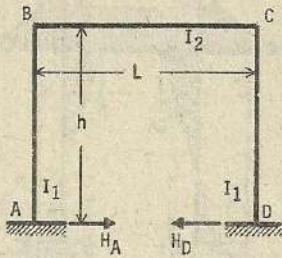
$$H_A = H_D = H = \frac{3Pe}{h^3} \cdot \frac{2ak(h-a) + a(2h-a)}{k+2} ; M_{BC} = M_{BA} = M_{CB} = M_{CD} = -Hh + M_A + Pe$$

$$M_{AB} = M_{DC} = \frac{Pe}{h^2} \cdot \frac{k(4ah - 3a^2 - h^2) + (6ah - 2h^2 - 3a^2)}{k+2}$$

Esfuerzo de flexión en la columna AE : $M_y = M_A - Hy$

columna EB : $M_y = M_A + Pe - Hy$

Esfuerzo por cambio de temperatura : (223)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = 0$$

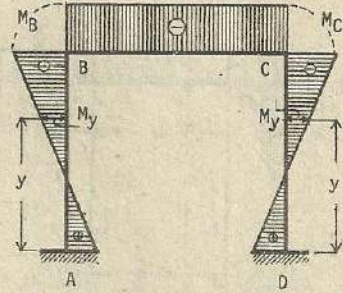
$$H_A = H_D = \frac{3E\epsilon t I_2 (2k+1)}{k(k+2)h^2}$$

$$M_A = M_D = \frac{3E\epsilon t I_2 (k+1)}{k(k+2)h}$$

$$M_B = M_C = -\frac{3E\epsilon t I_2}{(k+2)h}$$

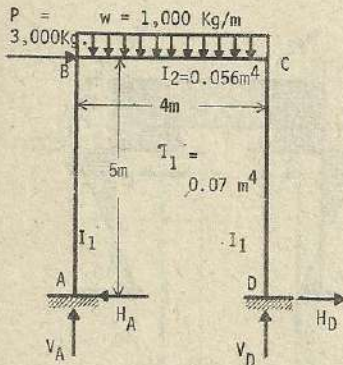
Esfuerzo de flexión en la columna : $M_y = \frac{3E\epsilon t I_2}{k(k+2)h} \cdot \left[(k+1) - (2k+1) \cdot \frac{y}{h} \right]$

en la viga : $M_x = -\frac{3E\epsilon t I_2}{(k+2)h}$ (constante)



Nota : El presente problema corresponde al caso de aumento de temperatura, en caso de descenso, los esfuerzos se anotan con los signos contrarios.

Solución numérica :



$$k = \frac{I_2 h}{I_1 L} = \frac{0.056 \times 5}{0.07 \times 4} = 1$$

Debido a la carga uniformemente distribuida (C.U.D)

$$V'_A = \frac{wL}{2} = \frac{1,000 \times 4}{2} = 2,000 \text{ Kg}$$

Debido a la carga concentrada (C.C.) (215)

$$V''_A = \frac{3k}{(6k+1)L} \cdot Ph = -\frac{3 \times 1}{(6 \times 1 + 1) \times 4} \times 3,000 \times 5$$

$$V_A = 2,000 \text{ Kg} - 1,607 \text{ Kg} = 393 \text{ Kg. (hacia arriba)}$$

$$V_D = wL - V_A = 1,000 \times 4 - 393 = 3,607 \text{ (hacia arriba)}$$

Debido a C.U.D (206) : $H'_A = \frac{wL^2}{4(k+2)h} = \frac{1,000 \times 4 \times 4}{4(1+2) \times 5} = 267 \text{ Kg (hacia la derecha)}$

Debido a C.C. (215) : $H''_A = -\frac{P}{2} = -\frac{3,000}{2} = -1,500 \text{ Kg (hacia la izquierda)}$

$$H_A = 267 - 1,500 = -1,233 \text{ Kg (hacia la izquierda)}$$

$$H_D = P - H_A = 3,000 - 1,233 = 1,767 \text{ Kg (hacia la derecha +)}$$

Debido a C.U.D. (206) : $M' = \frac{wL^2}{12(k+2)} = \frac{1,000 \times 4 \times 4}{12 \times 3} = 444 \text{ Kg-m}$

Debido a la C.C. : $M_A^H = - \frac{(3k+1)}{2(6k+1)} Ph = - \frac{(3 \times 1 + 1)}{2(6 \times 1 + 2)} \times 3,000 \times 5 = - 4,286 \text{ Kg-m}$

$M_A = 444 - 4,286 = -2,842 \text{ Kg-m}$

$M_D = \frac{(3k+1)}{2(6k+1)} Ph + \frac{wL^2}{12(k+2)} = 4,286 + 444 = 4,730 \text{ Kg-m}$

Debido a la C.U.D. : $M_B^U = - \frac{1}{6(k+2)} wL^2 = - \frac{1,000 \times 4 \times 4}{6(1+2)} = - 889 \text{ Kg-m}$

Debido a la C.C. : $M_B^H = \frac{3k}{2(6k+1)} Ph = \frac{3}{2(6 \times 1 + 1)} \times 3,000 \times 5 = 3,214 \text{ Kg-m}$

$M_B = - 889 + 3,214 = 2,325 \text{ Kg-m}$

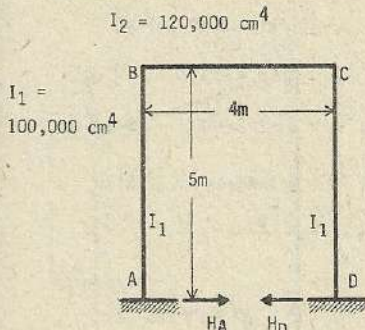
$M_C = - \frac{1}{6(k+2)} \cdot wL^2 - \frac{3k}{2(6k+1)} \cdot Ph = - 889 - 3,214 = - 4,103 \text{ Kg-m}$

Considerando como (x_m) la distancia entre (B) y el punto donde la cortante es cero :

$V_A = wx_m \longrightarrow 393 = 1,000x_m \quad \therefore x_m = 0.393 \text{ m}$

$M_{\text{máx.}} = M_B + V_A x_m - w \frac{x_m^2}{2}$

$2,325 + 393 \times 0.393 - 1,000 \times \frac{0.393 \times 0.393}{2} = 2,402 \text{ Kg-m}$



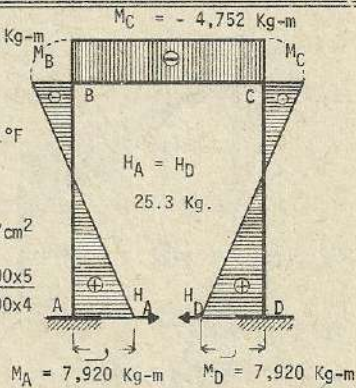
$\epsilon = 0.0000055/1^\circ\text{F}$

$t = 30^\circ$

$E = 140,000 \text{ Kg/cm}^2$

$k = \frac{I_2 h}{I_1 L} = \frac{120,000 \times 5}{100,000 \times 4}$

$= 1.5$



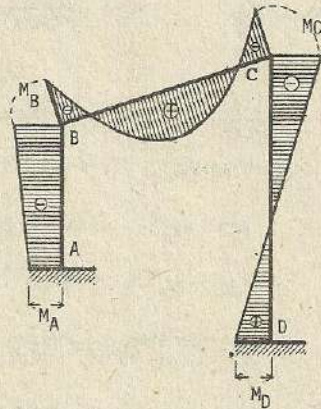
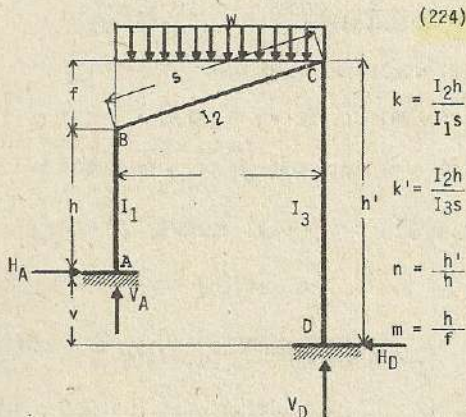
$M_A = M_D = \frac{3E\epsilon t I_2 (k+1)}{k(k+2)h}$

$= \frac{2 \times 140,000 \times 0.0000055 \times 30 \times 120,000 \times (1.5+1)}{1.5(1.5+2) \times 500} = 792,000 \text{ Kg-cm} = 7,920 \text{ Kg-m}$

$M_B = M_C = - \frac{3E\epsilon t I_2}{(k+2)h} = - M_A \times \frac{k}{1+k} = - 7,920 \times \frac{1.5}{1+1.5} = - 4,752 \text{ Kg-m}$

$H_A = H_D = \frac{3E\epsilon t I_2 (2k+1)}{k(k+2)h^2} = M_B \times \frac{2k+1}{kh} = 4,752 \times \frac{2 \times 1.5 + 1}{1.5 \times 500} = 25.3 \text{ Kg}$

Nota : En caso de descenso de temperatura, se modifican los signos.



$$N = (1+k+k'+3kk') (k+n^2k') + 3kk' (1+n+n^2)$$

$$\alpha_1 = 2k + 2n^2k' (1+k') + (8+3n) k'k ; \quad \alpha_2 = k [k + 2n (3+2n) k']$$

$$\alpha_3 = 3k [1 + 2 (2+n) k'] ; \quad \beta_1 = k' [n^2k' + 2 (2+3n) k]$$

$$\beta_2 = 2n^2k' + 2k (1+k) + n (3+8n) kk' ; \quad \beta_3 = 3k' [n + 2 (1+2n) k]$$

$$\gamma_1 = nkk' (2+4n+3nk') ; \quad \gamma_2 = nkk' (4+2n+3k) ; \quad \gamma_3 = k [2+2k+6kk' + (8+3n) k']$$

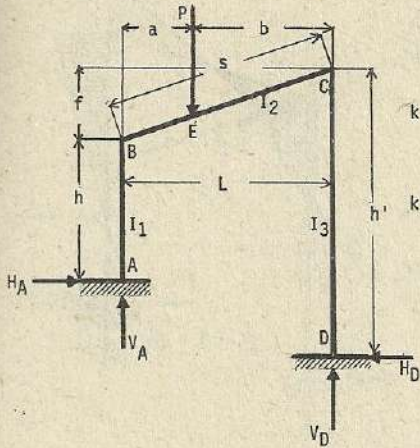
$$\delta_1 = k (2k+6kk' - nk') ; \quad \delta_2 = k (2nk'+6kk'-k) ; \quad \delta_3 = k(1+4k+4k' + 12kk')$$

$$H_A = H_D = H = \frac{wL^2}{24Nh} (\alpha_3 + \beta_3) ; \quad V_A = \frac{wL}{2} + \frac{M_C - M_B}{L} + \frac{Hf}{L}$$

$$V_D = \frac{wL}{2} - \frac{M_C - M_B}{L} - \frac{Hf}{L} ; \quad M_B = -\frac{wL^2}{24N} (\alpha_1 + \beta_1) ; \quad M_A = M_B + Hh$$

$$M_C = -\frac{wL^2}{24} (\alpha_2 + \beta_2) ; \quad M_D = M_C + Hh'$$

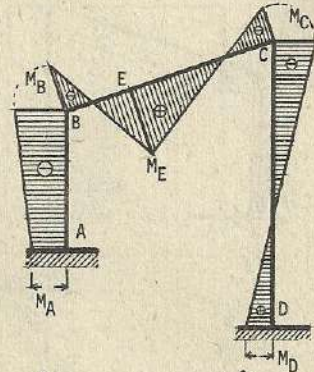
Nota : Los datos deducidos son también utilizables para los problemas que siguen.



(225)

$$k = \frac{I_2 h}{I_1 s}$$

$$k' = \frac{I_2 h'}{I_3 s}$$



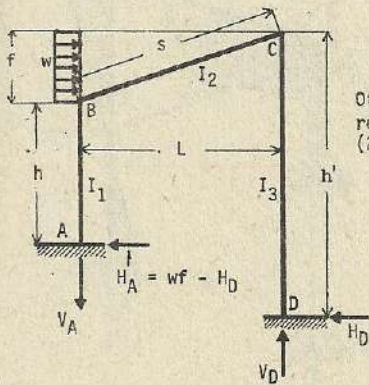
$$C_{BC} = -\frac{Pab^2}{L^2} ; C_{CB} = -\frac{Pa^2b}{L^2}$$

$$M_B = \frac{1}{2N} (\alpha_1 C_{BC} + \beta_1 C_{CB}) ; M_A = M_B + Hh ; M_C = \frac{1}{2N} (\alpha_2 C_{BC} + \beta_2 C_{CB})$$

$$M_D = M_C + Hh' ; H_A = H_D = H = -\frac{1}{2Nh} (\alpha_3 C_{BC} + \beta_3 C_{CB})$$

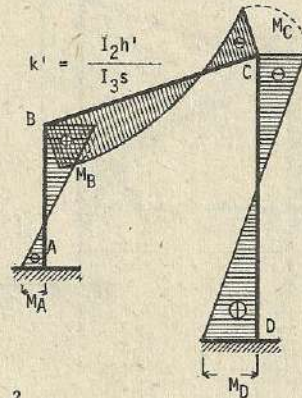
$$V_A = \frac{1}{L} (Pb - M_B + M_C + Hf) ; V_D = P - V_A$$

(226)



$$k = \frac{I_2 h}{I_1 s}$$

Otros datos,
referencia
(224)



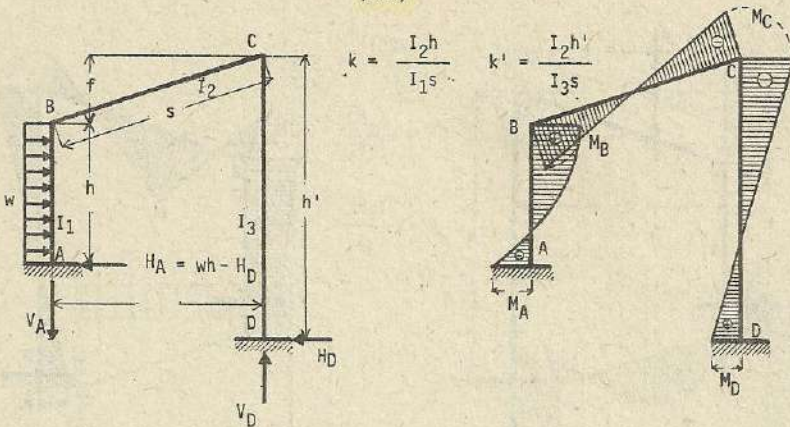
$$M_B = -\frac{wf^2}{24N} [12m\gamma_1 - (\alpha_1 + \beta_1)]$$

$$M_C = -\frac{wf^2}{24N} [12m\gamma_2 + (\alpha_2 + \beta_2)] ; H_D = \frac{wf^2}{12Nh} [12m\gamma_3 + (\alpha_3 + \beta_3)] ; H = wf - H_D$$

$$M_A = M_B - (wf - H_D)h ; M_D = M_C + H_D h'$$

$$V_A = V_D = V = \frac{1}{L} (w \cdot \frac{f^2}{2} + M_B - M_C - H_D f)$$

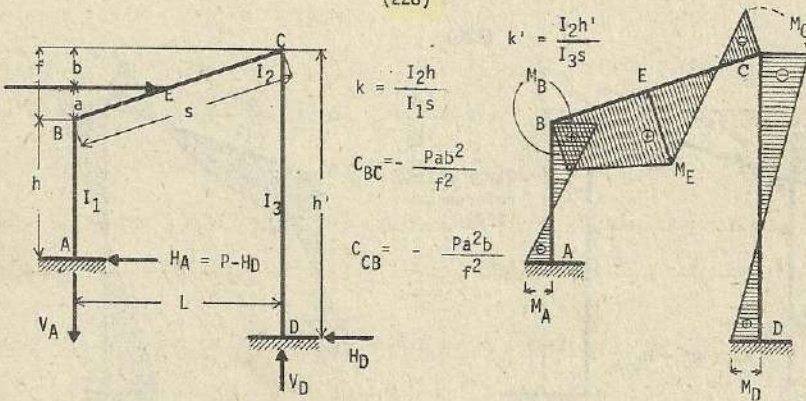
(227)



$$M_B = \frac{wh^2}{24N} (4 \gamma_1 - \delta_1) \quad ; \quad M_A = M_B + H_D h - \frac{wh^2}{2} \quad ; \quad M_C = -\frac{wh^2}{24N} (4 \gamma_2 + \delta_2)$$

$$M_D = M_C + H_D h' \quad ; \quad H_D = -\frac{wh}{24N} (4 \gamma_3 + \delta_3) \quad ; \quad V_A = V_D = V = \frac{1}{L} (M_B - M_C - H_D f)$$

(228)

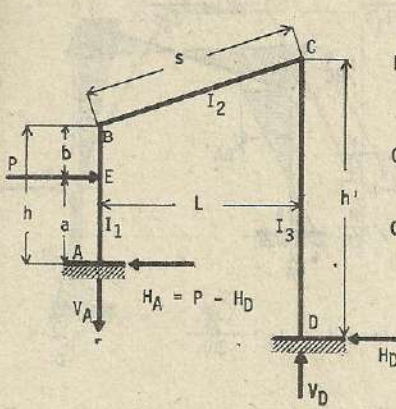


$$M_B = \frac{1}{2N} (\alpha_1 C_{BC} + \beta_1 C_{CB} + \gamma_1 Ph)$$

$$M_C = \frac{1}{2N} (\alpha_2 C_{BC} + \beta_2 C_{CB} - \gamma_2 Ph) \quad H_D = \frac{1}{2Nh} (\alpha_3 C_{BC} - \beta_3 C_{CB} + \gamma_3 Ph)$$

$$H_A = P - H_D \quad ; \quad M_A = M_B - (P - H_D)h$$

$$M_D = M_C + H_D h' \quad ; \quad V_A = V_D = V = \frac{1}{L} (M_B - M_C + Pa - H_D f)$$

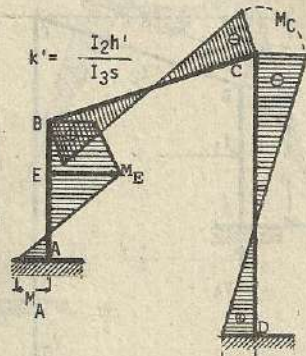


(229)

$$k = \frac{I_2 h}{I_1 s}$$

$$C_{AB} = -\frac{P a b^2}{h^2}$$

$$C_{BA} = -\frac{P a^2 b}{h^2}$$

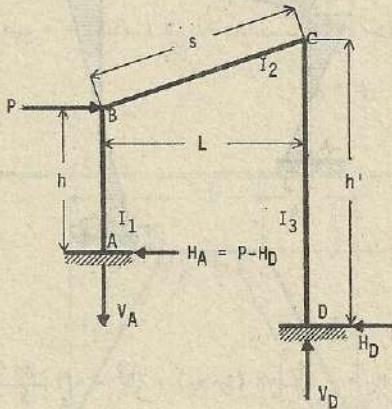


$$M_B = \frac{1}{2N} [\gamma_1 (Pa + C_{AB} + C_{BA}) - \delta_1 C_{BA}]$$

$$M_C = -\frac{1}{2N} [\gamma_2 (Pa + C_{AB} + C_{BA}) - \delta_2 C_{BA}] \quad ; \quad H_D = \frac{1}{2Nh} [\gamma_3 (Pa + C_{AB} + C_{BA}) - \delta_3 C_{BA}]$$

$$H_A = P - H_D \quad ; \quad M_A = M_B + H_D h - Pa \quad ; \quad M_D = M_C + H_D h'$$

$$V_A = V_D = V = \frac{1}{L} \cdot (M_B - M_C - H_D f)$$



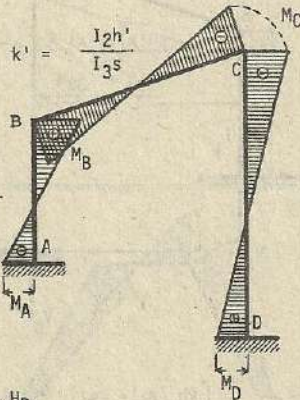
(230)

$$k = \frac{I_2 h}{I_1 s}$$

$$M_B = \gamma_1 \frac{Ph}{2N}$$

$$M_C = -\gamma_2 \cdot \frac{Ph}{2N}$$

$$H_D = \alpha_3 \frac{P}{2N}$$

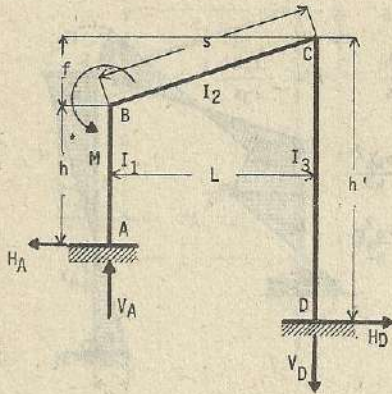


$$H_A = P - H_D$$

$$M_A = M_B - (P - H_D)h$$

$$M_D = M_C + H_D h'$$

$$V_A = V_D = V = \frac{1}{L} \cdot (M_B - M_C - H_D f)$$



$$(231) \quad k = \frac{I_2 h}{I_1 s}$$

$$k' = \frac{I_2 h'}{I_3 s}$$

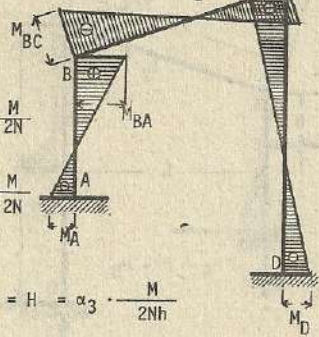
$$M_{BA} = \alpha_1 \frac{M}{2N}$$

$$M_C = \alpha_2 \frac{M}{2N}$$

$$H_A = H_D = H = \alpha_3 \frac{M}{2Nh}$$

$$M_A = M_{BA} - H_D h \quad ; \quad M_{BC} = M_{BA} - M$$

$$M_D = M_C - H_D h' \quad ; \quad V_A = V_D = \frac{1}{L} (M + M_C - M_B - H_D f)$$

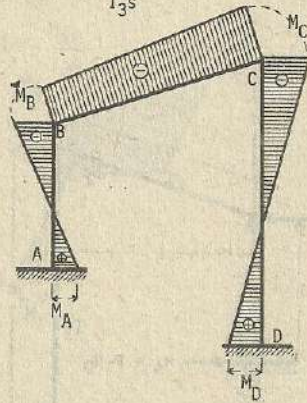
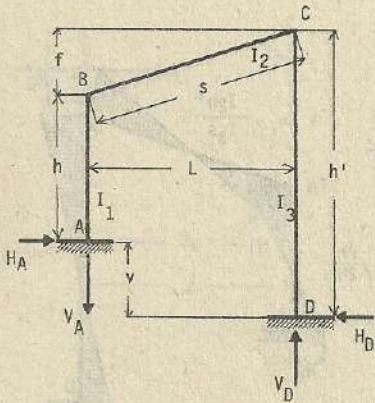


(232)

Esfuerzo por cambio de temperatura

$$k = \frac{I_2 h}{I_1 s}$$

$$k' = \frac{I_2 h'}{I_3 s}$$

Otros datos
referencia
(224)

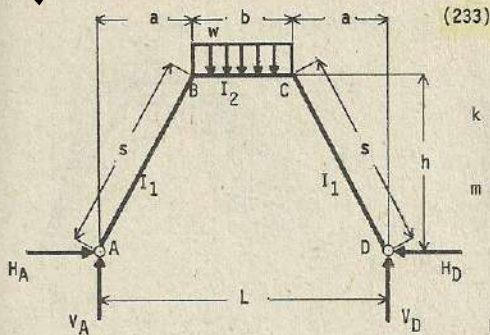
$$M_B = -\frac{T}{N} \left[v h (\beta_1 - \alpha_1) + (L^2 - v f) \frac{\delta_1}{k} \right]; \quad M_C = -\frac{T}{N} \left[v h (\beta_2 - \alpha_2) + (L^2 - v f) \frac{\delta_2}{k} \right]$$

$$H_A = H_D = H = \frac{T}{Nh} \left[v h (\beta_3 - \alpha_3) + (L^2 - v f) \frac{\delta_3}{k} \right]$$

$$V_A = V_D = V = \frac{M_B - M_C - H f}{L} \quad \text{Aquí } T = \frac{3EI_2 \epsilon t}{sLh}$$

Nota: El presente problema corresponde al incremento de temperatura.

En caso de descenso, los esfuerzos se anotan con los signos contrarios.



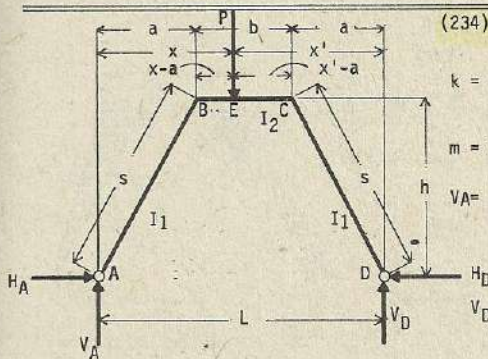
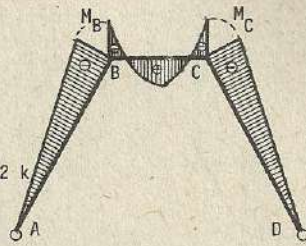
$$M_B = M_C = -\frac{wb^2}{4m}$$

$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

$$H_A = H_D = H = \frac{wb^2}{2h} \cdot \left(\frac{a}{b} + \frac{1}{2m} \right)$$

$$V_A = V_D = V = \frac{wb}{2}$$



$$k = \frac{I_2 s}{I_1 b}$$

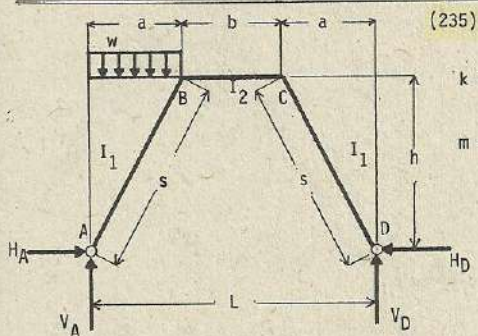
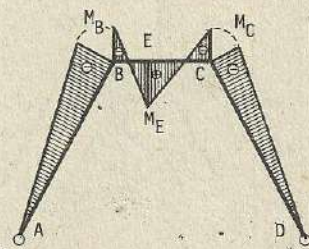
$$m = 3 + 2k$$

$$V_A = \frac{Px'}{L}$$

$$V_D = \frac{Px}{L} ; \quad M_B = V_A a - Hh ; \quad M_C = V_D a - Hh$$

$$H_{BC} = -\frac{P(x-a)(x'-a)}{2b^2} \cdot (b+x-a) ; \quad H_{CB} = -\frac{P(x-a)(x'-a)}{2b^2} (b+x-a)$$

$$H_A = H_D = H = \frac{Pa}{2h} - \frac{H_{BC} + H_{CB}}{mh}$$

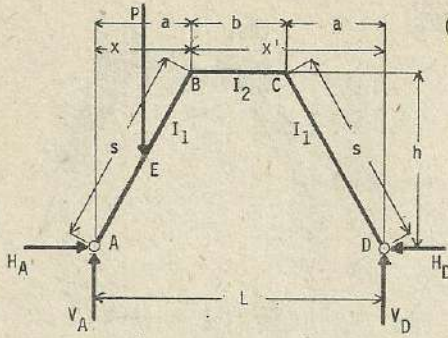


$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

$$V_A = \frac{wa(2L-a)}{2L} ; \quad V_D = \frac{wa^2}{2L}$$

$$H_A = H_D = H = \frac{wa}{4h} \cdot \left(1 + \frac{k}{2m} \right) ; \quad M_B = \frac{wa^2}{4} \cdot \left(\frac{b}{L} - \frac{k}{2m} \right) ; \quad M_C = \frac{wa^2}{4} \cdot \left(\frac{b}{L} - \frac{k}{2m} \right)$$



(236)

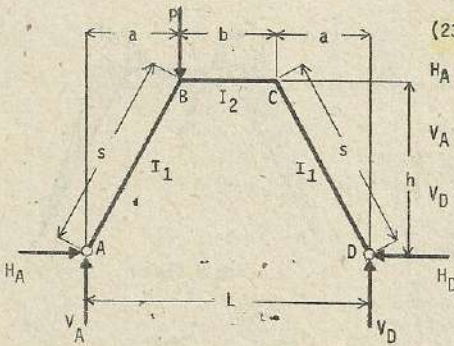
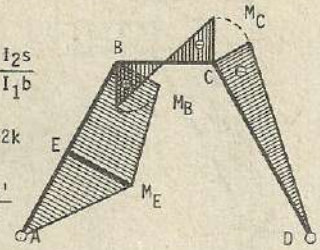
$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

$$V_A = \frac{Px'}{L}$$

$$V_D = \frac{Px}{L} ; \quad H_{BA} = - \frac{Px(a^2 - x^2)}{2a^2}$$

$$H_A = H_D = H = \frac{Px}{2h} - \frac{kH_{BA}}{mh} ; \quad M_B = \frac{Pbx}{2L} + \frac{kH_{BA}}{m} ; \quad M_C = - \frac{Pbx}{2L} + \frac{kH_{BA}}{m}$$

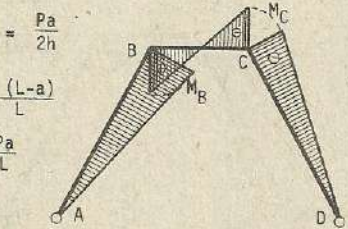


(237)

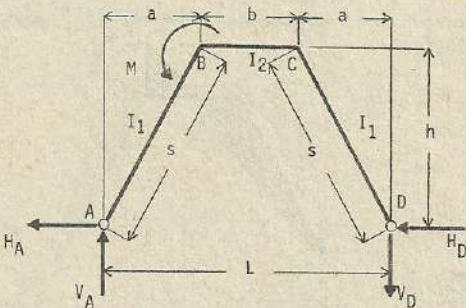
$$H_A = H_D = \frac{Pa}{2h}$$

$$V_A = \frac{P(L-a)}{L}$$

$$V_D = \frac{Pa}{L}$$



$$M_B = \frac{Pab}{2L} \quad ; \quad M_C = - \frac{Pab}{2L}$$



(238)

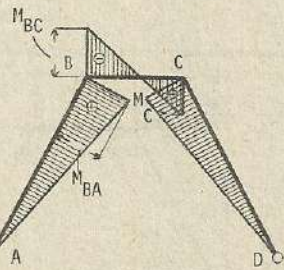
$$k = \frac{I_2 s}{I_1 b}$$

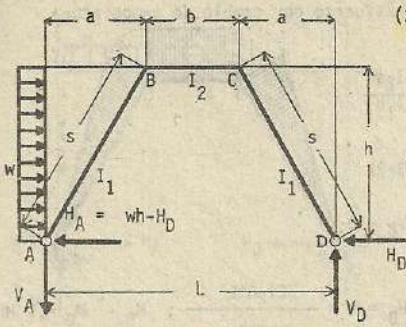
$$m = 3 + 2k$$

$$H_A = H_D = H = \frac{3M}{2mh}$$

$$M_{BA} = Hh + Va \quad ; \quad M_C = Hh - Va \quad ; \quad V_A = V_D = V = \frac{M}{L}$$

$$M_{BC} = M_{BA} - M$$





(239)

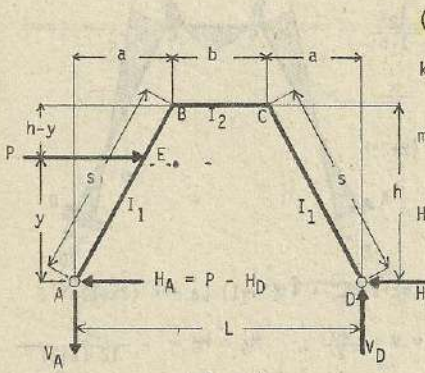
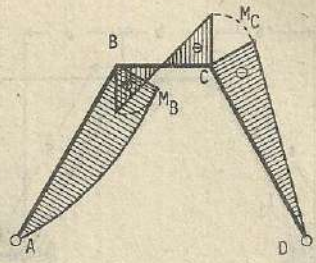
$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

$$H_D = \frac{wh}{4} \left(1 + \frac{k}{2m} \right) ; \quad V_A = V_D = V = \frac{mh^2}{2L}$$

$$M_B = \frac{wh^2}{4} \left(\frac{b}{L} - \frac{k}{2m} \right)$$

$$M_C = \frac{wh^2}{4} \left(-\frac{b}{L} - \frac{k}{2m} \right)$$



(240)

$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

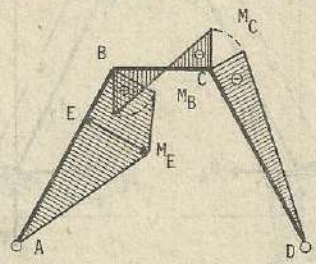
$$H_A = P - H_D$$

$$H_D = \frac{Py}{2h} - \frac{kH_{BA}}{mh} ; \quad V_A = V_D = V = \frac{Py}{L}$$

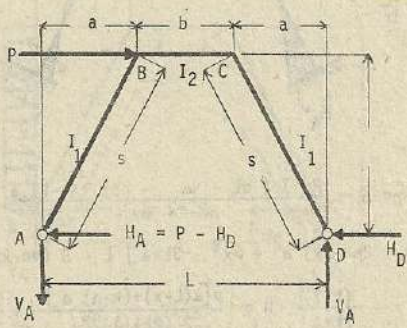
$$H_{BA} = -\frac{Py(h^2 - y^2)}{2h^2}$$

$$M_B = \frac{Pyb}{2L} + \frac{kH_{BA}}{m}$$

$$M_C = -\frac{Pyb}{2L} - \frac{kH_{BA}}{m}$$



(241)



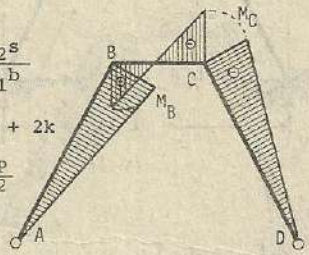
$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

$$H_D = \frac{P}{2}$$

$$V_A = V_D = V = \frac{Ph}{L}$$

$$M_B = -M_C = \frac{Pbh}{2L}$$



(242) Esfuerzo por cambio de temperatura

$k = \frac{I_2 s}{I_1 b}$
 $m = 3 + 2k$
 $V_A = V_D = 0$
 $H_A = H_D = H = \frac{3EI_2 \epsilon t L}{mh^2 b}$; $M_B = M_C = -Hh$

Nota: El presente problema corresponde al caso donde existe incremento de temperatura.

(243)

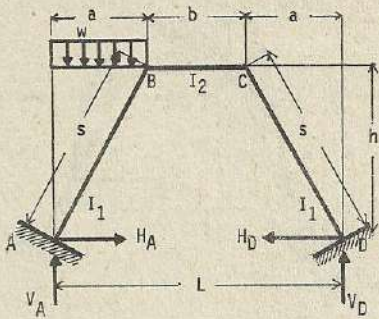
$k = \frac{I_2 s}{I_1 b}$
 $\alpha = \frac{x}{a}$
 $\beta = (k+2)h$
 $\gamma = (6k+1)L^2 - 4(3k+1)La + 4(2k+1)a^2$

$H_A = H_D = H = \frac{wb}{4} \cdot \frac{L + 2(k+1)a}{\beta}$; $V_A = V_D = v = \frac{wb}{2}$; $M_A = M_D = -\frac{wL^2}{12(k+2)}$

(244)

$k = \frac{I_2 s}{I_1 b}$
 $\alpha = \frac{x}{b}$
 $n = \frac{a}{b}$

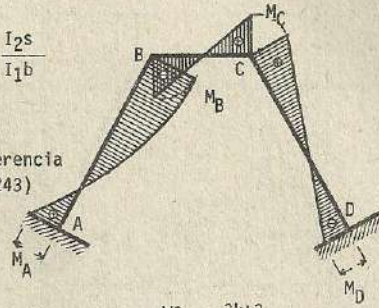
$H_A = H_D = H = \frac{P}{2} \cdot \frac{3(\alpha L - \alpha a - na) + (k-1)a}{\beta}$
 $v = \frac{P}{LY} \left[\alpha L^3 + 3[(k-n)a - \alpha x]L^2 + 2[(3\alpha + 2n - 2k)a^2 + \alpha x^2 - 3k\alpha a]L - 8(\alpha a - kx)a^2 \right]$
 $V_A = \frac{P(L-x)}{L} + v$; $V_D = \frac{Px}{L} - v$; $M_A = -\frac{(k+1)h}{2k+1} \cdot H + \frac{P[\alpha(L-x) + (k-n)a]}{2(2k+1)} + \frac{L}{2}v$
 $M_D = -M_A - vL$



(245)

$$k = \frac{I_2 s}{I_1 b}$$

referencia (243)

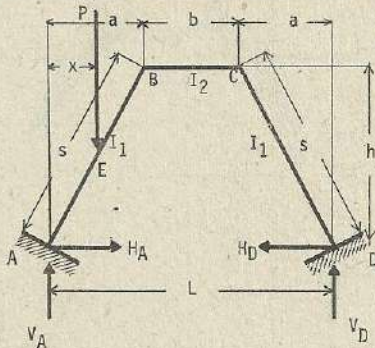


$$H_A = H_D = H = \frac{wa}{8} \cdot \frac{3k+3}{\beta}$$

$$V_A = \frac{wa^2}{2L} (2L-a) + v$$

$$v = \frac{wa^2}{2L} \cdot \frac{(4k+1)L^2 - (11k+4)La + 4(2k+1)a^2}{\gamma}$$

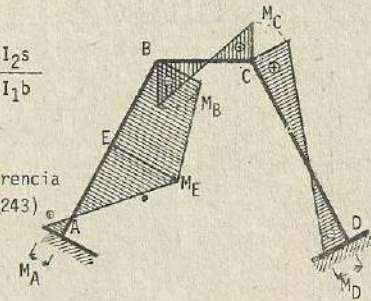
$$M_A = \frac{wa^2 (k+3)}{24 (k+2)} + \frac{L}{2} v; \quad M_D = M_A - vL; \quad V_D = \frac{wa^2}{2L} - v$$



(246)

$$k = \frac{I_2 s}{I_1 b}$$

referencia (243)

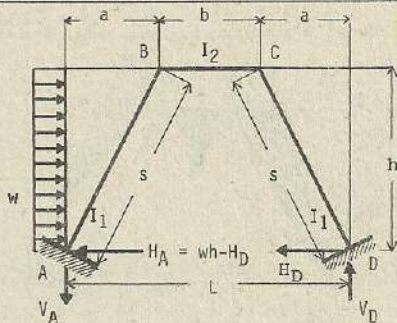


$$H_A = H_D = H = \frac{Px}{2} \cdot \frac{3\alpha (k+1) - \alpha^2 (2k+1)}{\beta}$$

$$V_A = \frac{P(L-x)}{L} + v$$

$$v = \frac{Px}{I_1} \cdot \frac{(6k-3\alpha+1)L^2 - 2(6k-k\alpha^2+2)La + 4(2k+1)a^2}{\gamma}$$

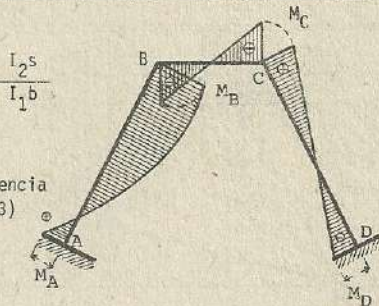
$$M_A = -\frac{(k+1)h}{2k+1} \cdot H + \frac{Px(2k-k\alpha-1)}{2(2k+1)} + \frac{I_1}{2} v; \quad M_D = M_A - vL; \quad V_D = \frac{Px}{L} - v$$



(247)

$$k = \frac{I_2 s}{I_1 b}$$

referencia (243)

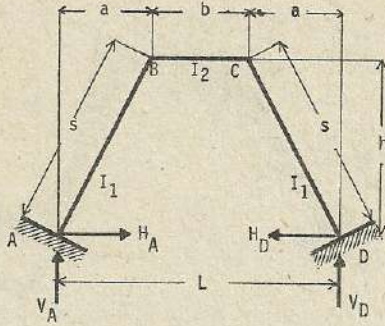


$$H_D = \frac{wh^2}{8} \cdot \frac{2k+3}{\beta};$$

$$V_A = V_D = V = \frac{wh^2}{2} \cdot \frac{k(2L-a)}{\gamma}$$

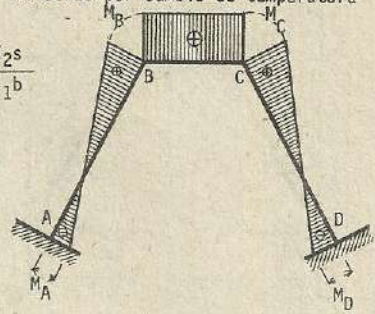
$$M_A = \frac{wh^2 (7k+15)}{24 (k+2)} - \frac{L}{2} V$$

$$M_D = -\frac{wh^2 (5k+9)}{24 (k+2)} + \frac{L}{2} V$$



(248) : Esfuerzo por cambio de temperatura

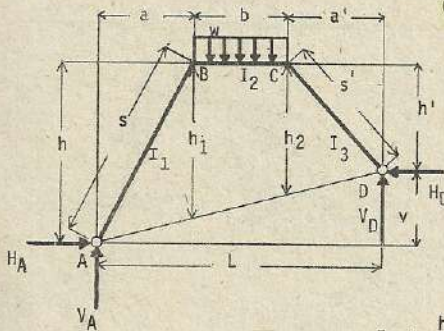
$$k = \frac{I_2 s}{I_1 b}$$



$$H_A = H_D = H = \frac{\pm 3(2k+1) E \alpha t I_2}{2(k+2) h^2}$$

$$M_A = M_D = \frac{\pm 3(k+1) E \alpha t I_2}{s(k+2) h}$$

Nota : El presente problema corresponde al incremento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.



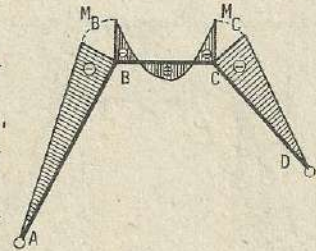
(249)

$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

$$\alpha_1 = \frac{a}{L}$$

$$\alpha_2 = \frac{a'}{L}$$



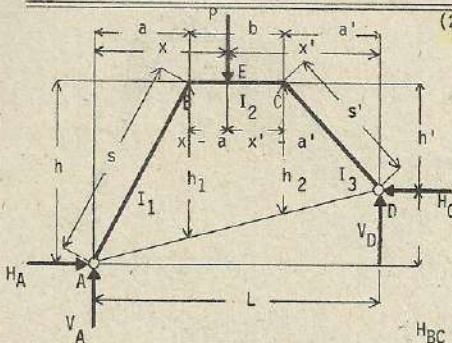
$$n = \frac{h'}{h} ; n_1 = \frac{h_1}{h} ; n_2 = \frac{h_2}{h} ; m = \frac{n_1}{n_2}$$

$$\phi_1 = n + 2m(1+k) ; \phi_2 = m + 2n(1+k') ; \lambda_1 = \phi_1(1-\alpha_1) + \phi_2 \alpha_2$$

$$N = \frac{nw}{2} (m\phi_1 + n\phi_2) = n_2 [m^2(1+k) + mn + n^2(1+k')] ; \lambda_2 = \phi_1 \alpha_1 + \phi_2(1-\alpha_2)$$

$$H_A = H_D = H = \frac{wb^2}{8nh} \left[2\alpha_1 \phi_1 \left(1 + 2\frac{a'}{b}\right) + 2\alpha_2 \phi_2 \left(1 + 2\frac{a}{b}\right) + m + n \right]$$

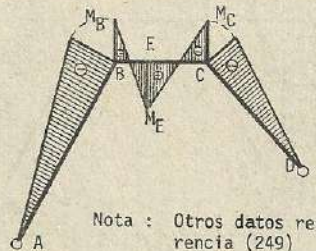
$$V_A = \frac{wb}{2} (1 - \alpha_1 - \alpha_2) + \frac{Hv}{L} ; V_D = wb - V_A ; M_B = V_A a - Hh ; M_C = V_D a' - Hh'$$



(250)

$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$



Nota : Otros datos referencia (249)

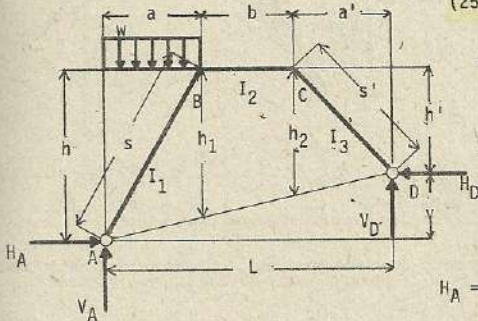
$$H_{BC} = - \frac{P(x-a)(x'-a')}{2b^2} (b+x'-a')$$

$$V_A = \frac{Px' + Hv}{L} ; V_D = \frac{Px - Hv}{L} ; H_{CB} = - \frac{P(x-a)(x'-a')}{2b^2} (b+x-a)$$

$$H_A = H_D = H = \frac{P(\alpha_1 \sin x' + \alpha_2 \sin 2x') - 2mH_{BC} - 2nH_{CB}}{2Nh}$$

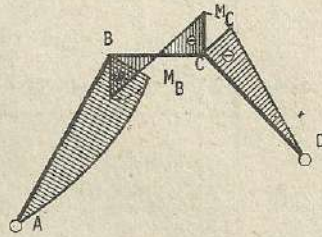
$$M_B = \alpha_1 Px' - Hh_1 \quad M_C = \alpha_2 Px - Hh_2$$

(251)



$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

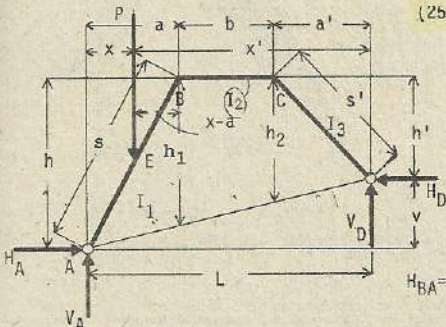


$$H_A = H_D = H = \frac{wa^2}{8Nh} \cdot (2\lambda_1 + mk)$$

$$V_A = \frac{wa}{2} \cdot (2 - \alpha_1) + \frac{Hv}{L} ; \quad V_D = \frac{wa}{2} \cdot \alpha_1 - \frac{Hv}{L} ; \quad M_B = \frac{wa^2}{2} \cdot (1 - \alpha_1) - Hh_1$$

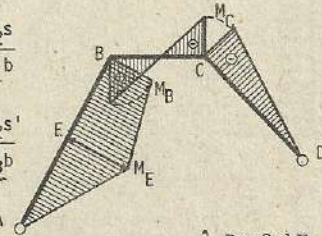
$$M_C = \frac{wa^2}{2} \cdot \alpha_2 - Hh_2$$

(252)



$$k = \frac{I_2 s}{I_1 b}$$

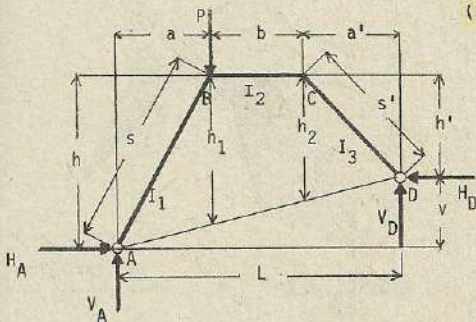
$$k' = \frac{I_2 s'}{I_3 b}$$



$$H_{BA} = -\frac{Px(a^2 - x^2)}{2a^2} ; \quad H_A = H_D = H = \frac{\lambda_1 Px - 2mkH_{BA}}{2Nh}$$

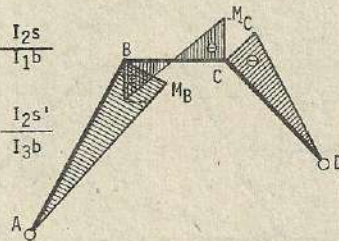
$$V_A = \frac{Px' + Hv}{L} ; \quad V_D = \frac{Px - Hv}{L} ; \quad M_B = (1 - \alpha_1)Px - Hh_1 ; \quad M_C = \alpha_2 Px - Hh_2$$

(253)



$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

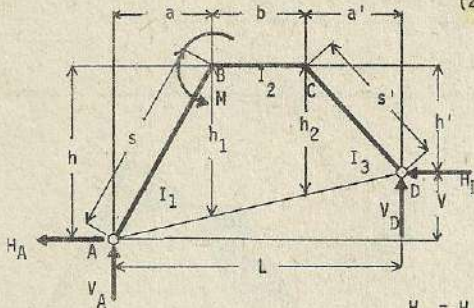


$$H_A = H_D = H = \frac{\lambda_1 Pa}{2Nh}$$

$$V_A = (1 - \alpha_1)P + \frac{Hv}{L} ; \quad V_D = \alpha_2 \cdot P - \frac{Hv}{L} ; \quad M_B = (1 - \alpha_2)Pa - Hh_1 ; \quad M_C = \alpha_2 Pa - Hh_2$$

Nota : Para otros datos se toma como referencia (249)

(254)



$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

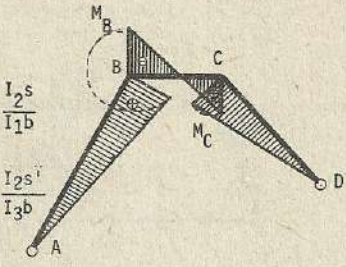
$$H_A = H_D = H = \frac{\lambda_1 P a}{2Nh}$$

$$V_A = (1 - \alpha_1) P + \frac{Hv}{L}$$

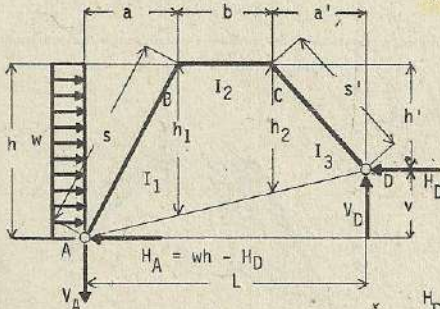
$$M_B = (1 - \alpha_1) Pa - Hh_1$$

$$M_C = \alpha_2 Pa - Hh_2$$

$$V_D = \alpha_2 P - \frac{Hv}{L}$$



(255)



$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

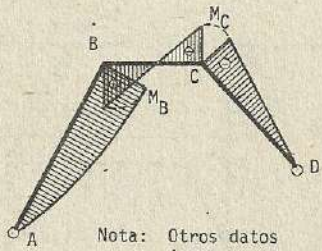
$$H_D = \frac{wh}{8h} \cdot (2\lambda_1 + mk)$$

$$H_A = wh - H_D$$

$$V_A = V_D = v = \frac{wh^2}{2L} - \frac{Hv}{L}$$

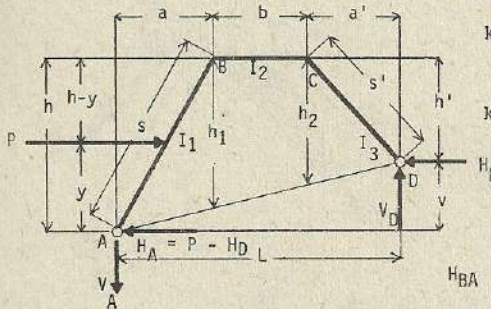
$$M_B = \frac{wh^2}{2} \cdot (1 - \alpha_1) - Hh_1$$

$$M_C = \frac{wh^2}{2} \cdot \alpha_2 - Hh_2$$



Nota: Otros datos (249)

(256)



$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

$$H_{BA} = - \frac{Py (h^2 - y^2)}{2h^2}$$

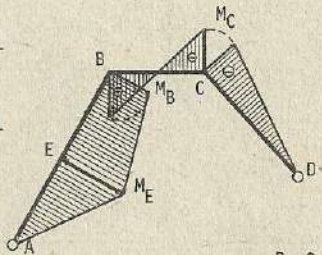
$$H_D = \frac{\lambda_1 Py - 2mkH_{BA}}{2Nh}$$

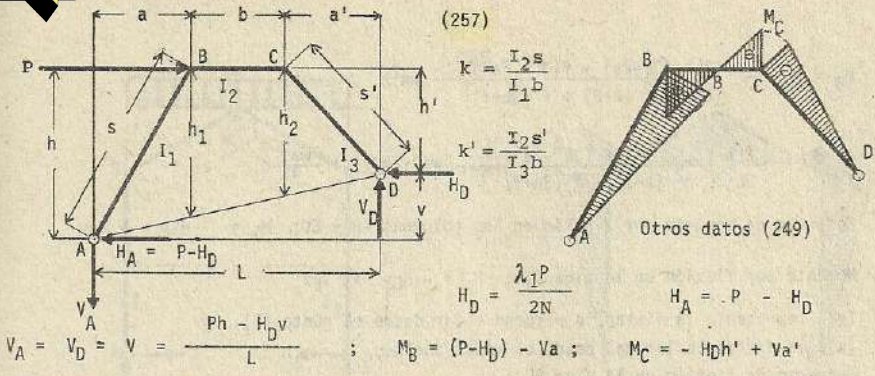
$$H_A = P - H_D$$

$$V_A = V_D = v = \frac{Py - Hv}{L}$$

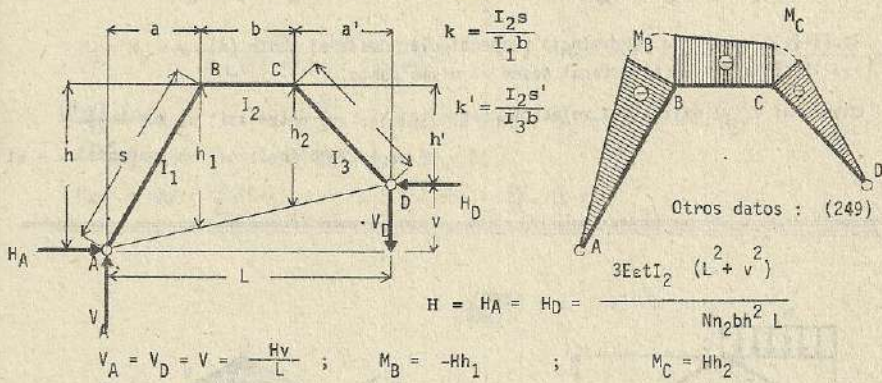
$$M_B = (1 - \alpha_1) Py - Hh_1$$

$$M_C = \alpha_2 Py - Hh_2$$



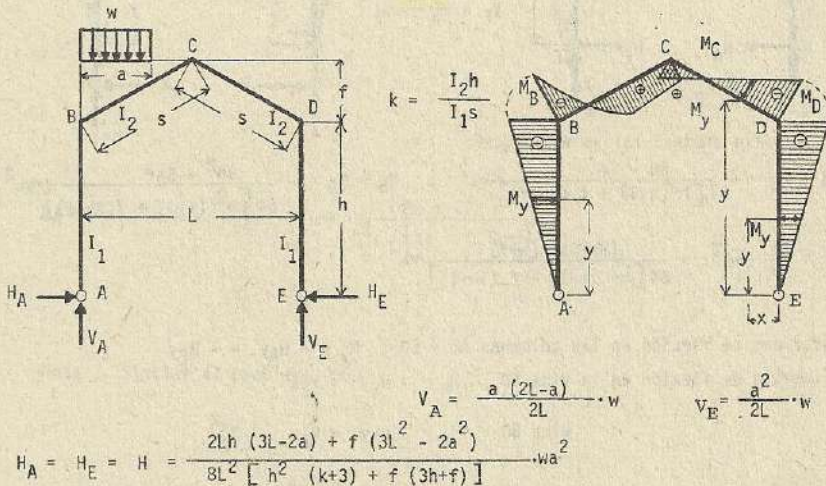


Esfuerzos por cambio de temperatura : (259)



Nota : En caso de descenso de temperatura, los esfuerzos se anotan con los signos contrarios.

(259)



$$M_B = M_D = - \frac{2Lh(3L-2a) + f(3L^2-2a^2)}{8L^2[h^2(k+3) + f(3h+f)]} \cdot wa^2h$$

$$M_C = - \frac{2Lh(3L-2a) + f(3L^2-2a^2)}{8L^2[h^2(k+3) + f(3h+f)]} \cdot wa^2 \cdot (h+f) + \frac{wa^2}{4}$$

Esfuerzo de momento por flexión en las columnas AB y ED: $M_y = -H_A y = -H_E y$

Momento por flexión en la viga CD: $M_x = -H_E y + V_E x$

(y) representa la distancia perpendicular desde el punto (E), y
(x) la distancia lateral desde el mismo punto.

Esfuerzo de flexión en la viga BC:

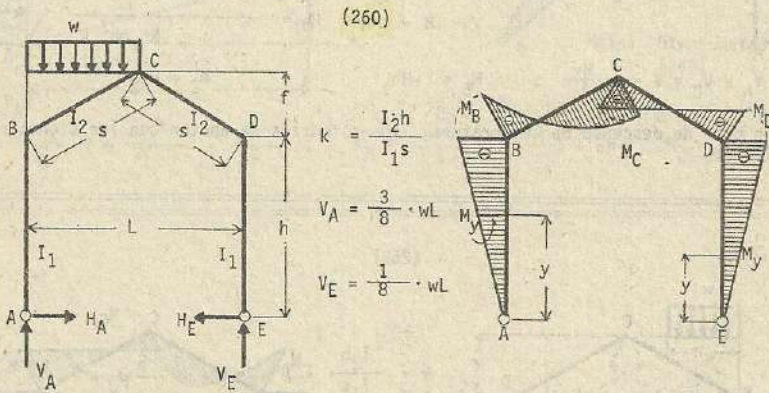
$$M_x = -H_A y + V_A x - \frac{wx^2}{2} \quad x \leq a$$

$$M_x = -H_A y + V_A x - \frac{wa(2x-a)}{2} \quad x \geq a$$

Aquí, (y) representa la distancia perpendicular desde el punto (A), y
(x) la distancia lateral desde el mismo punto.

Entre (x) e (y) existe una relación constante: $xf = \frac{L}{2}(y-h) \therefore x = \frac{L}{2f}(y-h)$

o también: $y = \left(\frac{2xf}{L} + h\right)$



$$H_A = H_E = \frac{8h + 5f}{64[h^2(k+3) + f(3h+f)]} \cdot wL^2 ; \quad M_B = M_D = - \frac{8h^2 + 5hf}{64[h^2(k+3) + f(3h+f)]} \cdot wL^2$$

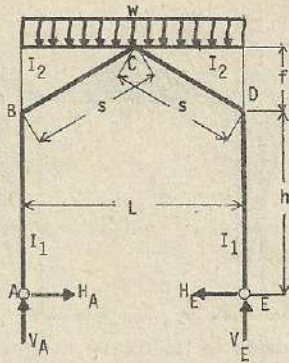
$$M_C = \frac{1}{16} \cdot wL^2 - \frac{(8h+5f)(h+f)}{64[h^2(k+3) + f(3h+f)]} \cdot wL^2$$

Esfuerzos de flexión en las columnas AB y ED: $M_y = -H_A x = -H_E y$

Esfuerzos de flexión en la viga CD: $M_x = -H_E y + V_E x$

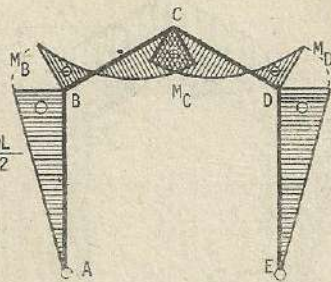
viga BC: $M_x = -H_A y + V_A x - \frac{wx^2}{2}$

(261)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{wL}{2}$$



$$H_A = H_E = H = \frac{8h + 5f}{32 [h^2(k+3) + f(3h+f)]} \cdot wL^2$$

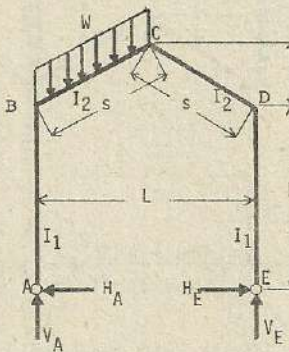
$$M_B = M_D = -\frac{8h^2 + 5hf}{32 [h^2(k+3) + f(3h+f)]} \cdot wL^2 \quad ; \quad M_C = \frac{wL^2}{8} - \frac{(8h+5f)(h+f)}{32 [h^2(k+3) + f(3h+f)]} \cdot wL^2$$

Esfuerzos de flexión en las columnas AB y ED : $M_y = -H_A y = -H_E y$

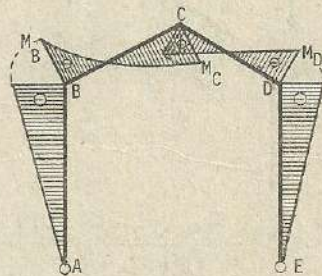
Esfuerzos de flexión en las vigas BC y DC :

$$M_x = -H_A y + \frac{wx}{2} \cdot (L-x) = -H_A \left(\frac{2fx}{L} + h \right) + \frac{wx}{2} \cdot (L-x)$$

(262)



$$k = \frac{I_2 h}{I_1 s}$$



W : Resultante de las fuerzas externas

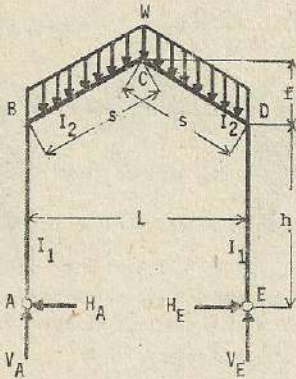
$$H_A = H_E = H = -\frac{WL}{64} \cdot \frac{8h + 5f}{h^2(k+3) + f(3h+f)}$$

$$V_E = \frac{WL}{4}$$

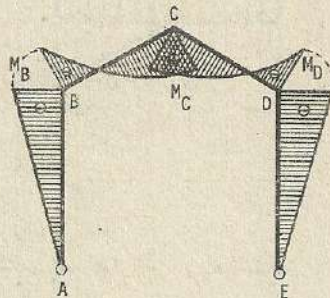
$$V_A = W - V_E$$

Nota : Similar al problema (260)

(263)



$$k = \frac{I_2 h}{I_1 s}$$

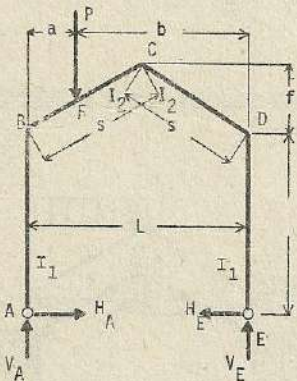


W : Resultante de las fuerzas externas

$$H_A = H_E = H = -\frac{WL}{3E} \cdot \frac{8h + 5f}{h^2(k+3) + f(2h+f)} \quad V_A = V_E = \frac{WL}{2}$$

Problema similar a (261)

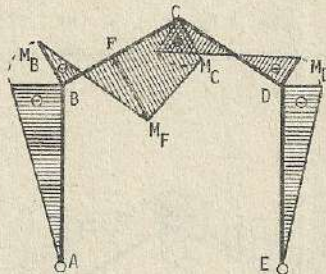
(264)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = \frac{Pb}{L}$$

$$V_E = \frac{Pa}{L}$$



$$H_A = H_E = \frac{a [6hbL + f(3L^2 - 4a^2)]}{4L^2 [h^2(k+3) + f(3h+f)]} \cdot P \quad ; \quad M_B = M_D = -\frac{ah [6hbL + f(3L^2 - 4a^2)]}{4L^2 [h^2(k+3) + f(3h+f)]} \cdot P$$

$$M_C = \frac{Pa}{2} - \frac{a [6hbL + f(3L^2 - 4a^2)] (h+f)}{4L^2 [h^2(k+3) + f(3h+f)]} \cdot P; \quad M_F = \frac{Pab}{L} - \frac{a(hL+2fa) [6hbL + f(3L^2 - 4a^2)]}{4L^3 [h^2(k+3) + f(3h+f)]} \cdot P$$

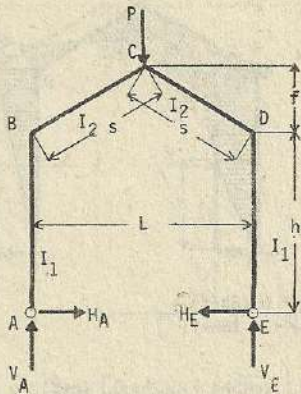
Esfuerzo de flexión en las columnas AB y ED : $M_y = -H_A x = -H_E y$

$$\text{viga CD : } M_x = V_{EX} - H_E \left(h + \frac{2fx}{L} \right)$$

viga BC :

$$M_x = V_A x - H_A \left(h + \frac{2fx}{L} \right) \quad x \leq a$$

$$M_x = V_A x - H_A \left(h + \frac{2fx}{L} \right) - P(x-a) \quad x \geq a$$

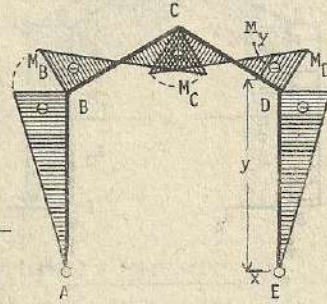


(265)

$$k = \frac{I_2 h}{I_1 s}$$

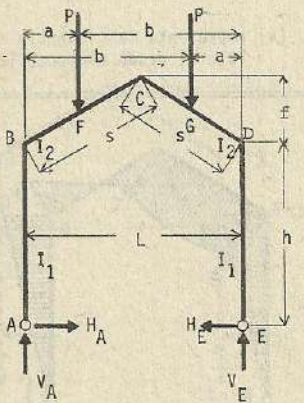
$$V_A = V_E = \frac{P}{2}$$

$$H_A = H_E = \frac{3h + 2f}{8 [h^2(k+3) + f(3h+f)]} \cdot PL$$



$$M_B = M_D = - \frac{3h^2 + 2hf}{8 [h^2(k+3) + f(3h+f)]} \cdot PL ; \quad M_C = \frac{PL}{4} - \frac{(3h+2f)(h+f)}{8 [h^2(k+3) + f(3h+f)]} \cdot PL$$

Esfuerzo de flexión en las columnas AB o ED : $M_y = -H_A y = -H_E y$
 Viga BC o DC : $M_x = \frac{P}{2} x - H_A (h + \frac{2fx}{L})$

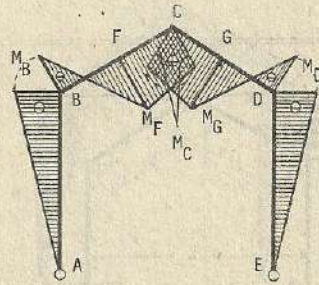


(266)

$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = P$$

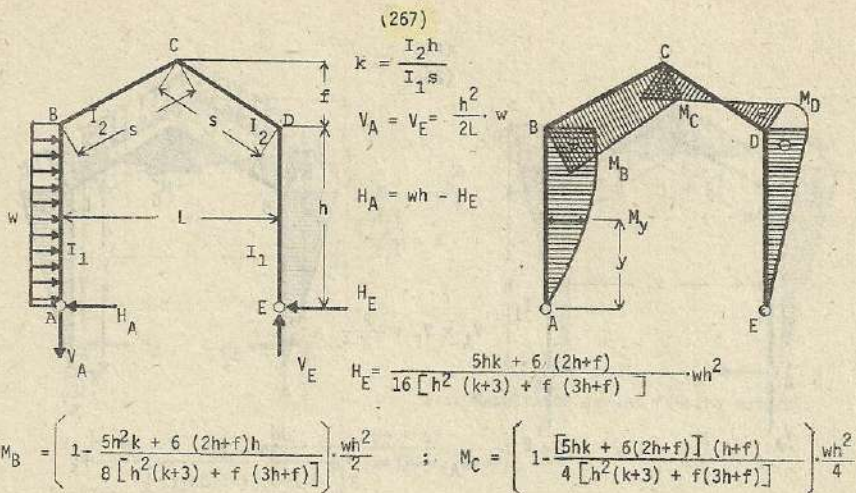
$$H_A = H_E = \frac{a [6hL + f(3L^2 - 4a^2)]}{2L^2 [h^2(k+3) + f(3h+f)]} P$$



$$M_B = M_D = - \frac{ah [6hL + f(3L^2 - 4a^2)]}{2L^2 [h^2(k+3) + f(3h+f)]} \cdot P ; \quad M_C = Pa - H_A(h+f)$$

$$M_F = Pa - H_A (h + \frac{2af}{L})$$

Esfuerzo de flexión en la columna AB o ED : $M_y = -H_A y = -H_E y$
 viga BC o DC : $M_x = Pa - H_A (h + \frac{2fx}{L}) \quad | \quad x \leq a$
 $M_x = Pa - H_A (h + \frac{2fx}{L}) - P(x-a) \quad | \quad x \geq a$



Esfuerzo de flexión
columna AB : $M_y = H_A y - \frac{wy^2}{2}$

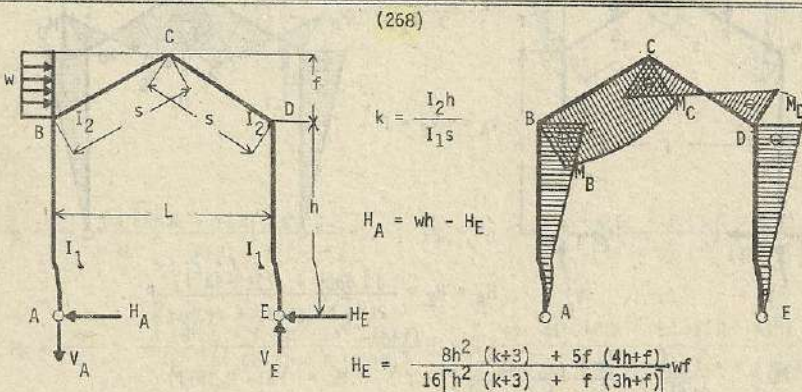
columna ED : $M_y = - H_E y$

Esfuerzo de flexión viga BC :

$$M_x = H_A \left(h + \frac{2fx}{L} \right) - wh \left(\frac{h}{2} + \frac{2fx}{L} \right) - V_A x \quad (x) \text{ representa la distancia lateral desde el punto (A)}$$

viga CD :

$$M_x = - H_E \left(h + \frac{2fx}{L} \right) + V_E x \quad \text{aquí. (x) representa la distancia lateral desde el punto (E)}$$



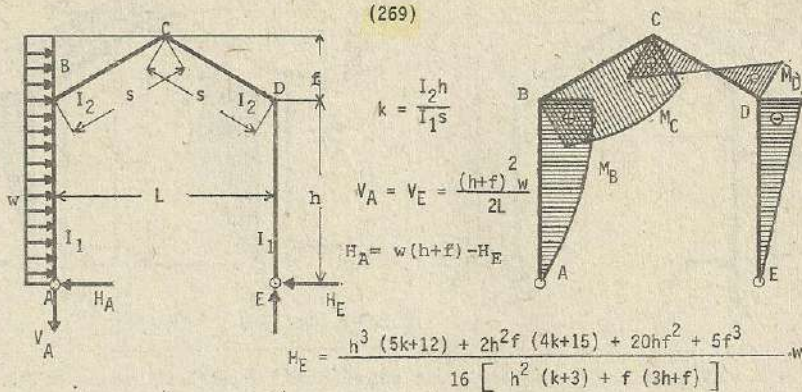
$$V_A = V_E = \frac{wf(2h+f)}{2L}$$

$$M_B = (wf - H_E)h ; \quad M_C = \frac{wf(2h+f)}{4} - H_E(h+f) ; \quad M_D = - H_E h$$

Esfuerzo de flexión columna AB ; $M_y = (wf - H_E)y$, $M_y = - H_E y$ (columna ED)

Esfuerzo de flexión viga ; BC ; $M_x = (wf - H_E)y - \frac{w(y-h)^2}{2} - V_A \frac{L(y-h)}{2f}$

viga ; $M_x = - H_E y + V_E \cdot \frac{L(y-h)}{2f}$
(CD)



$$M_B = w \cdot \left(\frac{h}{2} + f \right) h - H_E h ; \quad M_C = \frac{w (h+f)^2}{4} - H_E (h+f) ; \quad M_D = - H_E h$$

Esfuerzo de flexión en la columna AB : $M_y = w (h+f) y - H_E y - \frac{w y^2}{2}$

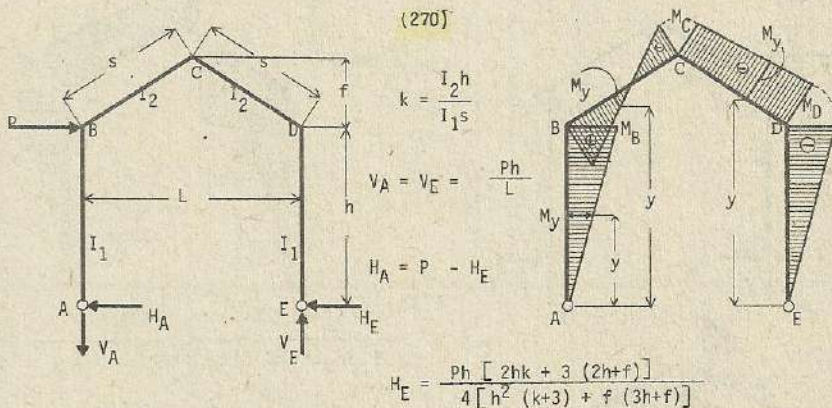
columna ED : $M_y = - H_E y$

Esfuerzo de flexión en la viga BC :

$$M_x = w (h+f) y - H_E y - \frac{w y^2}{2} - V_A \frac{L (y-h)}{2f}$$

viga DC :

$$M_x = - H_E y + V_E \cdot \frac{L (y-h)}{2f}$$



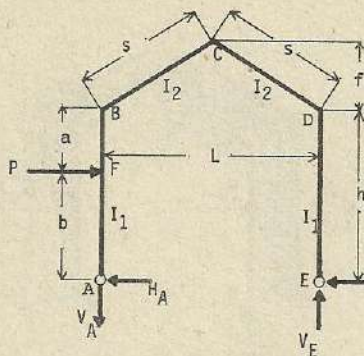
$$M_B = (P - H_E) h ; \quad M_C = \frac{Ph}{2} - H_E (h+f) ; \quad M_D = - H_E h$$

Esfuerzo de flexión en la columna AB : $M_y = P y - H_E y$

columna ED : $M_y = - H_E y$

viga BC : $M_x = Ph - H_E y - \frac{Ph (y-h)}{2f}$

viga CD : $M_x = - H_E y + \frac{Ph (y-h)}{2f}$



(271)

$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{Pb}{L}$$

$$H_A = P - H_E$$

$$H_E = \frac{Pb \left[k \left(3h - \frac{b^2}{h} \right) + 3(2h+f) \right]}{4 \left[h^2(k+3) + f(3h+f) \right]}$$

$$M_F = H_A b ; \quad M_B = Pb - H_E h ; \quad M_C = \frac{Pb}{2} - H_E(h+f) ; \quad M_D = -H_E h$$

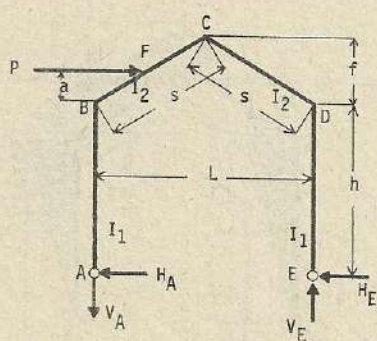
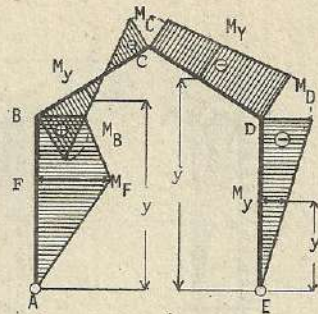
$$\text{Esfuerzo de flexión en la columna AB : } M_y = (P - H_E) y \quad y \leq b$$

$$M_y = Pb - H_E y \quad y > b$$

$$\text{columna ED : } M_y = -H_E y$$

$$\text{viga BC : } M_x = Pb - H_E y - V_A \cdot \frac{(y-h)L}{2f}$$

$$\text{viga CD : } M_x = -H_E y + V_E \cdot \frac{(y-h)L}{2f}$$



(272)

$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{h+a}{L} \cdot P$$

$$H_A = P - H_E$$

$$H_E = \frac{2h^2 k + 3(h+a)(2h+f) - \frac{a^2}{f}(3h+a)}{4 \left[h^2(k+3) + f(3h+f) \right]} \cdot P$$

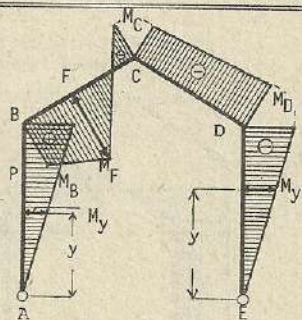
$$M_F = H_A(h+a) - \frac{P(h+a)a}{2f} ; \quad M_C = \frac{h+a}{2} \cdot P - H_E(h+f) ; \quad M_D = -H_E h$$

$$\text{Esfuerzo de flexión en la columna AB : } M_y = H_A y ; \quad \text{columna ED : } M_y = -H_E y$$

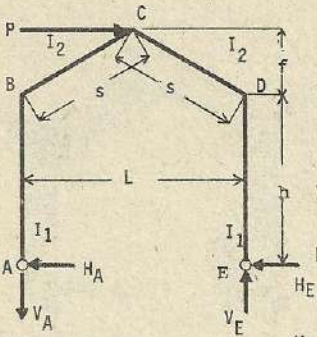
$$\text{viga BC : } M_x = H_A y - V_A \cdot \frac{(y-h)L}{2f} \quad y \leq h+a$$

$$M_x = H_A y - V_A \cdot \frac{(y-h)L}{2f} - P \cdot \frac{(y-h-a)L}{2f} \quad y > h+a$$

$$\text{viga DC : } M_x = -H_E y + V_E \cdot \frac{(y-h)L}{2f}$$



(273)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{(h+f) \cdot P}{L}$$

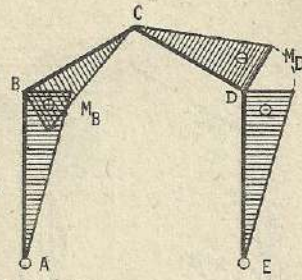
$$H_A = H_E = \frac{P}{2}$$

$$M_B = \frac{Ph}{2} ; \quad M_C = 0 ; \quad M_D = -\frac{Ph}{2}$$

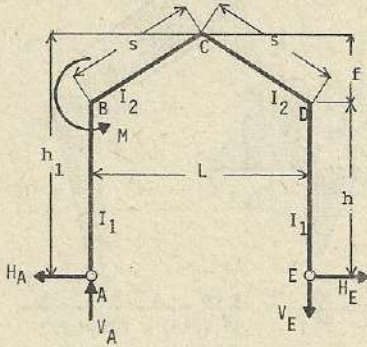
Esfuerzo de flexión en la columna AB : $M_y = \frac{P}{2}y$; columna ED : $M_y = -\frac{P}{2}y$

Esfuerzo de flexión en la viga BC : $M_x = \frac{Py}{2} - \frac{P(h+f)(y-h)}{2f}$

viga DC : $M_x = -\frac{Py}{2} + \frac{P(h+f)(y-h)}{2f}$



(274)



$$k = \frac{I_2 h}{I_1 s}$$

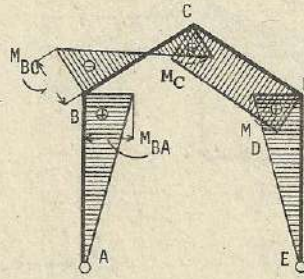
$$n = \frac{f}{h}$$

$$h_1 = (1+n)h$$

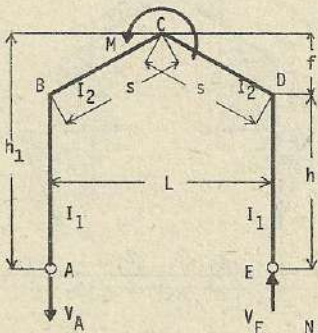
$$N = 3 + k + n(3+n)$$

$$H_A = H_E = H = \frac{3(2+n)M}{4Nh} ; \quad V_A = V_E = V = \frac{M}{L}$$

$$M_{BA} = Hh ; \quad M_{BC} = Hh - M ; \quad M_C = Hh_1 - \frac{M}{2} ; \quad M_D = Hh$$



(275)



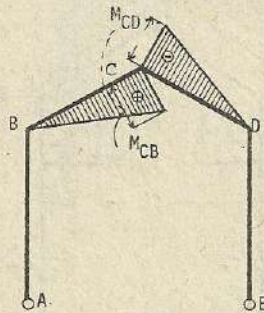
$$k = \frac{I_2 h}{I_1 s}$$

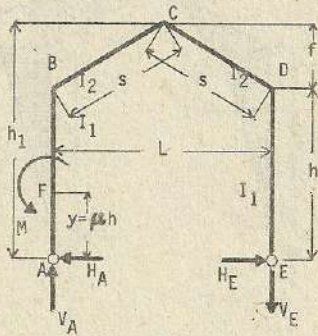
$$n = \frac{f}{h}$$

$$h_1 = (1+n)h$$

$$N = 3 + k + n(3+n)$$

$$H = 0 ; \quad V_A = V_E = V = \frac{M}{L} ; \quad M_B = M_D = 0 ; \quad M_{CB} = -M_{CD} = \frac{M}{2}$$





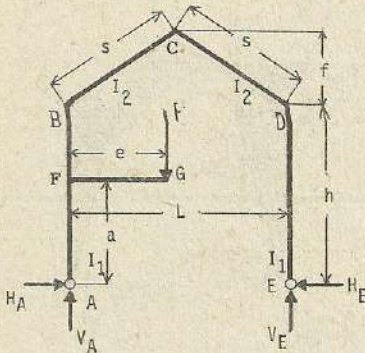
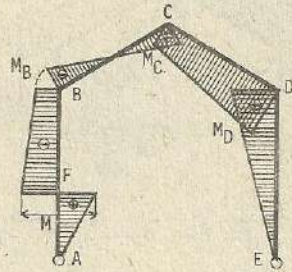
(276)

$$k = \frac{I_2 h}{I_1 s}$$

$$V_E = \frac{M}{L}$$

$$H_A = H_E = H = \frac{3}{4} \cdot M \cdot \frac{2h + f + kh(1 - \mu^2)}{h^2(k+3) + f(3h+f)}$$

$$M_B = Hh - M ; \quad M_C = H(h+f) - \frac{M}{2} ; \quad M_D = Hh$$



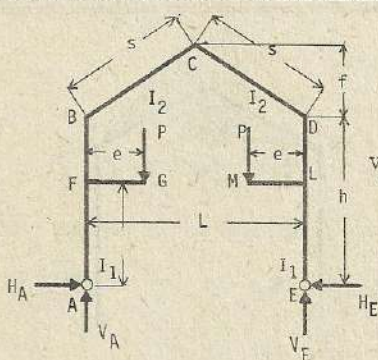
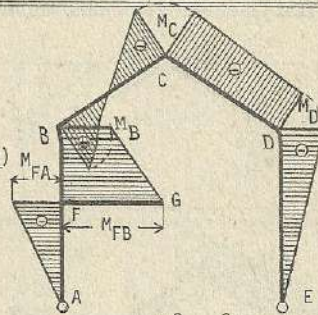
(277)

$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = \frac{P(L-e)}{L}$$

$$H_A = H_E = H = \frac{3Pe}{4h} \cdot \frac{k(h^2 - a^2) + h(2h+f)}{h^2(k+3) + f(3h+f)}$$

$$M_{FA} = -Ha ; \quad M_{FB} = Pe - Ha ; \quad M_B = Pe - Hh ; \quad M_C = \frac{Pe}{2} H(h+f) ; \quad M_D = Hh$$



(278)

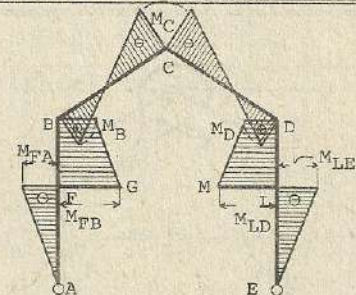
$$k = \frac{I_2 h}{I_1 s}$$

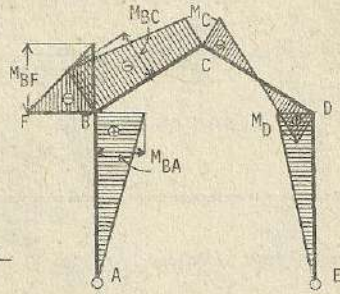
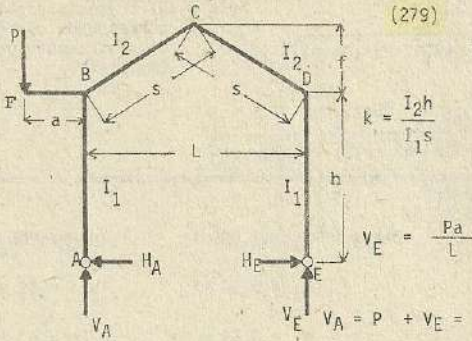
$$V_A = V_E =$$

$$V = P$$

$$H_A = H_E = H = \frac{3Pe}{2h} \cdot \frac{k(h^2 - a^2) + h(2h+f)}{h^2(k+3) + f(3h+f)}$$

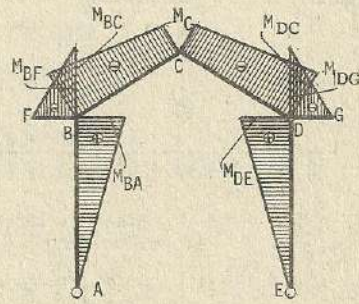
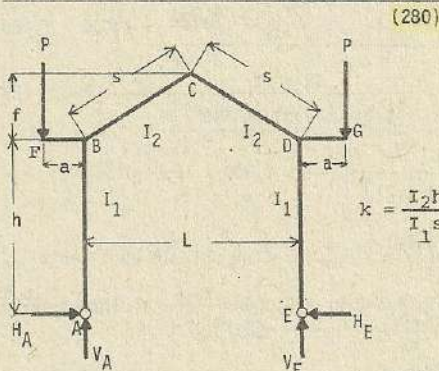
$$M_{FA} = -Ha ; \quad M_{FB} = Pe - Ha ; \quad M_B = M_D = Pe - Hh ; \quad M_C = Pe - H(h+f)$$





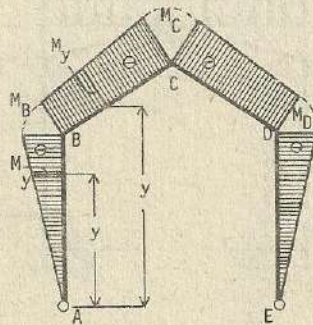
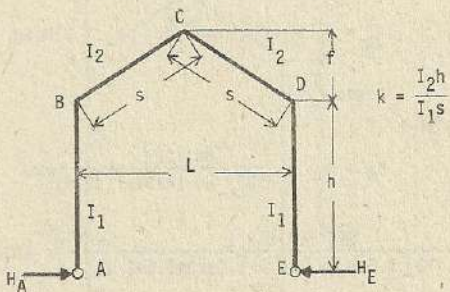
$$H_A = H_E = H = \frac{3}{4} \cdot \frac{Pa(2h+f)}{h^2(k+3) + f(3h+f)} ; M_{BA} = Hh ; M_{BC} = Hh - V_E L$$

$$M_C = H(h+f) - V_E \cdot \frac{L}{2} ; M_D = Hh$$



$$H_A = H_E = H = \frac{3}{2} \cdot \frac{Pa(2h+f)}{h^2(k+3) + f(3h+f)} ; V_A = V_E = P$$

Esfuerzos por cambio de temperatura : (281)



$$H_A = H_E = \frac{3E\epsilon t I_2 L}{2s [h^2(k+3) + f(3h+f)]} ; M_B = M_D = - \frac{3E\epsilon t I_2 L h}{2s [h^2(k+3) + f(3h+f)]}$$

$$M_C = - \frac{3E\epsilon t I_2 L (h+f)}{2s [h^2(k+3) + f(3h+f)]}$$

Esfuerzos de flexión en las columnas AB y ED :

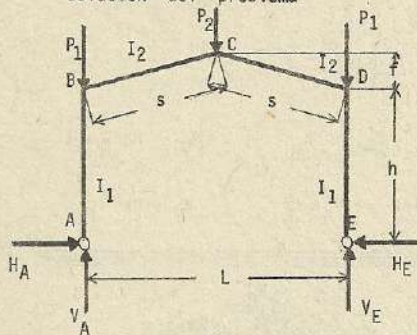
$$M_y = - \frac{3EtI_2Ly}{2s [h^2 (k+3) + f (3h+f)]}$$

Nota : el presente problema corresponde cuando - hay incremento de - temperatura

En las vigas BC y DC :

$$M_x = - \frac{3EtI_2Ly}{2s [h^2 (k+3) + f (3h+f)]}$$

Solución del problema

Datos : $P_1 = 1,000 \text{ Kg}$; $P_2 = 2,000 \text{ Kg}$.

$$I_1 = 0.18 \text{ m}^4 ; I_2 = 0.35 \text{ m}^4$$

$$s = 8.59 \text{ m} ; L = 16.6 \text{ m}$$

$$f = 2.2 \text{ m} ; h = 9.5 \text{ m}$$

$$k = \frac{I_2 h}{I_1 s} = \frac{0.35 \times 9.5}{0.18 \times 8.59} = 2.15$$

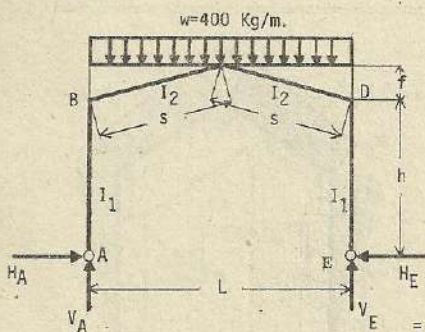
$$V_A = V_E = \frac{2,000}{2} + 1,000 = 2,000 \text{ Kg.}$$

$$H_A = H_E = \frac{3h + 2f}{8 [h^2 (k+3) + f (3h+f)]} \cdot P_2 L \quad (265)$$

$$= \frac{3 \times 9.5 + 2 \times 2.2}{8 \times [9.5 \times 9.5 \times 5.15 + 2.2 \times (3 \times 9.5 + 2.2)]} \times 2,000 \times 16.6 = 257 \text{ Kg.}$$

$$M_B = M_D = - H_A h = - H_E h = - 257 \times 9.5 = 2,441.5 \text{ Kg - m}$$

$$M_C = - H_A (h+f) + V_A \cdot \frac{L}{2} - P_1 \cdot \frac{L}{2} = - 257 \times (9.5+2.2) + 2,000 \times \frac{16.6}{2} - 1,000 \times \frac{16.6}{2} = 5,293.1 \text{ Kg - m}$$



$$k = \frac{I_2 h}{I_1 s} = \frac{0.35 \times 9.5}{0.18 \times 8.59} = 2.15$$

$$V_A = V_E = 400 \times \frac{16.6}{2} = 3,320 \text{ Kg.}$$

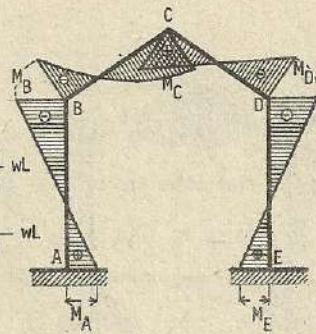
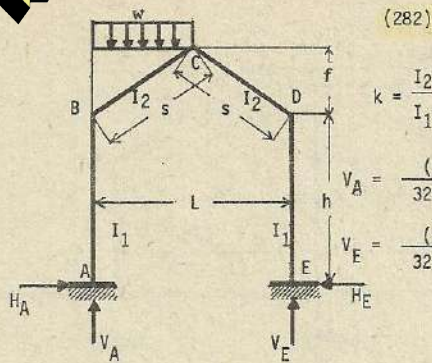
desde (261)

$$H_A = H_E = \frac{8h + 5f}{32 [h^2 (k+3) + f (3h+f)]} w L^2$$

$$= \frac{8 \times 9.5 + 5 \times 2.2}{32 [9.5 \times 9.5 \times 5.15 + 2.2 \times (3 \times 9.5 + 2.2)]} \times 400 \times 16.6^2 = 563 \text{ Kg}$$

$$M_B = M_D = - H_A h = - H_E h = 563 \times 9.5 = - 5,348.5 \text{ Kg - m}$$

$$M_C = - H_A (h+f) + V_A \cdot \frac{L}{2} - \frac{wL^2}{8} = - 563 (9.5+2.2) + 3,320 \times \frac{16.6}{2} - \frac{400 \times 16.6 \times 16.6}{8} = 7,190.9 \text{ Kg - m}$$



$$H_A = H_E = \frac{[k(4h+5f) + f] wL^2}{16 [(hk+f)^2 + 4k(h^2+hf+f^2)]} ; M_A = \left[\frac{hk(8h+15f) + (6h-f)f}{(hk+f)^2 + 4k(h^2+hf+f^2)} - \frac{3}{2(3k+1)} \right] \frac{wL^2}{96}$$

$$M_B = - \left[\frac{hk(16h+15f) + f^2}{(hk+f)^2 + 4k(h^2+hf+f^2)} + \frac{3}{2(3k+1)} \right] \cdot \frac{wL^2}{96}$$

$$M_D = - \left[\frac{hk(16h+15f) + f^2}{(hk+f)^2 + 4k(h^2+hf+f^2)} - \frac{3}{2(3k+1)} \right] \cdot \frac{wL^2}{96}$$

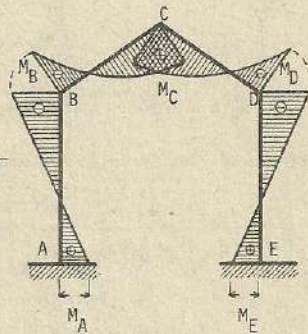
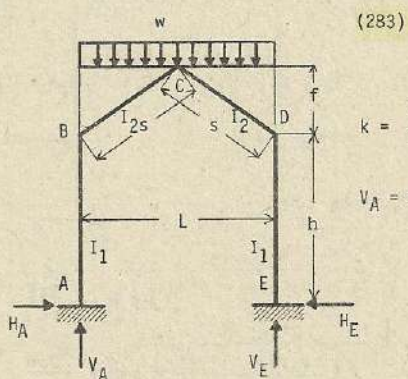
$$M_E = \left[\frac{hk(8h+15f) + 6(h-f)f}{(hk+f)^2 + 4k(h^2+hf+f^2)} + \frac{3}{2(3k+1)} \right] \cdot \frac{wL^2}{96}$$

$$M_C = - H_A(h+f) + M_E + V_E \cdot \frac{L}{2}$$

Esfuerzo de flexión en la columna AB : $M_y = M_A - H_A y$, columna ED : $M_y = M_E - H_E y$

viga BC : $M_x = M_A - H_A y + V_A \cdot \frac{(y-h)L}{2f} - \frac{(y-h)^2 wL^2}{2f^2}$

viga CD : $M_x = M_E - H_E y + V_E \cdot \frac{(y-h)L}{2f}$



$$H_A = H_E = \frac{k(4h+5f) + f}{8 [(hk+f)^2 + 4k(h^2+hf+f^2)]} wL^2$$

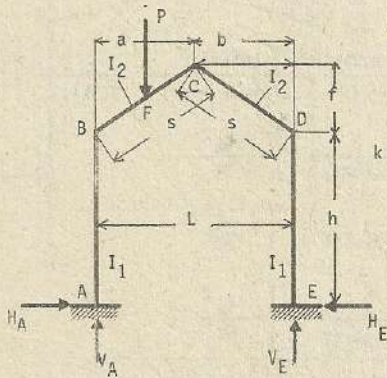
$$M_A = M_E = \frac{hk(8h+15f) + f(6h-f) \cdot wL^2}{48 [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

$$M_B = M_D = - \frac{hk(16h+15f) + f^2}{48 [(hk+f)^2 + 4k(h^2 + hf + f^2)]} wL^2$$

$$H_C = -H_A(h+f) + M_A + \frac{wL^2}{8}$$

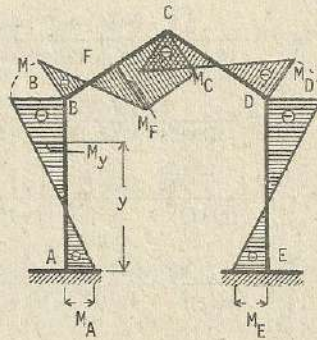
Esfuerzos de flexión en las columnas AB y ED; $M_y = M_A - H_A y = M_E - H_E y$

vigas BC y CD; $M_x = M_A - H_A y + \frac{wL^2(y-h)}{4f} - \frac{wL^2(y-h)^2}{8f^2}$



(284)

$$k = \frac{I_2 h}{I_1 s}$$



$$V_A = \frac{Pb [3kL^2 + b(L+2a)]}{(3k+1)L^3};$$

$$V_E = \frac{Pa [3kL^2 + a(L+2b)]}{(3k+1)L^3}$$

$$H_A = H_E = \frac{Pa [3kL^2(h+f) - 4a^2f(k+1) - 3aL(hk-f)]}{[(hk+f)^2 + 4k(h^2 + hf + f^2)]L^2}$$

$$M_A = M_E = \left[\frac{3kLbh^2 + 3hfL(2a+kL) - f^2L(L-4a) - 4a^2hf(k+2) - 4a^2f^2}{(hk+f)^2 + 4k(h^2 + hf + f^2)} + \frac{b(b-a)}{(3k+1)} \right] \cdot \frac{Pa}{2L^2}$$

$$M_B = -H_A h + M_A; \quad M_C = -H_A(h+f) + M_E + V_E \cdot \frac{L}{2}; \quad M_D = -H_E h + M_E$$

$$M_F = -H_A \left(h + \frac{2af}{L} \right) + M_A + V_A a$$

Esfuerzo de flexión en la columna AB : $M_y = M_A - H_A y$

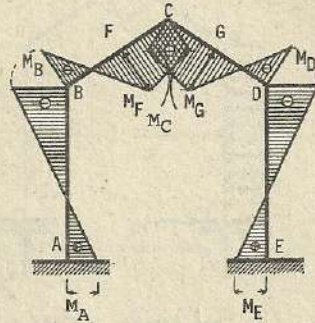
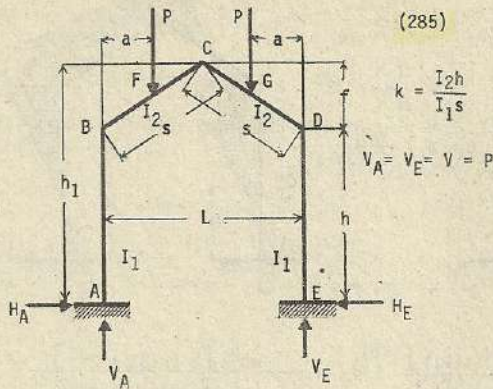
columna ED : $M_y = M_E - H_E y$

$$\text{viga BC : } M_x = M_A - H_A y + V_A \cdot \frac{(y-h)L}{2f} \quad y \leq h + \frac{2af}{L}$$

$$M_x = M_A - H_A y + V_A \cdot \frac{(y-h)L}{2f} - P \left[\frac{(y-h)L}{2f} - a \right] \quad y \geq h + \frac{2af}{L}$$

viga CD :

$$M_x = M_E - H_E y + V_E \cdot \frac{(y-h)L}{2f}$$

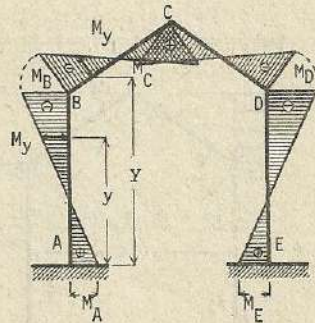
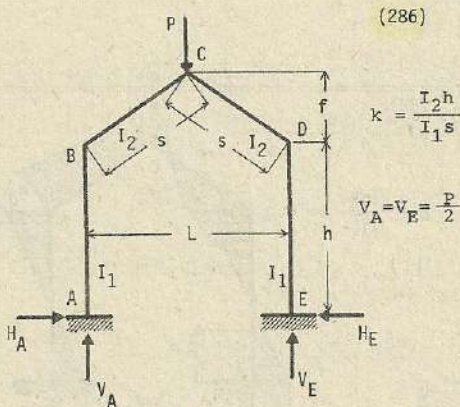


$$H_A = H_E = H = \frac{2Pa}{L^2} \cdot \frac{3L^2 k (h+f) + 4a^2 k (k+1) - 3aL (hk-f)}{(hk+f)^2 + 4k (h^2+hf+f^2)}$$

$$M_{AB} = M_{ED} = \frac{Pa}{L^2} \cdot \frac{1}{(hk+f)^2 + 4k (h^2+hf+f^2)} \times \left[2L^2 h^2 k + 3hLf(2a+Lk) - f^2 L (L+4a) - 4a^2 hf(k+2) - 4a^2 f^2 \right]$$

$$M_{BA} = M_{BC} = M_{DC} = M_{DE} = -Hh + \frac{M_{AB} L}{2} ; M_F = -H \left(h + \frac{2fa}{L} \right) + M_{BA} + Va$$

$$M_{CD} = M_{CB} = -H(h+f) + M_{AB} + V \cdot \frac{L}{2}$$



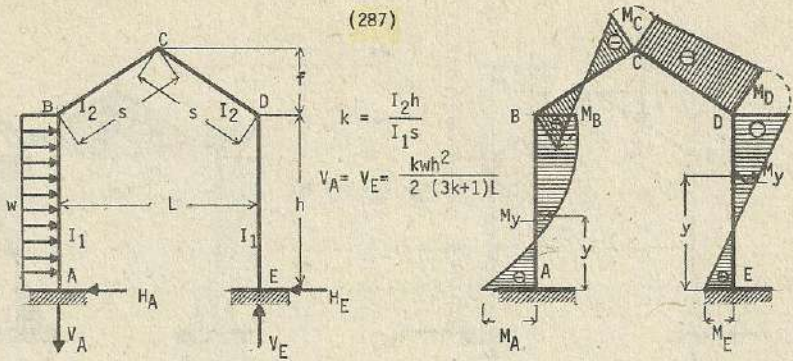
$$H_A = H_E = \frac{k(3h+4f) + f}{4 [(hk+f)^2 + 4k(h^2+hf+f^2)]} ; M_A = M_E = \frac{[h^2 k + hf(2k+1)] PL}{4 [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

$$M_B = M_D = -\frac{kh(h+f)PL}{2 [(hk+f)^2 + 4k(h^2+hf+f^2)]} ; M_C = -H_A(h+f) + M_A + \frac{PL}{4}$$

Esfuerzos de flexión en las columnas AB y ED : $M_y = M_A - H_A y = M_E - H_E y$

vigas BC y CD : $M_x = M_A - H_A y + \frac{P \cdot (y-h)L}{4f}$

(287)



$$H_A = \frac{wh \left[kh^2 (3k+13) + 22khf + 4f^2(4k+1) \right]}{4 \left[(hk+f)^2 + 4k(h^2+hf+f^2) \right]} ; H_E = \frac{wh^2k \left[h(k+3) + 2f^2 \right]}{4 \left[(hk+f)^2 + 4k(h^2+hf+f^2) \right]}$$

$$M_A = -\frac{wh^2}{24} \left[\frac{(12k+6)}{(3k+1)} + \frac{h^2 k(k+6) + kf(15h+16f) + 6f^2}{(hk+f)^2 + 4k(h^2+hf+f^2)} \right] ; M_B = M_A + \frac{wh^2}{2} - H_E h$$

$$M_E = \frac{wh^2}{24} \left[\frac{(12k+6)}{(3k+1)} - \frac{h^2 k(k+6) + kf(15h+16f) + 6f^2}{(hk+f)^2 + 4k(h^2+hf+f^2)} \right] ; M_C = M_E + V_E \cdot \frac{L}{2} - H_E(h+f)$$

$$M_D = M_E - H_E h$$

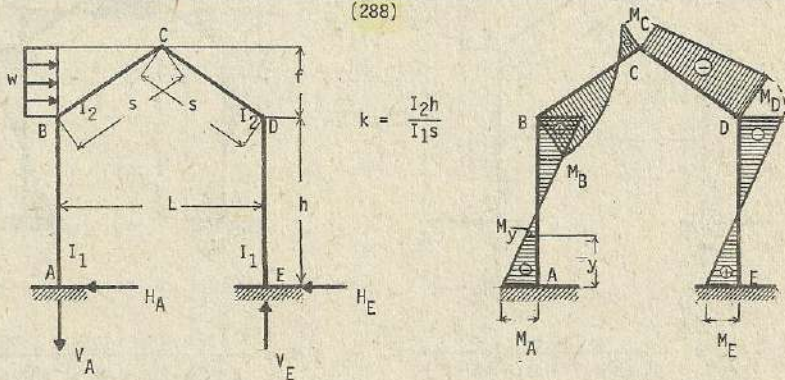
Esfuerzos de flexión en la columna AB : $M_y = M_A + H_A y - \frac{wy^2}{2}$

columna ED : $M_y = H_E - H_E y$

Viga BC : $M_x = M_A + H_A y - wh \left(y - \frac{h}{2} \right) - V_A \frac{(y-h)L}{2f}$

Viga CD : $M_x = M_E - H_E y + V_E \frac{(y-h)L}{2f}$

(288)



$$V_A = V_E = \frac{[5f + 12k(h+f)]}{8(3k+1)L} wf$$

$$H_A = \frac{[2kh^2(k+4) + 14khf + f^2(11k+3)]}{4 \left[(hk+f)^2 + 4k(h^2+hf+f^2) \right]} wf ; H_E = \frac{[5kf(2h+f) + 2kh^2(k+4) + f^2]}{4 \left[(hk+f)^2 + 4k(h^2+hf+f^2) \right]} wf$$

$$M_A = -\frac{wf}{24} \left[\frac{12h(3k+2) + 3f}{(6k+2)} + \frac{f \left[hk(4h+9f) + f(6h+f) \right]}{(hk+f)^2 + 4k(h^2+hf+f^2)} \right]$$

$$M_E = \frac{wf}{24} \left[\frac{12h(3k+2)}{(6k+2)} + 3f - \frac{f \{ hk(4h+9f) + f(6h+f) \}}{(hk+f)^2 + 4k(h^2+hf+f^2)} \right] \quad ; M_B = M_A + H_A h$$

$$M_C = M_E - H_E(h+f) + V_E \frac{L}{2} \quad M_D = M_E - H_E h$$

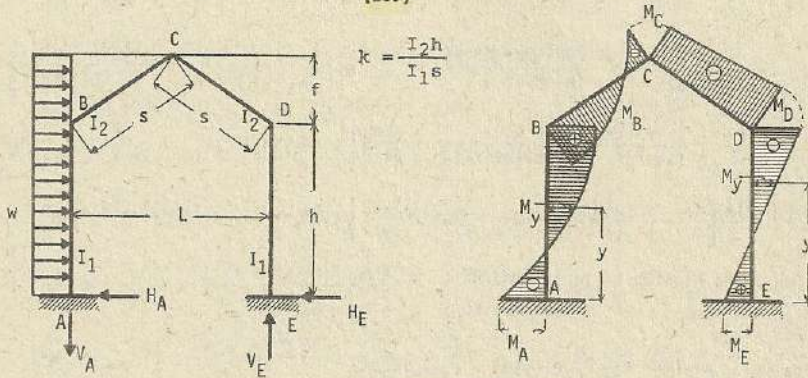
Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y$

columna ED : $M_y = M_E - H_E y$

$$\text{Viga BC : } M_x = M_A + H_A y - \frac{w(y-h)^2}{2} - V_A \frac{(y-h)L}{2f}$$

$$\text{Viga CD : } M_x = M_E - H_E y - V_E \frac{(y-h)L}{2f}$$

(289)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{\{ f^2 (5+12k) + 4kh (3f+h) \}}{8 (3k+1)L} w$$

$$H_A = \frac{1}{4 \{ (hk+f)^2 + 4k(h^2+hf+f^2) \}} \times \left[\{ kh^3 (3k+13) + f^3 (11k+3) + 2h^2 kf (k+15) \} \right. \\ \left. - \frac{2hf^2 (15k+2)}{2hf^2 (15k+2)} \right] w$$

$$H_E = \frac{\{ kh^3 (k+3) + f^3 (5k+1) + 10khf (h+f) + 2k^2 h^2 f \}}{4 \{ (hk+f)^2 + 4k(h^2+hf+f^2) \}} w$$

$$M_A = - \frac{w}{24} \left[\frac{12kh (2h+3f) + 12h (h+2f) + 3f^2}{(6k+2)} + \frac{kh^2 \{ h^2(k+6) + f(15h+16f) \}}{(hk+f)^2 + 4k(h^2+hf+f^2)} + \frac{f^2 \{ kh(4h+9f) + f(6h+f) + 6h^2 \}}{(hk+f)^2 + 4k(h^2+hf+f^2)} \right]$$

$$M_E = \frac{w}{24} \left[\frac{12kh (2h+3f) + 12h (h+2f) + 3f^2}{(6k+2)} - \frac{kh^2 \{ h^2(k+6) + f(15h+16f) \}}{(hk+f)^2 + 4k(h^2+hf+f^2)} + \frac{f^2 \{ kh(4h+9f) + f(6h+f) + 6h^2 \}}{(hk+f)^2 + 4k(h^2+hf+f^2)} \right]$$

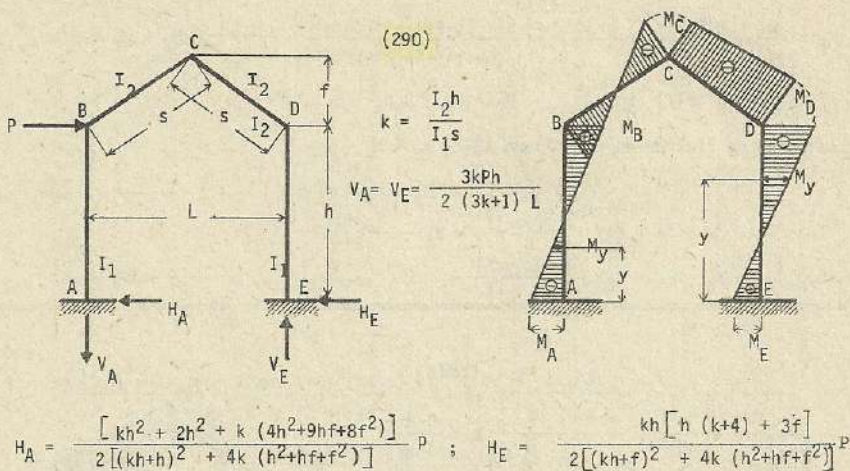
$$M_B = M_A + \frac{wh^2}{2} - H_E h \quad ; \quad M_C = M_E + V_E \frac{L}{2} - H_E (h+f) \quad ; \quad M_D = M_E - H_E h$$

Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y - \frac{wy^2}{2}$

columna ED : $M_y = M_E - H_E y$

$$\text{viga BC : } M_x = M_A + H_A y - \frac{wy^2}{2} - V_A \frac{(y-h)L}{2f}$$

$$\text{viga CD : } M_x = M_E - H_E y + V_E \frac{(y-h)L}{2f}$$



$$M_A = -\frac{Ph}{2} \left[\frac{f(kh+f+2kf)}{(kh+f)^2 + 4k(h^2 + hf + f^2)} + \frac{3k+2}{6k+2} \right] ; M_B = M_A + H_A h ; M_D = M_E - H_E h$$

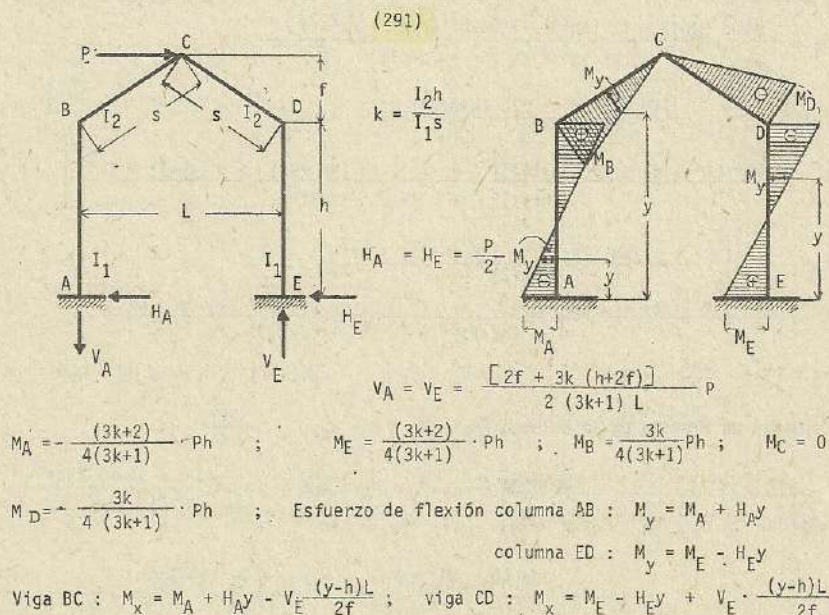
$$M_E = \frac{Ph}{2} \left[-\frac{f(kh+f+2kf)}{(kh+f)^2 + 4k(h^2 + hf + f^2)} + \frac{3k+2}{6k+2} \right] ; M_C = M_E - H_E(h+f) + V_E \frac{L}{2}$$

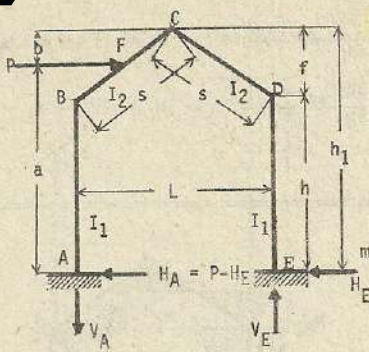
Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y$

columna ED : $M_y = M_E - H_E y$

Viga BC : $M_x = M_A + H_A y - P(y-h) - V_A \frac{(y-h)L}{2f}$

Viga CD : $M_x = M_E - H_E y + V_E \frac{(y-h)L}{2f}$



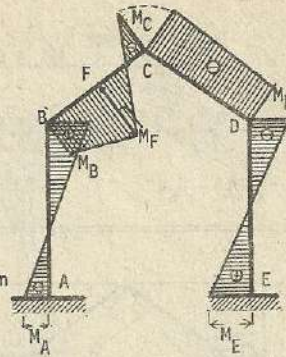


(292)

$$k = \frac{I_2 h}{I_1 s}$$

$$n = \frac{f}{h}$$

$$m = \frac{h_1}{h} = 1+n$$



$$N_1 = 4km^2 + (k-n)^2 ; N_2 = 1 + 3k$$

$$\alpha_0 = (1+k)(k-n)^2 ; \alpha_1 = 3k+4nk+n ; \alpha_2 = 3k+2nk-n$$

$$\beta_1 = k+2nk+n ; \beta_2 = (1+n)(k-n) ; \beta_3 = 2+3k$$

$$\gamma_1 = k(4+k+3n) ; \gamma_2 = k(2-k+3n) ; \gamma_3 = 2km+n+\gamma_1$$

$$C_{BC} = \frac{-P(f-b)b^2}{f^2} ; C_{CB} = -\frac{P(f-b)^2b}{f^2} ; H_{BC} = -\frac{P(f-b)b}{2f^2}(2f-b)$$

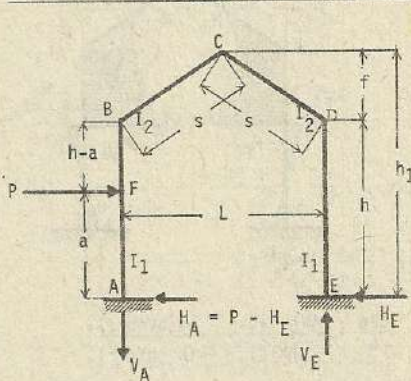
$$H_E = \frac{\alpha_0 P h + \alpha_1 (Pa - C_{CB}) - \alpha_2 C_{BC}}{2N_1 h}$$

$$H_A = P - H_E$$

$$M_A = -\frac{\beta_1 (Pb + C_{CB}) + \beta_2 C_{BC}}{2N_1} - \frac{\beta_3 P h - 2H_{BC}}{4N_2} ; M_B = (P - H_E)h + M_A$$

$$M_E = -\frac{\beta_1 (Pb + C_{CB}) + \beta_2 C_{BC}}{2N_1} + \frac{\beta_3 P h - 2H_{BC}}{4N_2} ; M_C = \frac{VL}{L} + M_E - H_E h_1$$

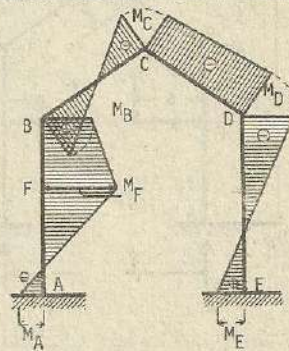
$$V_A = V_E = V = \frac{1}{L} (Pa + M_A - M_E) ; M_D = M_E - H_E h$$



(293)

$$k = \frac{I_2 h}{I_1 s}$$

$$n = \frac{f}{h}$$



Otros datos : referencia (292)

$$C_{BA} = -\frac{Pa^2(h-a)}{h^2} ; C_{AB} = -\frac{P(h-a)^2 a}{h^2} ; H_E = \frac{\gamma_1 (Pa + C_{AB}) + \gamma_2 C_{BA}}{2N_1 h}$$

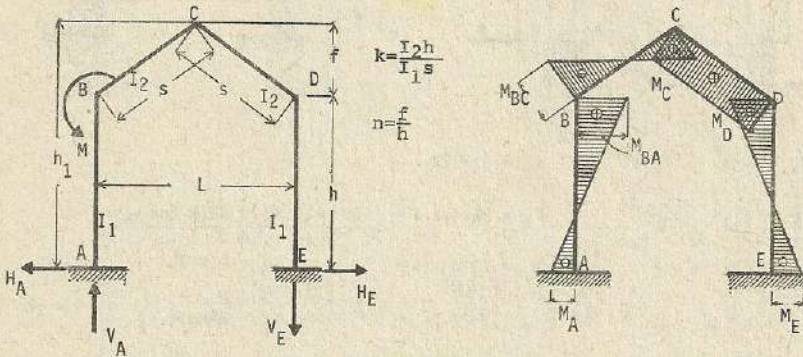
$$H_A = P - H_E ; V_A = V_E = V = \frac{Pa + M_A - M_E}{L}$$

$$M_A = \frac{n \beta_1 Pa - 2km^2 C_{AB} - \gamma C_{BA}}{2N_1} - \frac{\beta_3 Pa - 3k(C_{AB} + C_{BA})}{4N_2} ; M_B = Pa + M_A - H_E h$$

$$M_E = \frac{n \beta_1 Pa - 2km^2 C_{AB} - \gamma C_{BA}}{2N_1} + \frac{\beta_3 Pa - 3k(C_{AB} + C_{BA})}{4N_2} ; M_C = \frac{VL}{2} + M_E - H_E h_1$$

$$M_D = M_E - H_E h$$

(294)



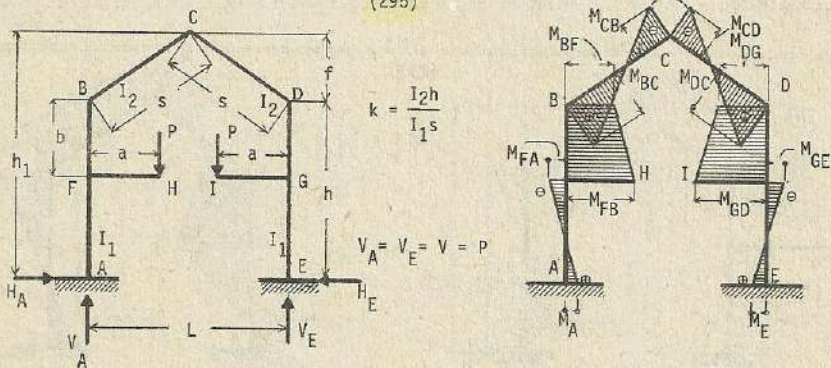
Otros datos : referencia (292)

$$H_A = H_E = H = \frac{3mkM}{N_1 h} ; V_A = V_E = V = \frac{3kM}{N_2 L} ; M_A = -\frac{M}{2} \left[\frac{\alpha_2 - \beta_2}{N_1} - \frac{1}{N_2} \right]$$

$$M_{BA} = M_A + Hh ; M_{BC} = M_{BA} - M ; M_E = -\frac{M}{2} \left[\frac{\alpha_2 - \beta_2}{N_1} + \frac{1}{N_2} \right]$$

$$M_C = \frac{1}{2} (M_A + M_E - M) + Hh_1 ; M_D = M_E + Hh$$

(295)



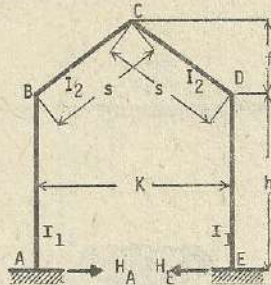
$$H_A = H_E = H = \frac{6Pa}{h} \cdot \frac{hk(h+f+bk) - bk(bk+b+f)}{(hk+f)^2 + 4k(h^2+hf+f^2)}$$

$$M_{AF} = M_{EG} = \frac{Pa}{h} \cdot \frac{1}{(hk+f)^2 + 4k(h^2+hf+f^2)} \left[h^2k(2bk+2h+3f) - bfk(6h+3b+4f) - h(3b^2k^2 + 9b^2k+f^2) \right]$$

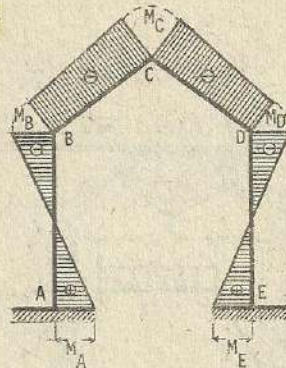
$$M_{FA} = M_{GE} = -H(h-b) + M_{AF} ; M_{BC} = M_{DC} = -Hh + M_{AF} + Pa$$

$$M_{FB} = M_{GD} = -H(h-b) + M_{AF} + Pa ; M_{CB} = -H(h+f) + M_{AB} + Pa + V_E \cdot \frac{1}{2}$$

Esfuerzos debido a cambio de temperatura : (296)



$$k = \frac{I_2 h}{I_1 s}$$



$$H_A = H_E = \frac{6EetI_2 L (k+1)}{s [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

$$M_A = M_E = \frac{3EetI_2 L [sh(k+2) + f]}{s [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

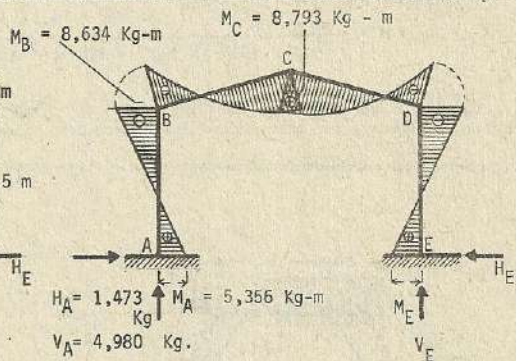
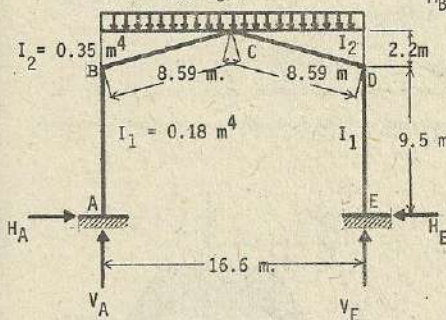
$$M_C = -\frac{3EetI_2 L [hk + f(2k+1)]}{s [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

$$; M_B = M_D = -\frac{3EetI_2 L (hk - f)}{s [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

Nota : El presente problema corresponde al incremento de temperatura. -

Problema numérico

$$w = 600 \text{ Kg/m}$$



$$k = \frac{I_2 h}{I_1 s} = \frac{0.35 \times 9.5}{0.18 \times 8.59} = 2.15 ; V_A = V_E = \frac{wL}{2} = \frac{600 \times 16.6}{2} = 4,980 \text{ Kg}$$

$$H_A = H_E = \frac{k(4h+5f) + f}{8 [(hk+f)^2 + 4k(h^2+hf+f^2)]} wL^2$$

$$= \frac{2.15 \times (4 \times 9.5 + 5 \times 2.2) + 2.2}{8 [(9.5 \times 2.15 + 2.2)^2 + 4 \times 2.15 \times (9.5^2 + 9.5 \times 2.2 + 2.2^2)]} \times 600 \times 16.6^2 = 1,473 \text{ Kg}$$

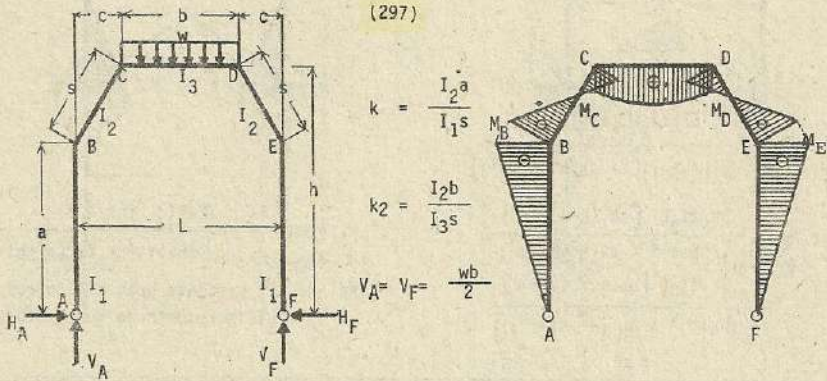
$$M_A = M_E = \frac{[hk(8h+15f) + f(6h-f)]wL^2}{48 [(hk+f)^2 + 4k(h^2+hf+f^2)]} = \frac{9.5 \times 2.15 \times (8 \times 9.5 + 15 \times 2.2) + 2.2 \times (6 \times 9.5 - 2.2)}{48 \times 1,509.4046} \times 600 \times 16.6^2 = 5,356 \text{ Kg-m}$$

$$M_B = M_D = -\frac{hk(16h+15f) + f^2}{48 [(hk+f)^2 + 4k(h^2+hf+f^2)]}$$

$$= -\frac{9.5 \times 2.15 \times (16 \times 9.5 + 15 \times 2.2) + 2.2^2}{48 \times 1,509.4046} \times 600 \times 16.6^2 = -8,634 \text{ Kg-m}$$

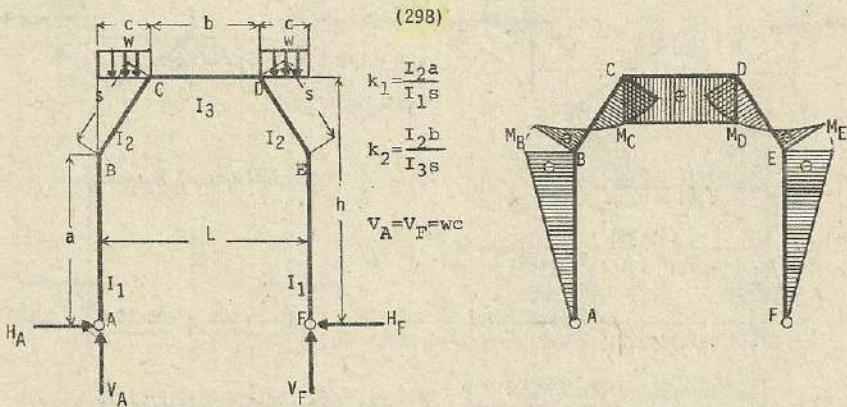
$$M_C = -H_A(h+f) + M_A + \frac{wL^2}{8}$$

$$= 1,473(9.5+2.2) + 5,356 + \frac{600 \times 16.6^2}{8} = 8,793 \text{ Kg} \cdot \text{m}$$



$$H_A = H_F = H = \frac{wb}{4} \cdot \frac{2(a+2h)c + hk_2(L+4c)}{2a^2(k_1+1) + 2ah + h^2(2+3k_2)} ; M_{BC} = M_{BA} = M_{EF} = M_{ED} = -Ha$$

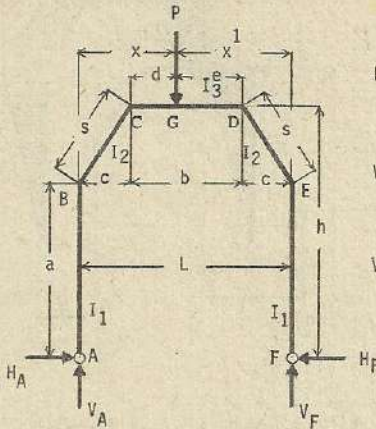
$$M_{CB} = M_{CD} = M_{DC} = M_{DE} = -Hh + \frac{wb}{2}c ; M_{\text{máx.}} = -Hh + \frac{wb}{2} \cdot \frac{L}{2}$$



$$H_A = H_F = H = \frac{wc^2}{4} \cdot \frac{5hk_2 + 5h + 3a}{2a^2(k_1+1) + 2ah + h^2(2+3k_2)}$$

$$M_{BA} = M_{BC} = M_{EF} = M_{ED} = -Ha ; M_{CB} = M_{CD} = M_{DC} = M_{DE} = -Hh + \frac{wc^2}{2}$$

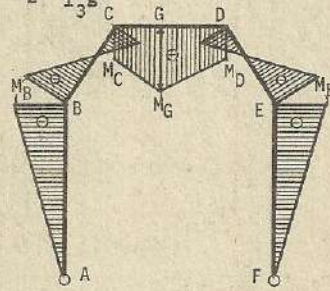
(299)



$$k_1 = \frac{I_2 a}{I_1 s} ; k_2 = \frac{I_2 b}{I_3 s}$$

$$V_A = \frac{p x'}{L}$$

$$V_F = \frac{p x}{L}$$



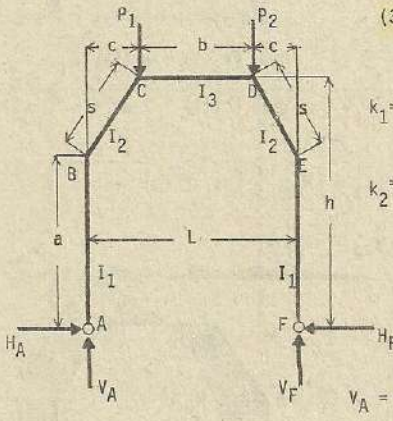
$$H_A = H_F = H = \frac{p}{2b} \frac{bc(2h+a) + 3hk_2(bx-d^2)}{2a^2(k_1+1) + 2ah + h^2(3k_2+2)}$$

$$\text{En el caso donde } d = e = \frac{b}{2} ; H = \frac{p}{8} \frac{4c(2h+a) + 3hk_2(4c+b)}{2a^2(k_1+1) + 2ah + h^2(3k_2+2)}$$

$$M_{BA} = M_{BC} = M_{EF} = M_{ED} = -Ha ; M_{DC} = M_{DE} = -Hh + V_F c$$

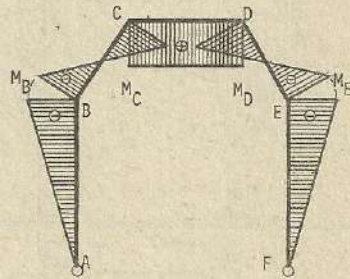
$$M_{CB} = M_{CD} = -Hh + V_A c ; M_G = -Hh + V_A x$$

(300)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$



$$V_A = \frac{P_1(L-c) + P_2 c}{L} ; V_F = \frac{P_2(L-c) + P_1 c}{L}$$

$$H_A = H_B = H = \frac{(P_1 + P_2) c [h(3k_2+2) + a]}{2a^2(k_1+1) + 2ah + h^2(3k_2+2)} ; M_{BA} = M_{BC} = M_{ED} = M_{EF} = -Hh$$

$$M_{CB} = M_{CD} = -Hh + V_A c ; M_{DC} = M_{DE} = -Hh + V_F c$$

(301)

$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$H_A = wf - H_F \quad ; \quad -V_A = V_F = \frac{wf(h - \frac{f}{2})}{L} \quad ; \quad H_A = wf - H_F$$

$$H_F = \frac{wf}{8} \cdot \frac{a^2(8k_1+9) + h[2a(3k_2+5) + h(6k_2+5)]}{2a^2(k_1+1) + 2ah + h^2(3k_2+2)} \quad ; \quad M_{ED} = M_{EF} = -H_F a$$

$$M_{BA} = M_{BC} = (wf - H_F)a \quad ; \quad M_{CB} = M_{CD} = V_F(L-c) - H_F h \quad ; \quad M_{DC} = M_{DE} = -H_F h - V_F c$$

(302)

$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$-V_A = V_F = \frac{wa^2}{2L}$$

$$H_A = wa - H_F$$

$$H_F = \frac{wa^2}{8} \cdot \frac{6h(k_2+1) + a(5k_1+6)}{2a^2(k_1+1) + 2ah + h^2(3k_2+2)}$$

$$M_{BA} = M_{BC} = (wa - H_F)a - \frac{wa^2}{2} \quad ; \quad M_{CB} = M_{CD} = V_F(L-c) - H_F h$$

$$M_{DC} = M_{DE} = -H_F h + V_F c \quad ; \quad M_{ED} = M_{EF} = -H_F a$$

(303)

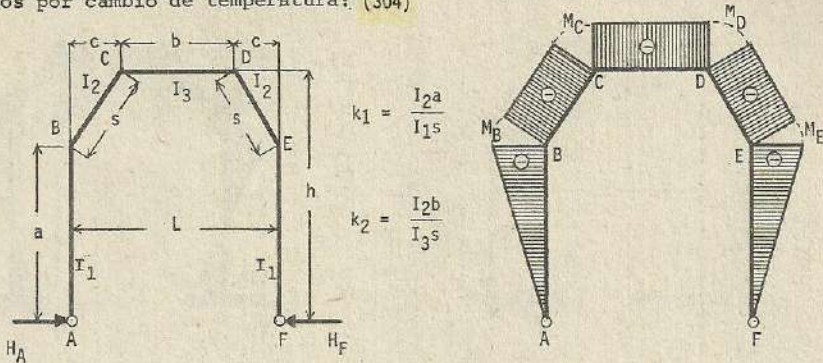
$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$H_A = \frac{1}{2} wa - H_F$$

$$H_F = \frac{wa^2}{40} \cdot \frac{10h(k_2+1) + a(9k_1+10)}{2(1+k_1)a^2 + 2ah + h^2(3k_2+2)}$$

Esfuerzos por cambio de temperatura: (304)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

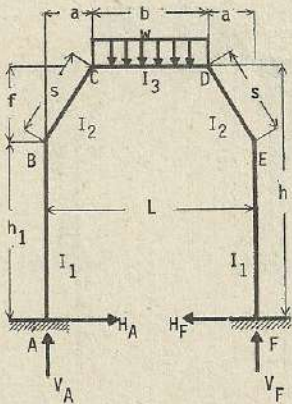
$$H_A = \frac{3E\epsilon t I_2 L}{s [2(k_1+1)a^2 + 2ha + h^2(3k_2+2)]}$$

$$M_B = M_E = -H_a$$

$$M_C = M_D = -Hh$$

Nota : El presente corresponde al problema de incremento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.

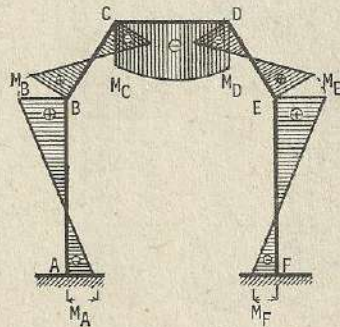
(305)



$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$x = a$$

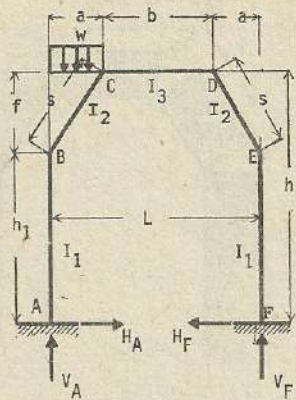


$$\beta = k_1 (k_1 + 2k_2 + 4) h_1^2 + 6k_1 (k_2 + 1) h_1 f + (6k_1 k_2 + 4k_1 + 2k_2 + 1) ; V_A = V_F = \frac{wb}{2}$$

$$\gamma = (6k_1 + k_2 + 6) L^2 - 4(k_2 + 3) \cdot La + 4(k_2 + 2) a^2$$

$$H_A = H_F = H = \frac{wb}{4\beta} \left[k_1 \{ k_2 L + 2(2k_2 + 3) a \} h_1 + k_2 (2k_2 + 1) L + 2(k_2 + 1) (4k_1 + 1) a \right] f$$

$$M_A = M_F = - \frac{(k_1 + k_2 + 2) h_1 + (k_2 + 1) f}{2k_1 + k_2 + 2} H + \frac{wb [k_2 L + 2(k_2 + 3) a]}{12(2k_1 + k_2 + 2)}$$

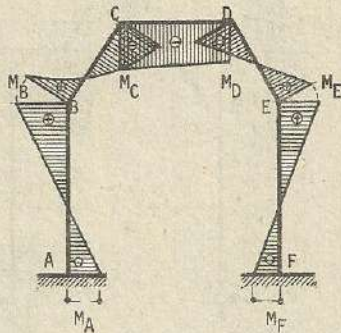


(306)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

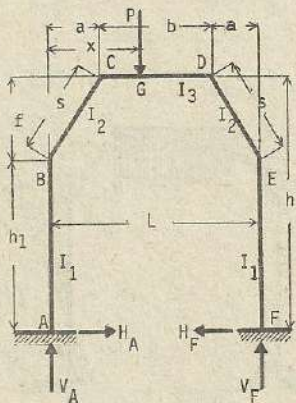
Otros datos
referencia
(305)



$$H_A = H_F = H = \frac{w a^2}{8} \cdot \frac{k_1 (6k_2 + 8) h_1 + (12k_1 k_2 + 10k_1 + 3k_2 + 2) f}{\beta}$$

$$v = \frac{w a^2}{2L} \cdot \frac{(k_2 + 4) L^2 - (4k_2 + 11) L a + 4_2 (k_2 + 2) a^2}{\gamma}; \quad v_A = \frac{w a (L - \frac{a}{2})}{L} + v; \quad v_F = \frac{w a^2}{2L} - v$$

$$M_A = - \frac{(k_1 + k_2 + 2) h_1 + (k_2 + 1) f}{2k_1 + k_2 + 2} H + \frac{w a^2 (3k_2 + 4)}{12 (2k_1 + k_2 + 2)} + \frac{L}{2} v; \quad M_F = -M_A - vL$$



(307)

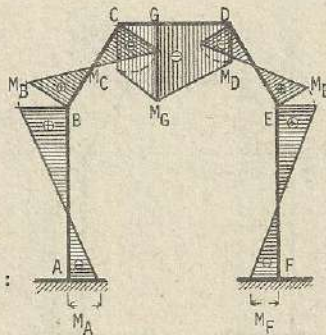
$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$x = aL$$

$$a = nb$$

otros datos :
(305)



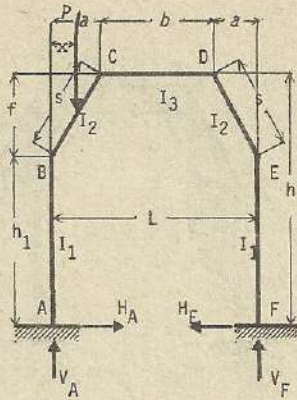
$$H_A = H_F = H = \frac{P}{2\beta} [3k_1 \{ k_2 \alpha (L-x) + (1-k_2)n a \} h_1 + \{ 3k_2 (2k_1+1) (\alpha L - \alpha x - na) + (4k_1 - k_2 + 1) a \} f]$$

$$v = \frac{P}{L\gamma} [k_2 \alpha L^3 + 3 \{ (1-k_2)n a - k_2 \alpha x \} L^2 + 2 \{ (3k_2 \alpha + 2k_2 n - 2) a^2 + k_2 \alpha x^2 - 3x a \} L - 8 (k_2 \alpha a - x) a^2]$$

$$v_A = \frac{P (L-x)}{L} + v; \quad v_F = \frac{Px}{L} - v$$

$$M_A = - \frac{(k_1 + k_2 + 2) h_1 + (k_2 + 1) f}{2k_1 + k_2 + 2} H + \frac{P [k_2 \alpha (L-x) + (1-k_2)n a]}{2 (2k_1 + k_2 + 2)} + \frac{L}{2} v$$

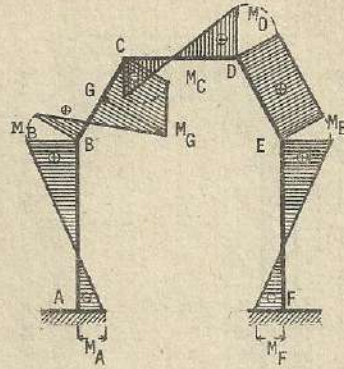
$$M_F = -M_A - vL$$



(308)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

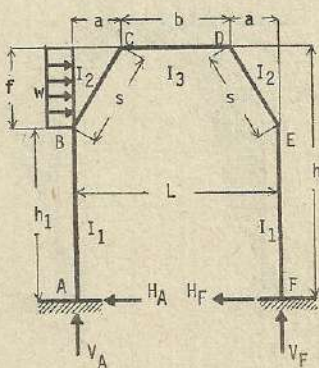
Otros datos:
(305)

$$H_A = H_F = H = \frac{Px}{2b} \left[3k_1 (2k_2 + 2 - a) h_1 + \left\{ 3(k_2 + 1)(2k_2 + a) - (2k_1 + k_2 + 2)a^2 \right\} f \right]$$

$$v = \frac{Px}{L} \frac{(k_2 + 6 - 3a)L^2 - 2(2k_2 + 6 - a^2)La + 4(k_2 + 2)a^2}{Y}$$

$$V_A = \frac{P(L-x)}{L} + v \quad ; \quad V_F = \frac{Px}{L} - v$$

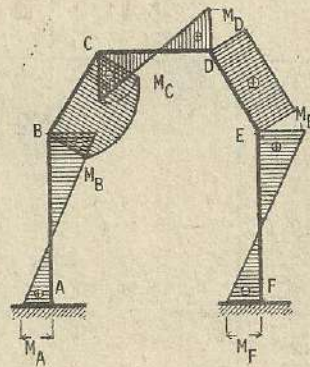
$$M_A = - \frac{(k_1 + k_2 + 2)h_1 + (k_2 + 1)f}{(2k_1 + k_2 + 2)} H + \frac{Px(k_2 + 2 - a)}{2(2k_1 + k_2 + 2)} + \frac{I_2}{2} v \quad ; \quad M_F = -M_A - vL$$



(309)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

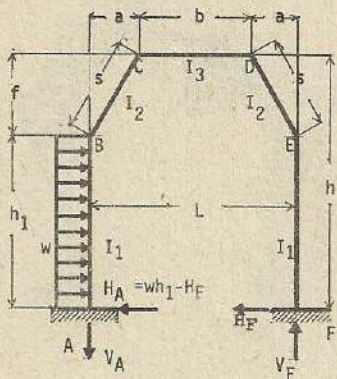
Otros datos:
(305)

$$H_A = H_F = H = \frac{wf}{8b} \left[4k_1(k_1 + 2k_2 + 4)h_1^2 + 2k_1(9k_2 + 10)h_1 f + \left\{ 3k_2(4k_1 + 1) + 2(5k_1 + 1) \right\} f^2 \right]$$

$$V_A = V_F = V = \frac{2 \left[3k_1 h_1 + (3k_1 + 1) f \right] L - fa}{Y} w$$

$$M_A = - \frac{(k_1 + k_2 + 2)h_1 + (k_2 + 1)f}{2k_1 + k_2 + 2} H + \frac{wf}{6} \frac{(9k_1 + 6k_2 + 12)h_1 + (3k_1 + 3k_2 + 5)f}{2k_1 + k_2 + 2} - \frac{L}{2} v$$

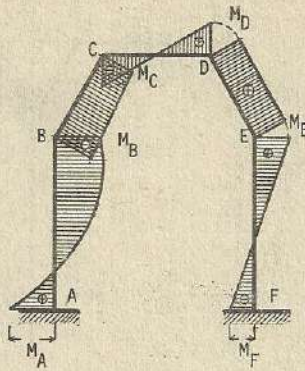
$$M_F = - \frac{1}{2} wf(2h_1 + f) + M_A + vL$$



(310)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

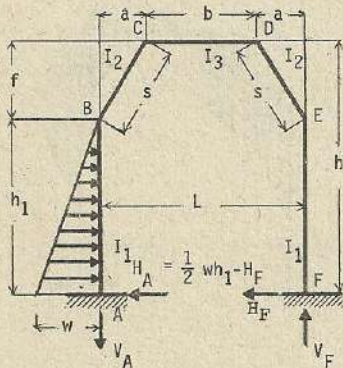
Otros datos
(305)

$$H_D = \frac{wh_1^2}{8} \cdot \frac{k_1(2k_1+3k_2+5)h_1 + 4k_1(k_2+1)f}{B} \quad ; \quad H_A = wh_1 - H_F$$

$$V_A = V_F = V = \frac{wh_1^2 k_1 L}{Y}$$

$$M_A = - \frac{(k_1+k_2+2)h_1 + (k_2+1)f}{2k_1+k_2+2} \cdot H + \frac{wh_1^2(5k_1+3k_2+6)}{6(2k_1+k_2+2)} - \frac{L}{2} V$$

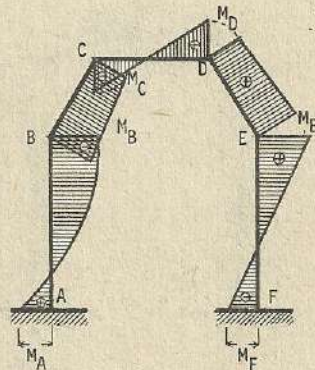
$$M_F = - \frac{1}{2} \cdot wh_1^2 + M_A + VL$$



(311)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

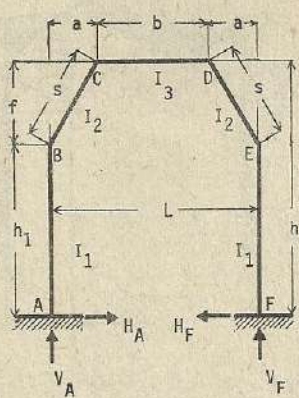
$$k_2 = \frac{I_2 h}{I_3 s}$$

otros datos
(305)

$$H_F = \frac{wh_1^2}{40} \cdot \frac{k_1(3k_1+4k_2+8)h_1 + 5k_1(k_2+1)f}{B} \quad ; \quad H_A = \frac{1}{2} wh_1 - H_F$$

$$V_A = V_F = V = \frac{wh_1^2 k_1 L}{4Y} \quad ; \quad M_F = - \frac{1}{6} wh_1^2 + M_A + VL$$

$$M_A = - \frac{(k_1+k_2+2)h_1 + (k_2+1)f}{2k_1+k_2+2} \cdot H + \frac{wh_1^2(7k_1+4k_2+8)}{24(2k_1+k_2+2)} - \frac{L}{2} V$$



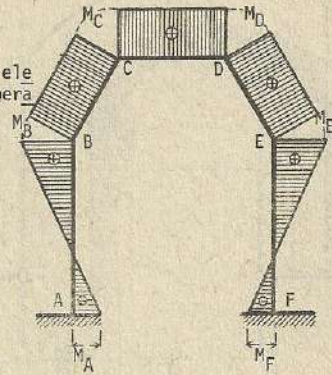
(312)

Esfuerzos por elevación de Temperatura

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

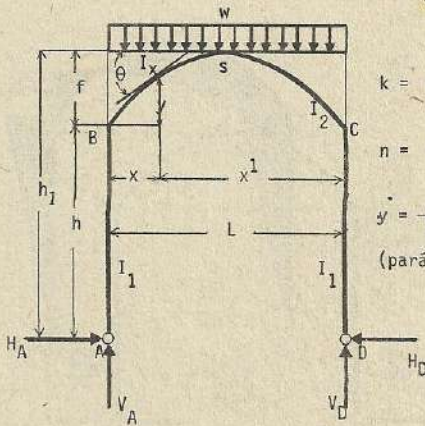
otros datos (305)



$$H_A = H_F = H = \frac{3(2k_1 + k_2 + 2)E\epsilon t L I_2}{s\beta} ; M_A = M_F = -\frac{3[(k_1 + k_2 + 2)h_1 + (k_2 + 1)f]E\epsilon t L I_2}{s\beta}$$

Nota : En caso de descenso de temperatura, los esfuerzos se anotan con los signos contrarios.

(313)

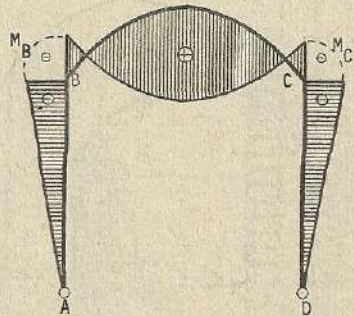


$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

$$y = \frac{4f}{L^2} x x'$$

(parábola)

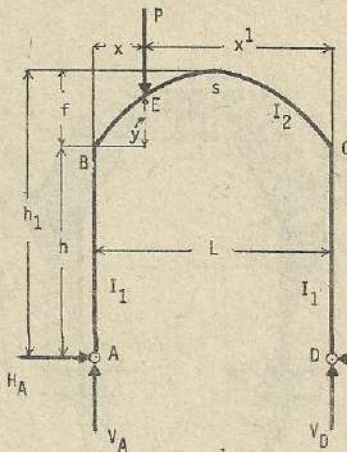


$$\alpha = 5(2k+3) + 4n(2n+5) ; I_x = I_2 \sec \theta$$

$$H_A = H_D = H = \frac{wL^2}{4\alpha h} (4n+5) = \frac{wL^2}{4} \cdot \frac{(5h+4f)}{[5h^2(2k+3) + 4f(5h+2f)]}$$

$$V_A = V_D = V = \frac{wL}{2}$$

$$M_B = M_C = -Hh ; M_s = \frac{wL^2}{8} - Hh_1 ; M_x = \frac{wx x'}{2} - H(h+y)$$



(314)

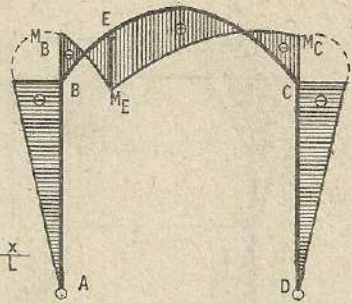
$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos

(313)

$$\beta = \frac{x^1}{L} \quad \gamma = \frac{x}{L}$$

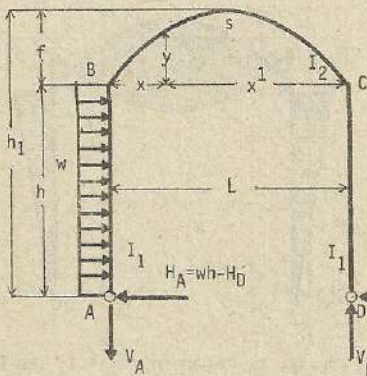


$$H_A - H_D - H = \frac{5Px^1}{2\alpha Lh} [3 + 2n(1 + \beta\gamma)] = \frac{5Px(3x^1 hL^2 + 2fL^2 - 4fx^2 L + 2fx^3)}{2L^3 [5h^2(2k+3) + 4f(5h+2f)]}$$

$$M_B = M_C = -Hh \quad ; \quad V_A = \beta P \quad ; \quad V_D = \gamma P \quad ; \quad M_s = \frac{Px}{2} - Hh_1$$

$$M_E = \frac{pxx^1}{L} - H(h + 4\beta\gamma f) \quad ; \quad M_x = M - H(h + \gamma) \quad ; \quad M : \text{esfuerzo de flexión de la viga BC}$$

(315)



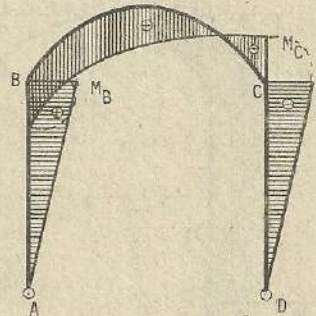
$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos

(313)

$$H_A = wh - H_D \quad ; \quad V_A = V_D = V = \frac{wh^2}{2L}$$



$$H_D = \frac{5wh}{8\alpha} (6 + 5k + 4n) = \frac{5wh^2}{8} \cdot \frac{[h(5k+6) + 4f]}{[5h^2(2k+3) + 4f(5h+2f)]}$$

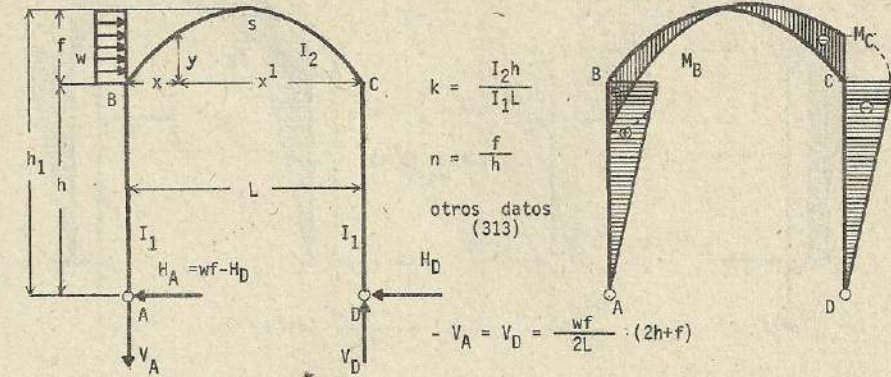
$$M_B = \frac{wh^2}{2} - H_D h \quad ; \quad M_C = -H_D h \quad ; \quad M_s = \frac{wh^2}{4} - H_D h_1$$

Esfuerzo de flexión en cada miembro : AB $\rightarrow M = wy(h - \frac{y}{2}) - H_D y$

$$BC \rightarrow M = \frac{wh^2}{2} \cdot \frac{L-x}{L} - H_D y$$

$$CD \rightarrow M = -H_D y$$

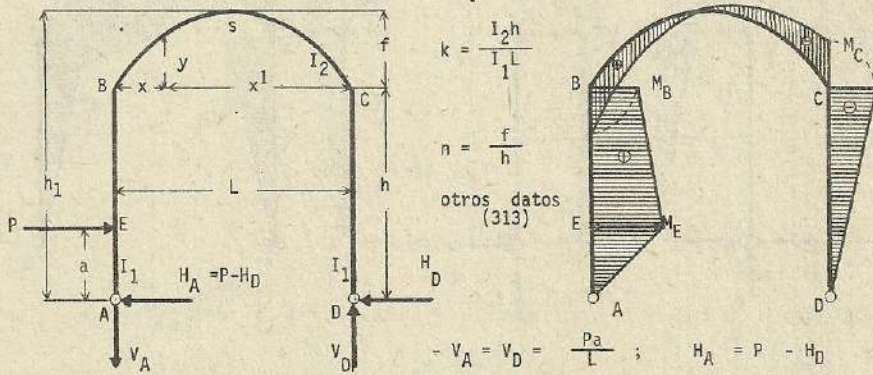
(316)



$$H_D = \frac{wf}{28} \cdot \frac{[70h^2(2k+3) + f(224h + 64f)]}{[5h^2(2k+3) + 4f(5h+2f)]} ; \quad H_A = wf - H_D$$

$$M_B = H_A h ; \quad M_C = -H_D h ; \quad M_s = \frac{V_D L}{2} - H_D(h+f)$$

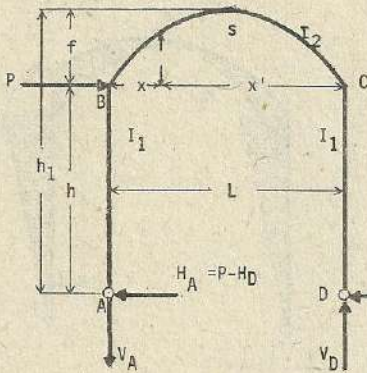
(317)



$$H_D = \frac{5Pa}{2h} \cdot \frac{[3h^2(k+1) + 2fh - a^2k]}{[5h^2(2k+3) + 4f(5h+2f)]} ; \quad M_E = H_A a ; \quad M_B = Pa - H_D h$$

$$M_C = -H_D h$$

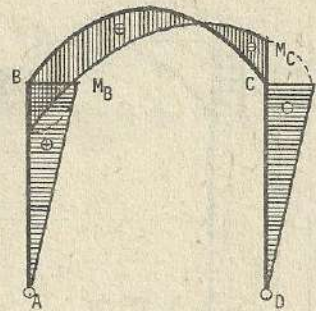
(318)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(313)



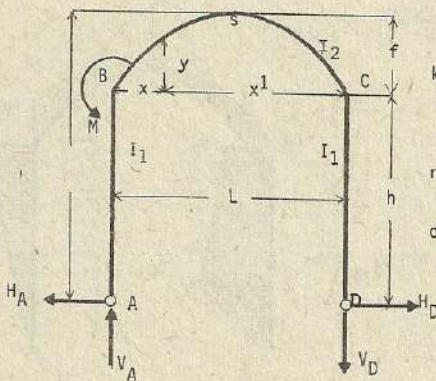
$$H_D = \frac{5P}{2\alpha} \cdot (3+2k + 2n)$$

$$M_B = (P - H_D) h$$

$$; \quad M_s = \frac{Ph}{2} - H_D h_1 \quad ; \quad M_D = -H_D h$$

$$- \quad V_A = V_D = \frac{Ph}{L}$$

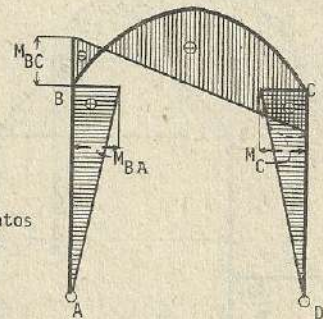
(319)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(313)



$$H_A = H_D = H = \frac{5(3+2n)M}{2\alpha h}$$

$$M_{BA} = M_C = Hh$$

;

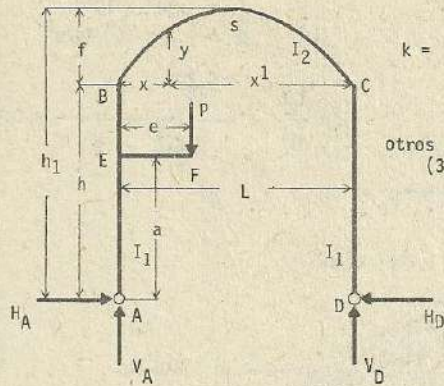
$$M_{BC} = Hh - M$$

;

$$M_s = Hh_1 - \frac{M}{2}$$

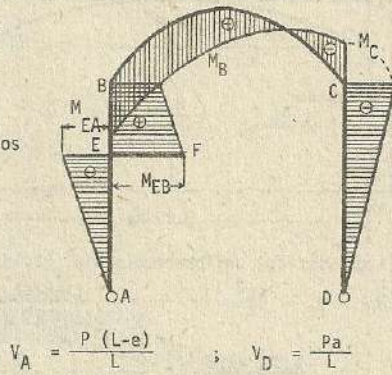
$$V_A = V_D = V = \frac{M}{L}$$

(320)



$$k = \frac{I_2 h}{I_1 L}$$

otros datos
(313)



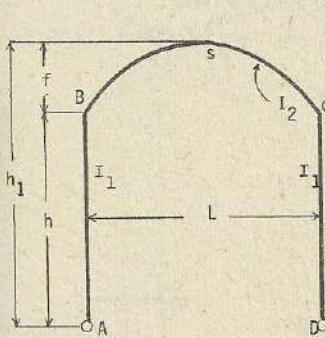
$$V_A = \frac{P(L-e)}{L} \quad ; \quad V_D = \frac{Pa}{L}$$

$$H_A = H_D = H = \frac{5Pe}{2h} \frac{[2k(h^2 - a^2) + h(3h + 2f)]}{[5h^2(2k+3) + 4f(5h+2f)]}$$

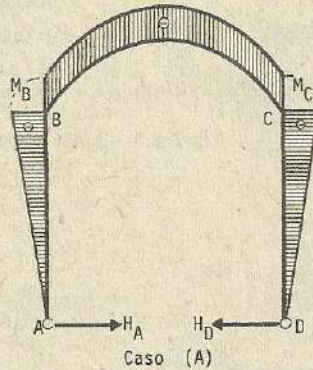
$$; \quad M_{EA} = -Ha \quad ; \quad M_{EB} = Pe - H_D$$

$$M_B = Pe - Hh \quad ; \quad M_C = -Hh$$

Esfuerzos por cambio de temperatura : (321)

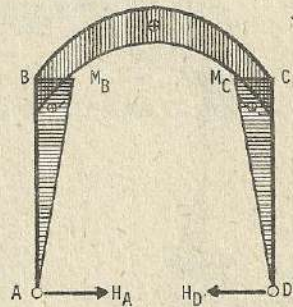


$$k = \frac{I_2 h}{I_1 L}$$



Caso (A)

incremento de temperatura

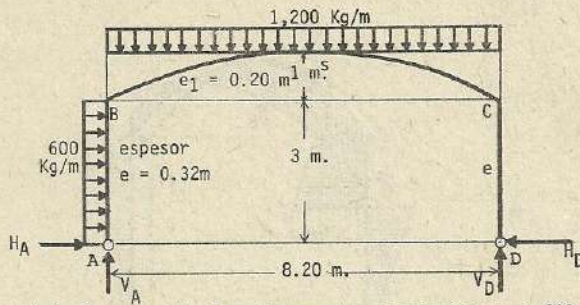


Caso (B)

descenso de temperatura

$$H_A = H_D = H = \frac{15E\epsilon I_2 t}{\alpha h^2} \frac{15E\epsilon I_2 t}{5h^2(2k+3) + 4f(5h+2f)} \quad ; \quad V = 0$$

$$M_B = M_C = -Hh \quad ; \quad M_s = -Hh_1$$



Datos : $t = + 20 \text{ }^{\circ}\text{C}$

$$\varepsilon = 1/10^5 = 10^{-5}$$

$$E = 2.1 \times 10^9 \text{ Kg/m}^2$$

$$k = \frac{I_2 h}{I_1 L} = \frac{\frac{0.20^3}{12} \times 3.0}{\frac{0.32^3}{12} \times 8.2} = 0.09$$

(a) Carga vertical uniformemente distribuida : 1,200 Kg/m.

$$H_A = H_D = H = \frac{wL^2}{4} - \frac{(5h+4f)}{5h^2(2k+3) + 4f(5h+2f)}$$

$$= \frac{1,200 \times 8.20^2}{4} \times \frac{5 \times 3 + 4 \times 1}{5 \times 3 \times 3 (2 \times 0.09 + 3) + 4 \times 1 (5 \times 3 + 2 \times 1)} = 1,816 \text{ Kg.}$$

$$M_B = M_C = -Hh = -1,816 \times 3 = -5,448 \text{ Kg-m}$$

$$M_S = \frac{wL^2}{8} - Hh_1 = \frac{1,200}{8} \times 8.2 \times 8.2 - 1,816 \times 4 = 2,822 \text{ Kg-m}$$

(b) Carga lateral uniformemente distribuida : $w = 600 \text{ Kg/m}$.

$$H_D = \frac{5wh^2}{8} \frac{[h(5k+6) + 4f]}{[5h^2(2k+3) + 4f(5h+2f)]}$$

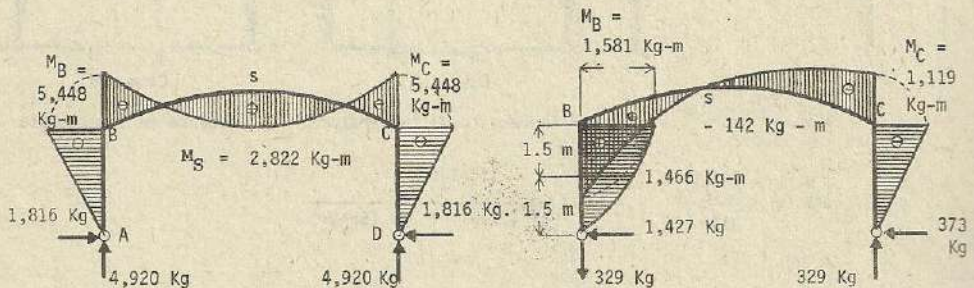
$$= \frac{5 \times 600 \times 3^2}{8} \frac{[3(5 \times 0.09 + 6) + 4 \times 1]}{[5 \times 3^2(2 \times 0.09 + 3) + 4 \times 1(5 \times 3 + 2 \times 1)]} = 373 \text{ Kg.}$$

$$H_A = wh - H_D = 600 \times 3 - 373 = 1,427 \text{ Kg.} ; \quad V_A = V_D = \frac{wh^2}{2L} = \frac{600 \times 3^2}{2 \times 8.2} = 329 \text{ Kg.}$$

$$M_B = \frac{wh^2}{4} - H_D h = \frac{600 \times 3^2}{4} - 373 \times 3 = 1,581 \text{ Kg-m} ; \quad M_C = -H_D h = -373 \times 3 = -1,119 \text{ Kg-m}$$

$$M_S = \frac{wh^2}{4} - H_D h_1 = \frac{600 \times 3^2}{4} - 373 \times 4 = -142 \text{ Kg-m.}$$

$$M_E = 600 \times 1.5 \left(3 - \frac{1.5}{2} \right) - 373 \times 1.5 = 1,466 \text{ Kg-m}$$



Caso de carga vertical

Caso de carga lateral

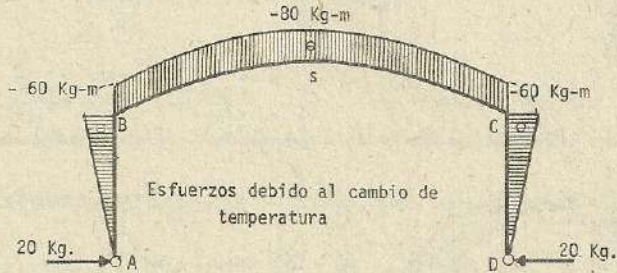
(c): Esfuerzos debido a cambio de temperatura

$$H_A = H_D = H = \frac{15EI_2\epsilon t}{5h^2(2k+3) + 4f(5h+2f)}$$

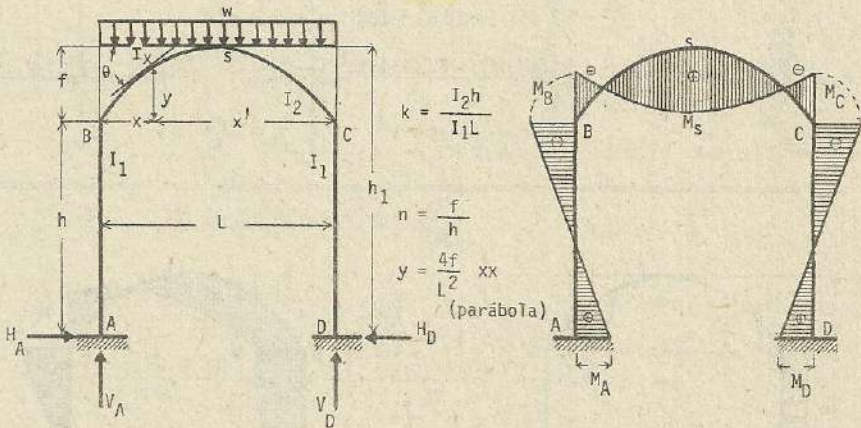
$$= \frac{15 \times 2.1 \times 10^9 \times \frac{0.2^3}{12} \times 10^{-5} \times 20}{5 \times 3^2(2 \times 0.09 + 3) + 4 \times 1(5 \times 3 + 2 \times 1)} = 20 \text{ Kg.}$$

$$M_B = -M_C = -Hh = -20 \times 3 = -60 \text{ Kg-m}$$

$$M_S = -Hh_1 = -20 \times 4 = -80 \text{ Kg-m}$$



(322)



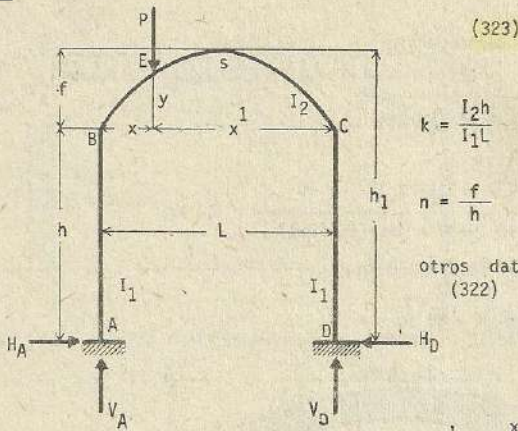
$$\lambda_1 = k(2+k+2n); \quad \lambda_2 = k(3+4n); \quad \mu_1 = 5k+12nk+2n; \quad \mu_2 = 15k+30nk-4n^2$$

$$m = 3+3k+2n; \quad \alpha_1 = 15k(2+k) + 12nk(5+4n) + 4n^2; \quad \alpha_2 = 1 + 6k$$

$$H_A = H_D = H = \frac{wL^2}{4\alpha_1 h} (15k+24nk+2n); \quad V_A = V_D = V = \frac{wL}{2}$$

$$M_A = M_D = -\frac{3(k+1)h+2f}{3(2k+1)} H + \frac{wL^2}{12(2k+1)}; \quad M_x = \frac{wxx^1}{2} - H(h+y) + M_A$$

$$M_B = M_C = M_A - Hh; \quad M_s = \frac{wL^2}{8} + M_A - Hh_1$$



(323)

$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(322)

$$\alpha^1 = \frac{x}{L} ; \beta^1 = \frac{x^1}{L}$$

$$H_A = H_D = H = \frac{15xx^1}{2\alpha_1 h L} \left[\lambda_2 + 2n\alpha^1 \beta^1 (2k+1) \right] ; M_s = \frac{1}{2} (Px + M_A + M_D) - Hh_1$$

$$M_A = \frac{Pxx^1}{2\alpha_1 L} \left[\mu_2 + 10n\alpha^1 \beta^1 m \right] - \frac{\alpha^1 \beta^1 P(x^1 - x)}{2\alpha_2} ; M_D = \frac{Pxx^1}{2\alpha_1 L} \left[\mu_2 + 10n\alpha^1 \beta^1 m \right] + \frac{\beta^1 \alpha^1 P(x^1 - x)}{2\alpha_2}$$

$$M_B = M_A - Hh ; M_C = M_D - Hh ; M_E = \frac{Pxx^1}{L} + \beta^1 M_A + \alpha^1 M_D + H(h + 4\alpha^1 \beta^1 f)$$

$$V_A = \beta^1 P - M_A + M_D ; V_D = \alpha^1 P + M_A - M_D$$

Otra forma de solución : $k = \frac{I_2 h}{I_1 L} ; x = \alpha^1 L$

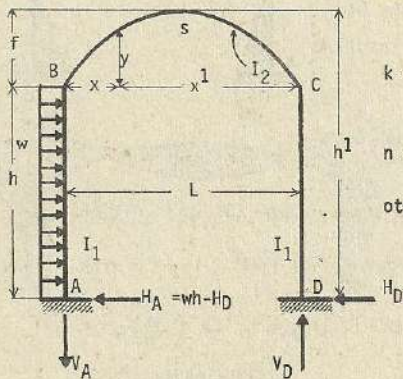
$$Y = 6k + 1$$

$$B = 15k(k+2)h^2 + 60kfh + 4(12k+1)f^2$$

$$H_A = H_D = H = \frac{15Px(L-x)}{2L} \cdot \frac{k(3h+4f) + 2(2k+1)f\alpha^1(1-\alpha^1)}{\beta} ; v = \frac{P(L-x)}{L} \cdot \frac{\alpha^1(1-2\alpha^1)}{Y}$$

$$V_A = \frac{Px^1}{L} + v ; V_D = \frac{Px}{L} - v$$

(324)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(322)

$$H_D = \frac{5(6\lambda_1 - \lambda_2)wh}{8\alpha_1} ; H_A = wh - H_D ; V_A = V_D = V = \frac{kw h^2}{\alpha_2 L}$$

$$M_A = -\frac{wh^2}{24\alpha_1} (\alpha_1 + 12n\mu_1 + \nu_2) - \frac{(1+4k)wh^2}{4\alpha_2}; \quad M_B = \frac{wh^2}{2} + M_A - H_D h$$

$$M_D = -\frac{wh^2}{24\alpha_1} (\alpha_1 + 12n\mu_1 + \nu_2) + \frac{(1+4k)wh^2}{4\alpha_2}; \quad M_C = M_D - H_D h$$

$$M_S = \frac{wh^2}{4} + \frac{M_A + M_D}{2} - H_D h_1$$

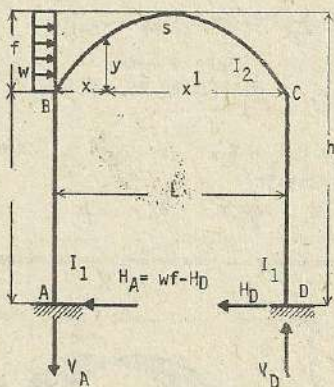
Uso de fórmula diferente: $H_D = \frac{5wh^3}{8} \cdot \frac{3k(2k+3)h + 8kf}{\beta}$

donde: $\beta = 15k(k+2)h^2 + 60kfh + 4(12k+1)f^2$

$$v = \frac{wh^2}{L} \cdot \frac{k}{\gamma} \quad \text{siendo} \quad \gamma = 6k+1$$

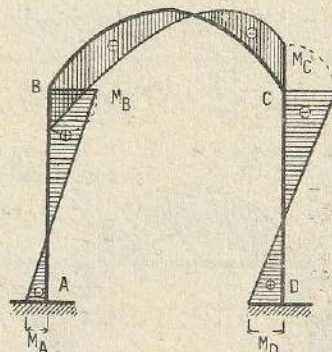
$$M_A = -\frac{3(k+1)h + 2f}{3(2k+1)} H_D + \frac{wh^2(3k+3)}{6(2k+1)} - \frac{L}{2} v; \quad M_D = -\frac{1}{2} wh^2 + M_A + vL$$

(325)



$$k = \frac{I_2 h}{I_1 L}$$

$$\gamma = 6k + 1$$



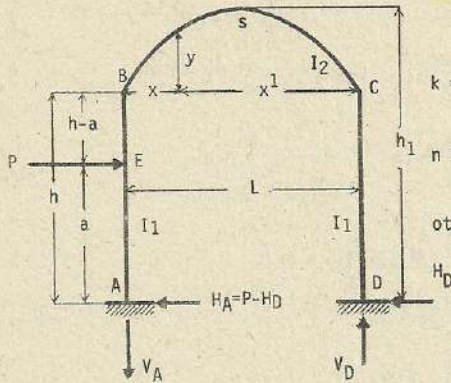
$$\beta = [15k(k+2)h^2 + 60kfh + 4(12k+1)f^2]$$

$$H_D = \frac{3wf}{14} \cdot \frac{35k(k+2)h^2 + 112kfh + 4(16k+1)f^2}{\beta}; \quad H_A = wf - H_D$$

$$v = -\frac{wf}{8L} \cdot \frac{12kh + (12k+1)f}{\gamma}; \quad M_D = -\frac{1}{2} wf(2h+f) + M_A + vL$$

$$M_A = -\frac{3(k+1)h + 2f}{3(2k+1)} + \frac{wf(10(3k+2)h + (10k+9)f)}{20(2k+1)} - \frac{L}{2} v$$

(326)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(322)

$$C_{AB} = -\frac{Pa(h-a)^2}{h^2} ; C_{BA} = -\frac{Pa^2(h-a)}{h^2}$$

$$H_D = \frac{15}{2\alpha_1} \frac{1}{h} [\lambda_1 (Pa + C_{AB} - C_{BA}) + \lambda_2 C_{BA}] ; H_A = P - H_D$$

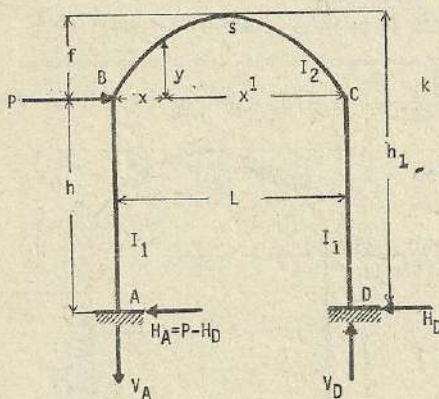
$$M_A = -\frac{1}{2} \frac{1}{\alpha_1} [2n u_1 (Pa + C_{AB} - C_{BA}) - \alpha_1 C_{AB} - u_2 C_{BA}] - \frac{1}{2\alpha_2} [(1+2k)Pa - 3k(C_{AB} + C_{BA})]$$

$$M_D = -\frac{1}{2\alpha_1} [2nu_1 (Pa + C_{AB} - C_{BA}) - \alpha_1 C_{AB} - u_2 C_{BA}] + \frac{1}{2\alpha_2} [(1+2k)Pa - 3k(C_{AB} + C_{BA})]$$

$$M_B = Pa + M_A - H_D h ; V_A = V_D = V = \frac{1}{L} (Pa + M_A - M_D) ; M_C = M_D - H_D h$$

$$M_x = (Pa + M_A) \frac{x^1}{L} + M_D \frac{x}{L} - H_D (h+y) ; M_s = \frac{1}{2} VL \pm M_D - H_D h$$

(327)



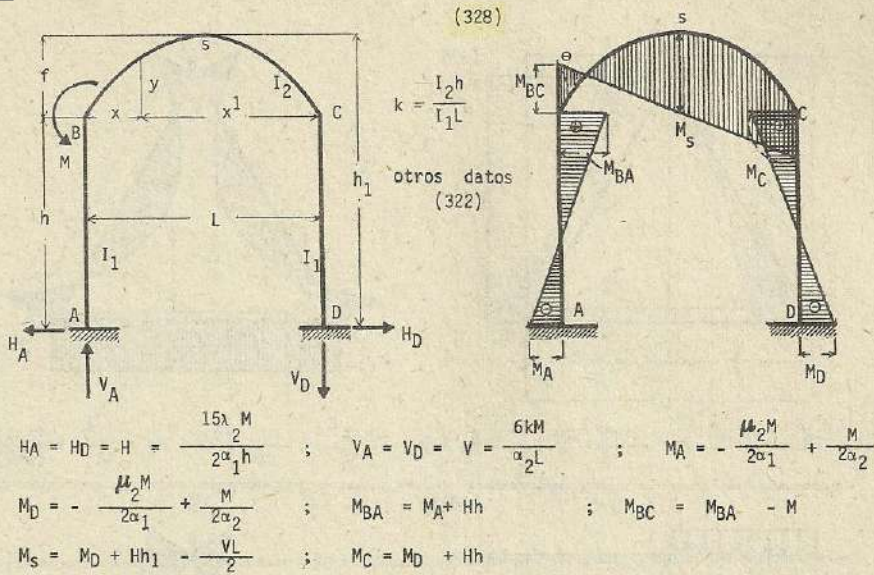
$$k = \frac{I_2 h}{I_1 L}$$

otros datos
(322)

$$H_D = \frac{15\lambda_1 P}{2\alpha_1} ; H_A = P - H_D ; V_A = V_D = V = \frac{3kPh}{\alpha_2 L}$$

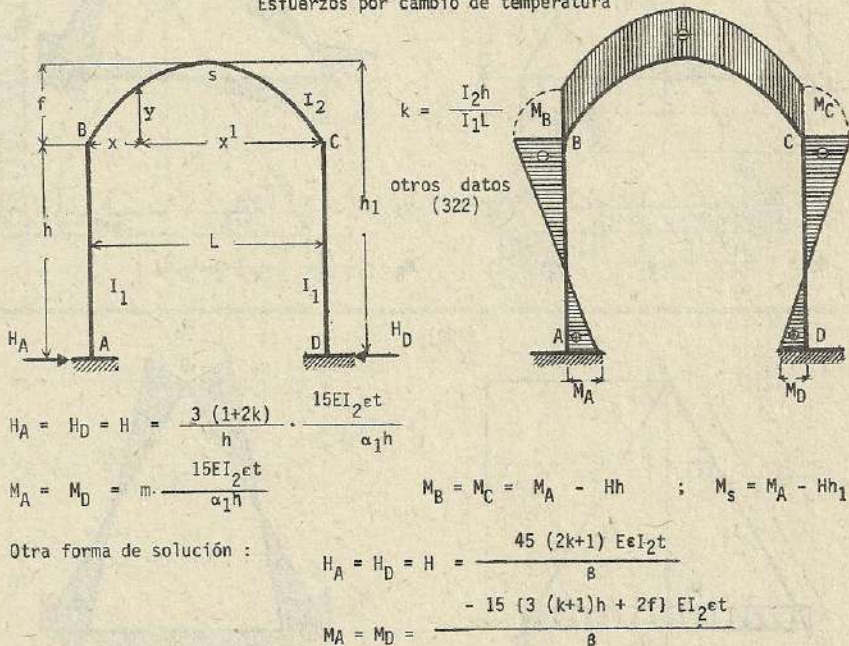
$$M_A = -\frac{u_1 Pf}{\alpha_1} - (1+3k) \frac{Ph}{2\alpha_2} ; M_D = -\frac{u_1 Pf}{\alpha_1} + (1+3k) \frac{Ph}{2\alpha_2} ; M_B = (P-H_D)h + M_A$$

$$M_C = M_D - H_D h ; M_s = M_D + \frac{1}{2} VL - H_D h_1$$

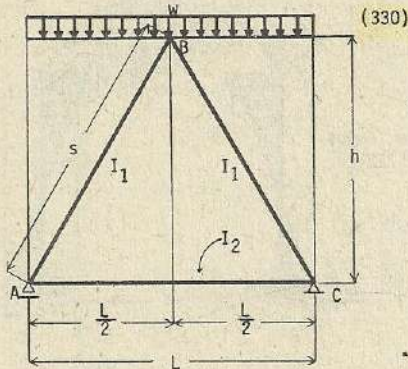


(329)

Esfuerzos por cambio de temperatura



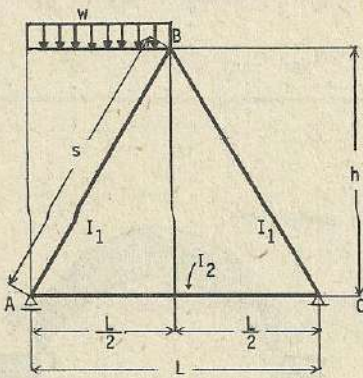
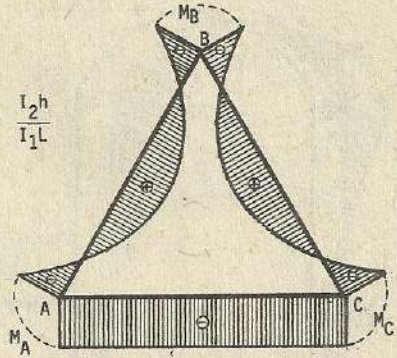
Nota : El presente problema corresponde al incremento de temperatura, en caso de descenso, los esfuerzos se anotan con los signos contrarios.



$$H = \frac{wL}{16} \cdot \frac{L}{h} \cdot \frac{2k+5}{k+2} ;$$

$$M_A = M_C = -\frac{wL^2}{48} k \frac{1}{(k+2)} ; \quad M_B = -\frac{wL^2}{48} k \frac{(3+k)}{(k+2)}$$

$$k = \frac{I_2 h}{I_1 L}$$



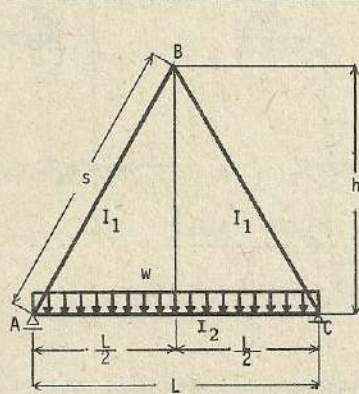
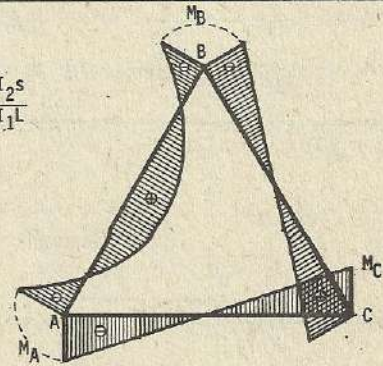
$$H = \frac{wL}{32} \cdot \frac{L}{h} \cdot \frac{(2k+5)}{(k+2)} ;$$

$$M_C = -\frac{wL^2}{96} \left[\frac{1}{(k+2)} - \frac{3}{2k+1} \right] k ;$$

$$k = \frac{I_2 s}{I_1 L}$$

$$M_A = -\frac{wL^2}{96} \left[\frac{1}{k+2} + \frac{3}{2k+1} \right] k$$

$$M_B = -\frac{wL^2}{96} \left[\frac{1}{k+2} + 1 \right]$$

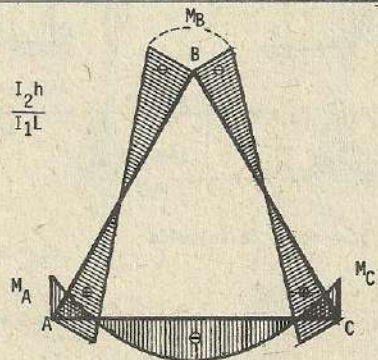


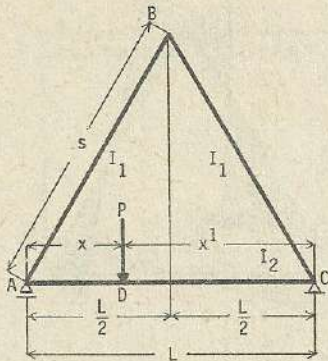
$$M_A = M_C = -\frac{wL^2}{6} \frac{1}{k+2} ;$$

$$k = \frac{I_2 h}{I_1 L}$$

$$H = \frac{wL}{4} \cdot \frac{L}{h} \cdot \frac{1}{k+2}$$

$$M_B = -\frac{wL^2}{12(k+2)}$$





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$$k = \frac{I_2 h}{I_1 L}$$

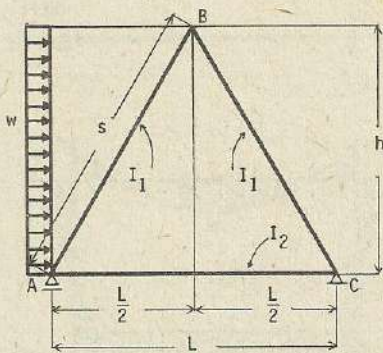
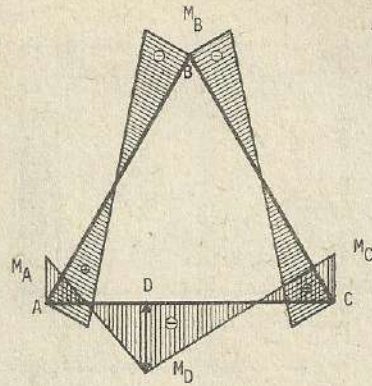
$$\alpha = \frac{x}{L}$$

$$\beta = \frac{x_1}{L}$$

$$H = -\frac{3}{2} P \frac{L}{h} \cdot \frac{\alpha \beta}{k+2} ;$$

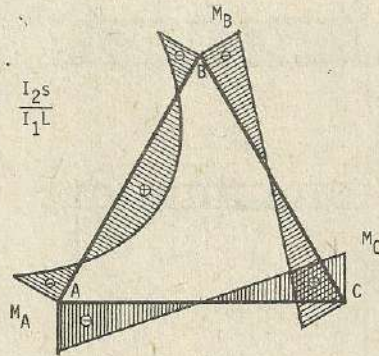
$$M_A = \frac{PL}{2} \alpha \beta \left[\frac{2}{k+2} + \frac{\beta - \alpha}{2k+1} \right]; \quad M_C = \frac{PL}{2} \left[\frac{2}{k+2} - \frac{\beta - \alpha}{2k+1} \right] \alpha \beta$$

$$M_B = -\frac{P x x_1}{L^2} \left[\frac{x + x_1}{2(k+2)} \right]$$



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$$k = \frac{I_2 s}{I_1 L}$$



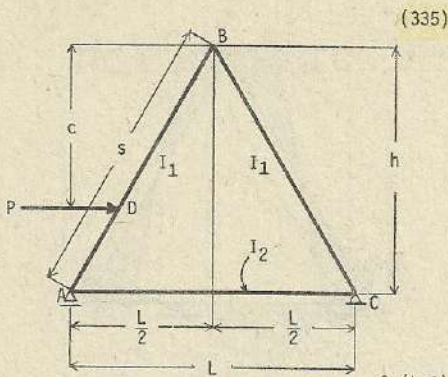
$$H_A = -\frac{wh}{2} \left[1 + \frac{2k+3}{4(k+2)} \right] ;$$

$$H_C = -\frac{wh}{2} \left[-1 + \frac{2k+3}{4(k+2)} \right]$$

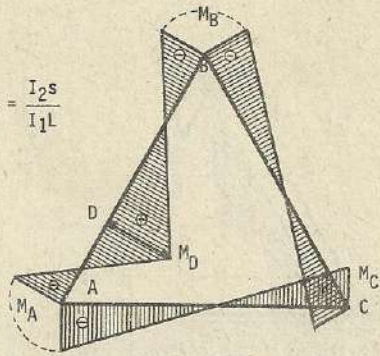
$$M_A = -\frac{kwh^2}{24} \left[\frac{1}{k+2} + \frac{3}{2k+1} \right] ;$$

$$M_C = -\frac{kwh^2}{24} \left[\frac{1}{k+2} - \frac{3}{2k+1} \right]$$

$$M_B = -\frac{(k+3)wh^2}{24(k+2)}$$



$$k = \frac{I_2 s}{I_1 L}$$

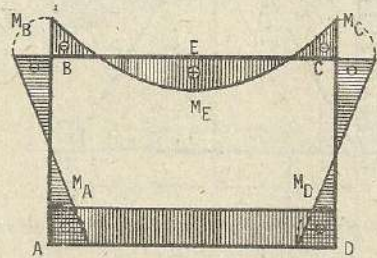
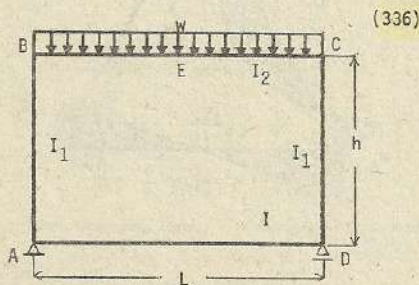


$$H_A = -\frac{P}{2} \left[1 + \left(\frac{c}{h} \right)^2 \frac{3(k+1) - \frac{c}{h}(2k+1)}{k+2} \right]$$

$$H_C = -\frac{P}{2} \left[-1 + \left(\frac{c}{h} \right)^2 \frac{3(k+1) - \frac{c}{h}(2k+1)}{k+2} \right]$$

$$M_A = -\frac{P}{2} k \left(1 - \frac{c}{h} \right) \left[\frac{\frac{c}{h}}{k+2} + \frac{\frac{c}{h} + 1}{2k+1} \right]$$

$$M_B = -\frac{P}{2} \frac{(h-c)c}{h^2} \left[\frac{c}{k+2} + (h-c) \right]; \quad M_C = -\frac{P}{2} k \left(1 - \frac{c}{h} \right) \left[\frac{\frac{c}{h}}{k+2} - \frac{\frac{c}{h} + 1}{2k+1} \right]$$



$$k = \frac{I_2 h}{I_1 L} \quad k^1 = \frac{I_2}{I_1}$$

$$\nu_1 = k\beta + k^1\alpha \quad \nu_2 = \gamma + \gamma^1$$

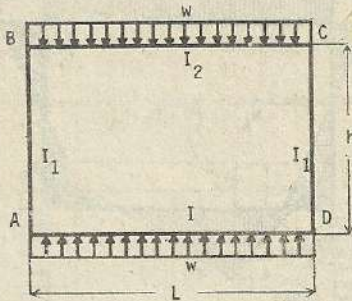
$$\alpha = 2k+3 \quad \beta = k+2$$

$$\gamma = 3k+1$$

$$\alpha^1 = 2k+3k^1 \quad \beta^1 = k+2k^1$$

$$\gamma^1 = 3k+k^1$$

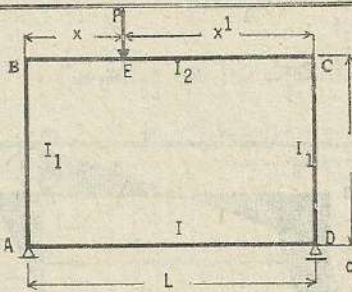
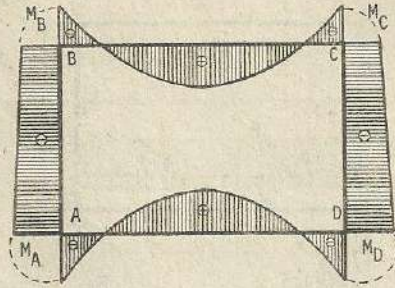
$$M_A = M_D = \frac{kwL^2}{12\nu_1}; \quad M_B = M_C = -\frac{\alpha^1 wL^2}{12\nu_1}; \quad M_E = \frac{wL^2}{8} - M_B$$



(337)

$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos, referencia (336)}$$

$$M_A = M_D = -\frac{wL^2}{12\mu_1} (3k^1 + 2kk^1 - K) ; \quad M_B = M_C = -\frac{wL^2}{12\mu_1} (3k^1 - kk^1 + 2k)$$



(338)

$$k = \frac{I_2 h}{I_1 L}$$

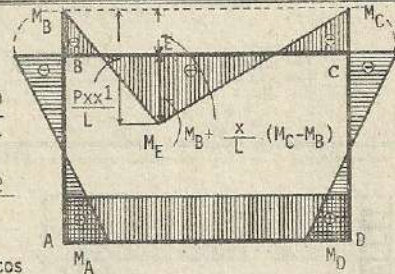
$$k^1 = \frac{I_2}{I}$$

otros datos (336)

$$C_{BC} = -\frac{Px(x^1)^2}{L^2} ; \quad C_{CB} = \frac{Px^2 x^1}{L^2}$$

$$M_D = -\frac{k(C_{BC} + C_{CB})}{2\mu_1} - \frac{C_{BC} + C_{CB}}{2\mu_2}$$

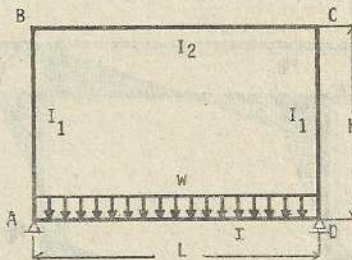
$$M_C = \frac{\alpha^1(C_{BC} - C_{CB})}{2\mu_1} - \frac{C_{BC} - C_{CB}}{2\mu_2}$$



$$M_A = -\frac{k(C_{BC} + C_{CB})}{2\mu_1} + \frac{C_{BC} - C_{CB}}{2\mu_2}$$

$$M_B = \frac{\alpha^1(C_{BC} + C_{CB})}{2\mu_1} + \frac{C_{BC} - C_{CB}}{2\mu_2}$$

$$M_E = \frac{Px x^1}{L} - \left[M_B + \frac{x}{L} (M_C - M_B) \right]$$



(339)

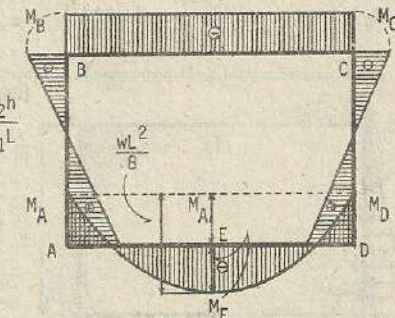
$$k = \frac{I_2 h}{I_1 L}$$

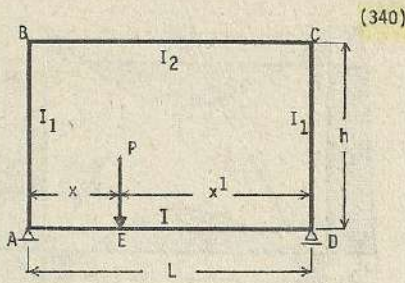
$$C_{AD} = C_{DA} = -\frac{wL^2}{12}$$

$$M_A = C_{AD} \frac{2k+3}{(k+1)(k+3)} = M_D$$

$$M_B = -C_{AD} \frac{k}{(k+1)(k+3)} = M_C$$

$$M_E = \frac{wL^2}{8} - M_A$$



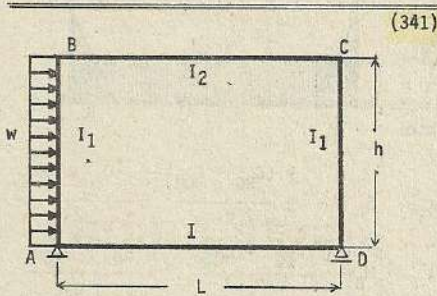
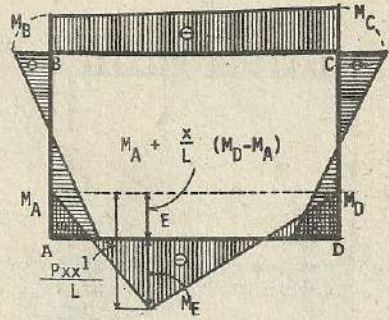


$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos (336)}$$

$$C_{AD} = \frac{Px \frac{x^2}{L^2}}{L^2} ; \quad C_{DA} = \frac{Px \frac{x^2}{L^2}}{L^2} ; \quad M_A = - \frac{\alpha k^1 (C_{AD} + C_{DA})}{2\mu_1} - \frac{k^1 (C_{AD} - C_{DA})}{2\mu_2}$$

$$M_D = - \frac{\alpha k^1 (C_{AD} + C_{DA})}{2\mu_1} + \frac{k^1 (C_{AD} - C_{DA})}{2\mu_2} ; \quad M_B = \frac{kk^1 (C_{AD} + C_{DA})}{2\mu_1} - \frac{k^1 (C_{AD} - C_{DA})}{2\mu_2}$$

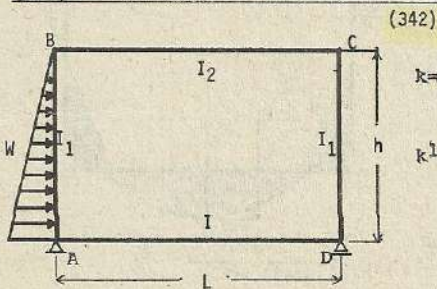
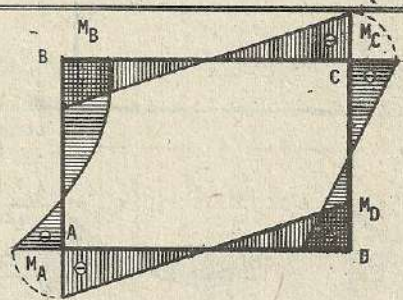
$$M_C = \frac{kk^1 (C_{AD} + C_{DA})}{2\mu_1} + \frac{k^1 (C_{AD} - C_{DA})}{2\mu_2} ; \quad M_E = \frac{Px x^1}{L} - \left[M_A + \frac{x}{L} (M_D - M_A) \right]$$



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos (336)}$$

$$M_A = \frac{wh^2}{4} \left[- \frac{k - (k+3)}{6\mu_1} - \frac{4k+1}{\mu_2} \right] ; \quad M_D = \frac{wh^2}{4} \left[- \frac{k - (k+3)}{6\mu_1} + \frac{4k+1}{\mu_2} \right]$$

$$M_B = M_C = \frac{wh^2}{4} \left[- \frac{k - (k+3)}{6\mu_1} \pm \frac{2k+k^1}{\mu_2} \right]$$

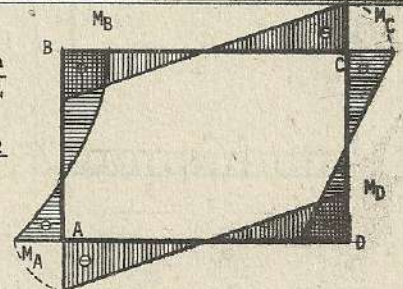


$$k = \frac{I_2 h}{I_1 L}$$

$$k^1 = \frac{I_2}{I}$$

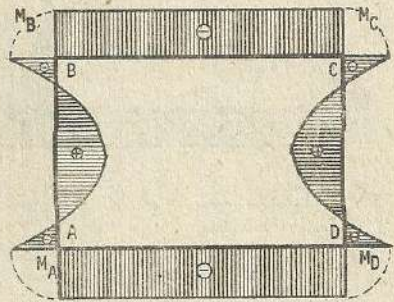
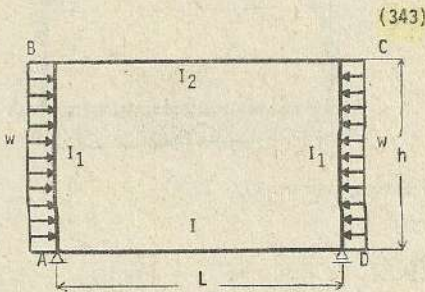
$$m = \frac{1}{1+6k+k^1}$$

$$n = \frac{1}{2+k + \frac{k^1}{k} (2k+3)}$$



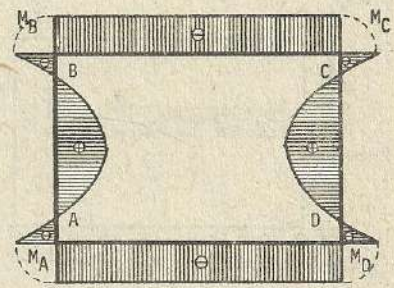
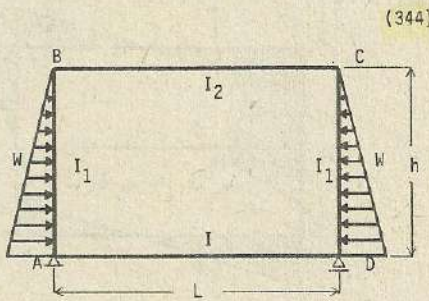
$$M_A = -\frac{Wh}{3} \left[\frac{3k+8}{20} n + \frac{9k+2}{4} m \right] ; \quad M_D = -\frac{Wh}{3} \left[\frac{3k+8}{20} n - \frac{9k+2}{4} m \right]$$

$$M_B = -\frac{wh}{2} \left[\frac{2k+7k^1}{20} n - \frac{3k+2k^1}{4} m \right] ; \quad M_C = -\frac{Wh}{2} \left[\frac{2k+7k^1}{20} n + \frac{3k+2k^1}{4} m \right]$$



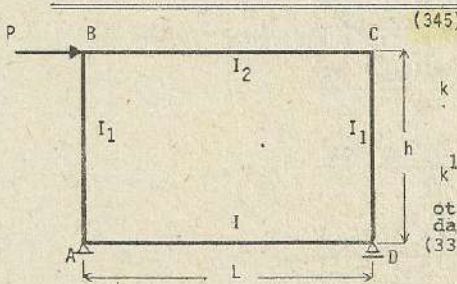
$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad n = \frac{1}{2+k + \frac{k^1}{k} (2k+3)}$$

$$M_A = M_D = -\frac{wh^2}{12} (k+3) n ; \quad M_B = M_C = -\frac{wh^2}{12} (3k^1+k) n$$

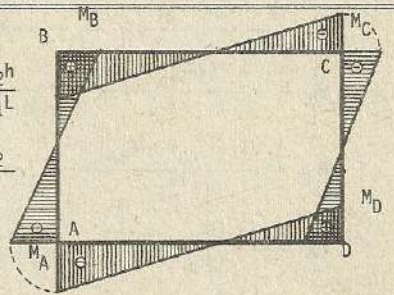


$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad n = \frac{1}{2+k + \frac{k^1}{k} (2k+3)}$$

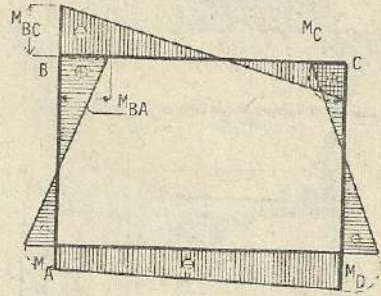
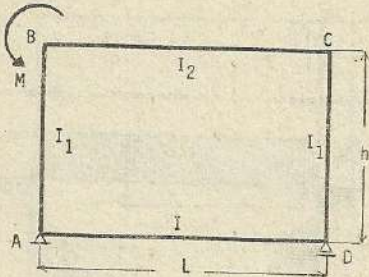
$$M_A = M_D = -\frac{Wh}{30} (3k+8) n ; \quad M_B = M_C = -\frac{Wh}{30} (2k+7k^1) n$$



otros datos (336)



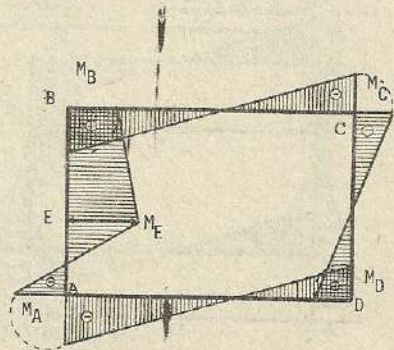
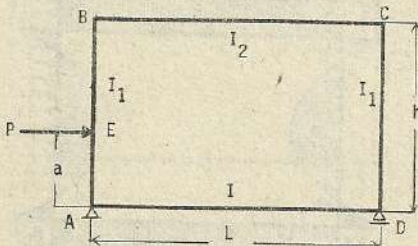
$$M_A = -\frac{\gamma Ph}{2\sqrt{2}} ; \quad M_B = \frac{\gamma^1 Ph}{2\sqrt{2}} ; \quad M_C = -\frac{\gamma^1 Ph}{2\sqrt{2}} ; \quad M_D = \frac{\gamma Ph}{2\sqrt{2}}$$



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos, referencia (336)}$$

$$M_A = - \frac{M}{2} \left[\frac{k}{\nu_1} - \frac{1}{\nu_2} \right] ; \quad M_D = - \frac{M}{2} \left[\frac{k}{\nu_1} + \frac{k}{\nu_2} \right]$$

$$M_{BA} = \frac{M}{2} \left[\frac{\alpha^1}{\nu_1} - \frac{1}{\nu_2} \right] ; \quad M_C = \frac{M}{2} \left[\frac{\alpha^1}{\nu_1} + \frac{1}{\nu_2} \right]; \quad M_{BC} = M_{BA} - M$$



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos (336)}$$

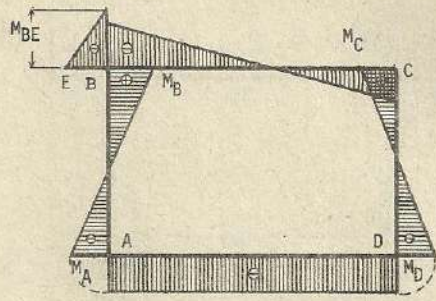
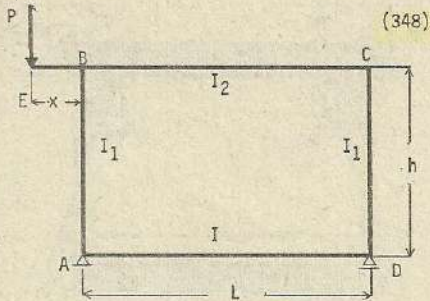
$$C_{AB} = - \frac{Pa(h-a)^2}{h^2} ; \quad C_{BA} = - \frac{Pa^2(h-a)}{h^2}$$

$$M_D = \frac{k(\beta C_{AB} + C_{BA})}{2\nu_1} + \frac{\gamma Pa - 3k(C_{AB} + C_{BA})}{2\nu_2}$$

$$M_B = \frac{k(k^1 C_{AB} + \beta^1 C_{BA})}{2\nu_1} + \frac{\gamma^1 Pa + 3k(C_{AB} + C_{BA})}{2\nu_2}$$

$$M_C = \frac{k(k^1 C_{AB} + \beta^1 C_{BA})}{2\nu_1} - \frac{\gamma^1 Pa + 3k(C_{AB} + C_{BA})}{2\nu_2}$$

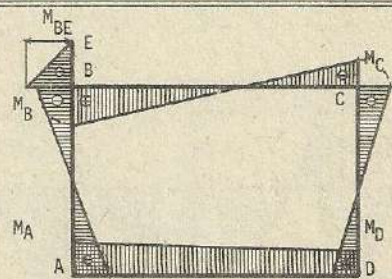
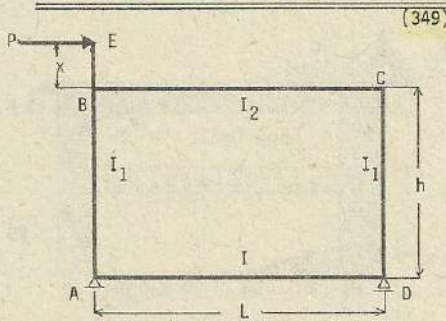
$$M_A = \frac{k(\beta C_{AB} + C_{BA})}{2\nu_1} - \frac{\gamma Pa - 3k(C_{AB} + C_{BA})}{2\nu_2}$$



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad m = \frac{1}{1+6k+k^1} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = -\frac{Px}{2}(n-m) ; \quad M_D = -\frac{Px}{2}(n+m)$$

$$M_{BA} = \frac{Px}{2} \left[(2+3\frac{k^1}{k})n+m \right] ; \quad M_C = \frac{Px}{2} \left[(2+3\frac{k^1}{k})n-m \right]$$

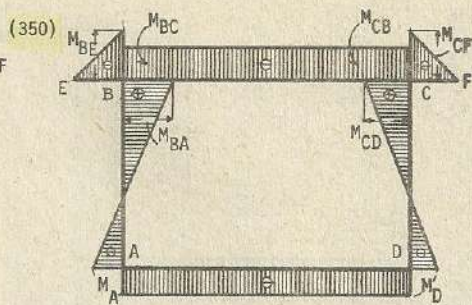
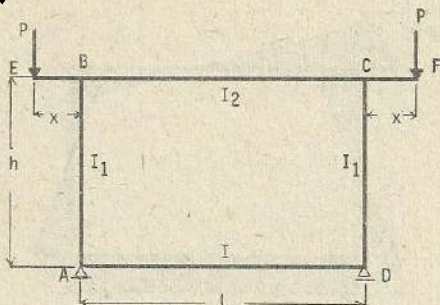


$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad m = \frac{1}{1+6k+k^1} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = \frac{Ph}{2} \left[\frac{x}{h}n - (1+3k+\frac{x}{h})m \right] ; \quad M_D = \frac{Ph}{2} \left[\frac{x}{h}n + (1+3k+\frac{x}{h})m \right]$$

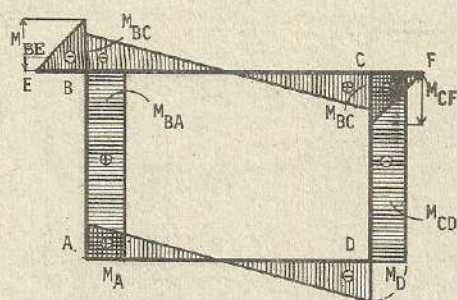
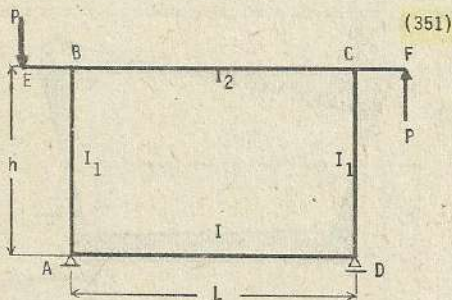
$$M_{BA} = -\frac{Ph}{2} \left[(2+3\frac{k^1}{k})\frac{x}{h}n - (k^1+3k-\frac{x}{h})m \right]$$

$$M_C = -\frac{Ph}{2} \left[(2+3\frac{k^1}{k})\frac{x}{h}n + (k^1+3k-\frac{x}{h})m \right]$$



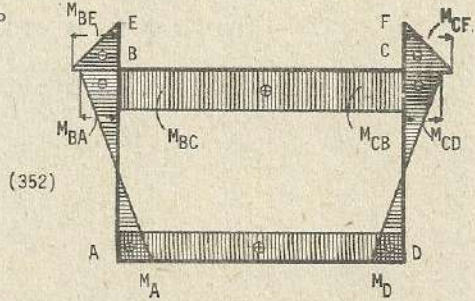
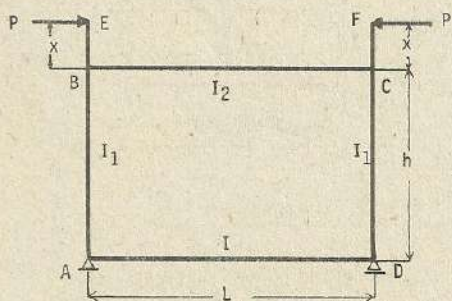
$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad m = \frac{1}{1+6k+k^1} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = M_D = Pxn ; \quad M_{BA} = Px(2+3\frac{k^1}{k})n ; \quad M_{BC} = M_{CB} = Pxn$$



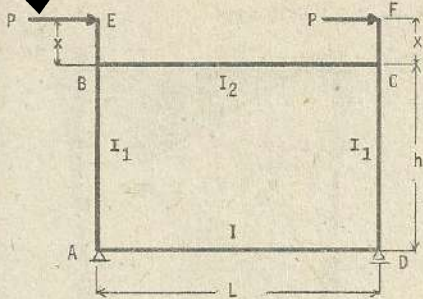
$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad m = \frac{1}{1+6k+k^1} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = Pxm = -M_D = M_{BA} = -M_{CD} ; \quad -M_{BC} = M_{CB} = M_{BA} = Pxn$$

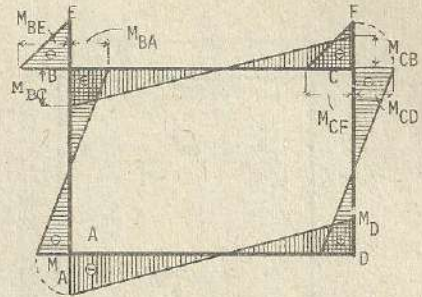


$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad m = \frac{1}{1+6k+k^1} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = Pxn = M_D ; \quad M_{BA} = -Px(2+3\frac{k^1}{k})n = M_{BC} = M_{CB} = M_{CD}$$



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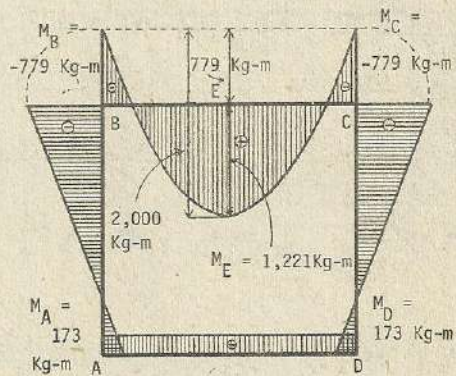
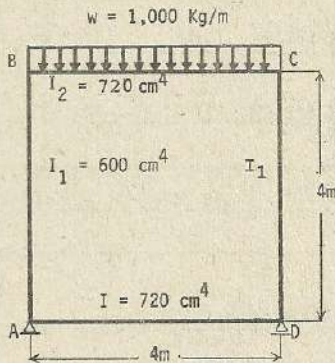


$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad m = \frac{1}{1+6k+k^1} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = -M_D = -P [h(1+3k) + x] m ; \quad M_{BA} = -M_{CD} = P [h(k^1+3k) - x] m$$

$$M_{BC} = -M_{CB} = M_{BA} + Px$$

Solución de problemas numéricos :



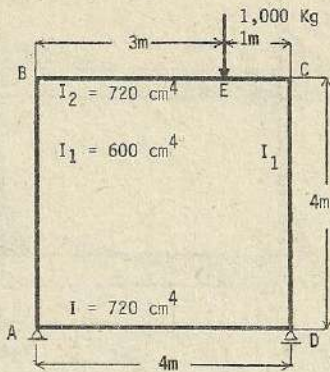
$$k = \frac{I_2 h}{I_1 L} = \frac{720 \times 4}{600 \times 4} = 1.2 ; \quad k^1 = \frac{I_2}{I} = \frac{720}{720} = 1$$

$$\alpha = 2k+3 = 2 \times 1.2 + 3 = 5.4 ; \quad \alpha^1 = 2k + 3k^1 = 2 \times 2.1 + 3 \times 1 = 5.4$$

$$\beta = k+2 = 1.2 + 2 = 3.2 ; \quad \mu_1 = k\beta + k^1\alpha = 1.2 \times 3.2 + 1 \times 5.4 = 9.24$$

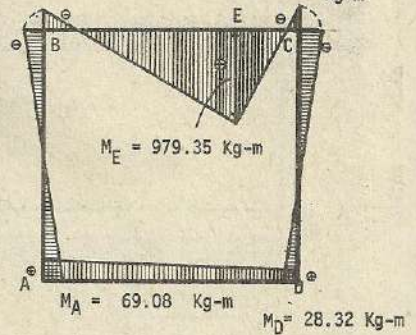
$$M_A = M_D = \frac{k w L^2}{12 \mu_1} = \frac{1.2 \times 1,000 \times 4 \times 4}{12 \times 9.24} = 173 \text{ Kg-m} ; \quad M_B = M_C = -\frac{\alpha^1 w L^2}{12 \mu_1} = -\frac{5.4 \times 1,000 \times 4 \times 4}{12 \times 9.24} = -779 \text{ Kg-m}$$

$$M_E = \frac{w L^2}{8} - M_B = \frac{1,000 \times 4 \times 4}{8} - 779 = 1,221 \text{ Kg-m}$$



$$M_B = -198.78 \text{ Kg-m}$$

$$M_C = -239.54 \text{ Kg-m}$$



$$k = \frac{I_2 h}{I_1 L} = \frac{720 \times 4}{600 \times 4} = 1.2 ; \quad k^1 = \frac{I_2}{I} = \frac{720}{720} = 1$$

$$C_{BC} = -\frac{Px(x^1)^2}{L^2} = -\frac{1,000 \times 3 \times 1 \times 1}{4 \times 4} = -187.5 \text{ Kg-m} ; \quad \gamma = 3k+1 = 3 \times 1.2 + 1 = 4.6$$

$$C_{CB} = -\frac{Px^2 x^1}{L^2} = -\frac{1,000 \times 3 \times 3 \times 1}{4 \times 4} = -562.5 \text{ Kg-m} ; \quad \gamma^1 = 3k^1 + 1 = 3 \times 1.2 + 1 = 4.6$$

$$\alpha = 2k+3 = 2 \times 1.2 + 3 = 5.4 ; \quad \alpha^1 = 2k+3k^1 = 2 \times 1.2 + 3 \times 1 = 5.4 ; \quad \beta = k+2 = 1.2 + 2 = 3.2$$

$$\nu_1 = k\beta + k^1 \alpha = 1.2 \times 3.2 + 1 \times 5.4 = 9.24 ; \quad \nu_2 = \gamma + \gamma^1 = 4.6 + 4.6 = 9.2$$

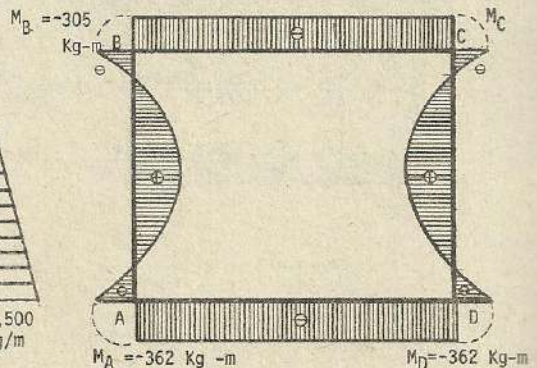
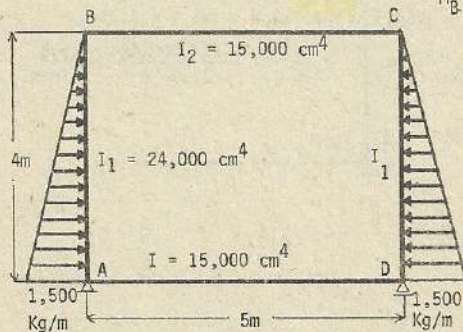
$$M_A = -\frac{k(C_{BC} + C_{CB})}{2\nu_1} + \frac{(C_{BC} + C_{CB})}{2\nu_2} + \frac{1.2 [(-187.5) + (-562.5)]}{2 \times 9.24} + \frac{(-187.5) - (-562.5)}{2 \times 9.2} = 69.08 \text{ Kg-m}$$

$$M_B = -\frac{\alpha^1 (C_{BC} + C_{CB})}{2\nu_1} + \frac{C_{BC} - C_{CB}}{2\nu_2} = +\frac{5.4 [(-187.5) + (-562.5)]}{2 \times 9.24} + \frac{(-187.5) - (-562.5)}{2 \times 9.2} = -198.78 \text{ Kg-m}$$

$$M_C = +\frac{\alpha^1 (C_{BC} + C_{CB})}{2\nu_1} - \frac{C_{BC} - C_{CB}}{2\nu_2} = +\frac{5.4 [(-187.5) + (-562.5)]}{2 \times 9.24} - \frac{(-187.5) - (-562.5)}{2 \times 9.2} = -239.54 \text{ Kg-m}$$

$$M_E = \frac{Px x^1}{I} - \left[M_B + \frac{x}{L} (M_C - M_B) \right] = \frac{1,000 \times 3 \times 1}{720} - \left[-198.78 + \frac{3}{4} (-239.54 + 198.78) \right] = 979.35 \text{ Kg-m}$$

$$M_D = -\frac{K(C_{BC} + C_{CB})}{2\nu_1} - \frac{C_{BC} - C_{CB}}{2\nu_2} = -\frac{1.2 [(-187.5) + (-562.5)]}{2 \times 9.24} - \frac{(-187.5) - (-562.5)}{2 \times 9.2} = 28.32 \text{ Kg-m}$$



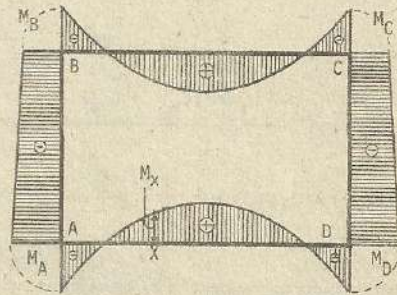
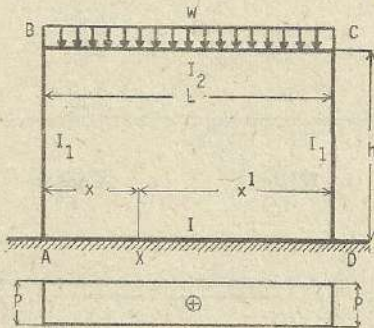
$$k = \frac{I_2 h}{I_1 L} = \frac{15,000 \times 4}{24,000 \times 5} = \frac{1}{2} = 0.5 ; \quad k^1 = \frac{I_2}{I} = \frac{15,000}{15,000} = 1$$

$$m = \frac{1}{1+6k+k^1} = \frac{1}{1+6 \times 0.5+1} = \frac{1}{5} ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)} = \frac{1}{2+0.5+\frac{1}{0.5}(2 \times 0.5+3)}$$

$$M_A = M_D = - \frac{Wh}{30} (3k+8) n = - \frac{1,500 \times 4 \times \frac{1}{2} \times 4}{30} (3 \times 0.5+8) \times \frac{1}{10.5} = - 362 \text{ Kg-m}$$

$$M_B = M_C = - \frac{Wh}{30} (2k+7k^1) n = - \frac{1,500 \times 4 \times \frac{1}{2} \times 4}{30} (2 \times 0.5+7 \times 1) \times \frac{1}{10.5} = - 305 \text{ Kg-m}$$

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$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ;$$

$$\alpha = 2k+3 ; \quad \beta = k+2 ;$$

$$\alpha^1 = 2k+3k^1 ; \quad \beta^1 = k+2k^1 ;$$

$$\lambda = \frac{x}{L} ; \quad \lambda^1 = \frac{x^1}{L}$$

$$\mu_1 = k\beta + k^1 \alpha ; \quad \mu_2 = \gamma + \gamma^1$$

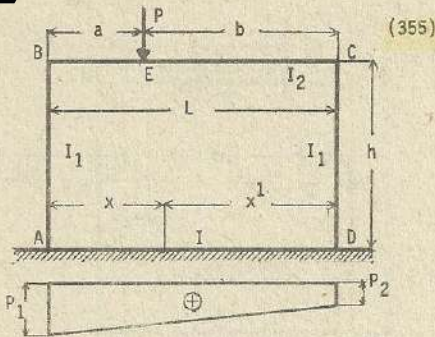
$$\gamma = 3k+1 ; \quad \delta = 3k+1 - \frac{k^1}{5}$$

$$\gamma^1 = 3k+k^1 ; \quad \delta^1 = 3k+k^1 - \frac{k^1}{5}$$

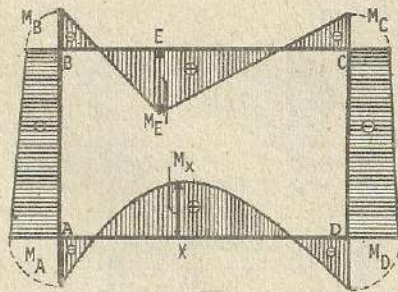
$$C_{BC} = - \frac{wL^2}{12} ; \quad C_{CB} = - \frac{wL^2}{12} ; \quad M_A = M_D = - \frac{\alpha k^1 wL + 6k(C_{BC} + C_{CB})}{12 \mu_1}$$

$$M_B = M_C = + \frac{k k^1 wL + 6\alpha^1 (C_{BC} + C_{CB})}{12 \mu_1}$$

$$M_x = \lambda^1 M_A + \lambda M_D + \frac{1}{6} (2w+p) x x^1 \quad p = w$$



(354)



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos, referencia (354)}$$

$$C_{BC} = - \frac{Pab^2}{L^2} ; \quad C_{CB} = - \frac{Pa^2b}{L^2}$$

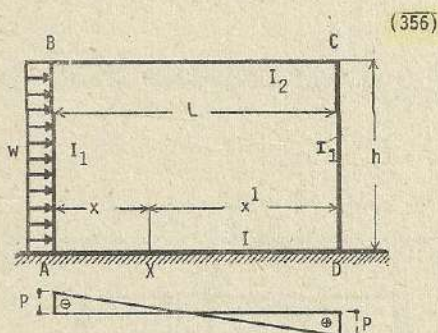
$$M_A = - \frac{\alpha k^1 PL + 6k(C_{BC} + C_{CB})}{12\mu_2} ; \quad \frac{k^1 P (b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_D = - \frac{\alpha k^1 PL + 6k(C_{BC} + C_{CB})}{12\mu_1} + \frac{k^1 P (b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

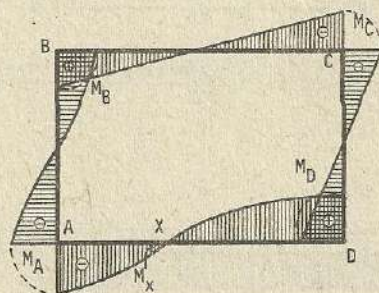
$$M_B = + \frac{kk^1 PL + 6\alpha^1(C_{BC} + C_{CB})}{12\mu_1} - \frac{k^1 P (b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_C = + \frac{kk^1 PL + 6\alpha^1(C_{BC} + C_{CB})}{12\mu_1} + \frac{k^1 P (b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_x = \lambda^1 M_A + \lambda M_D + \frac{1}{6} \left(\frac{2P}{L} + P_x \right) x x^1 ; \quad P_1 = \frac{2P(2b-a)}{L^2} ; \quad P_2 = \frac{2P(2a-b)}{L^2}$$



(355)



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos (354)}$$

$$M_A = \frac{wh^2}{4} \left[- \frac{k(k+3)}{6\mu_1} - \frac{5+20k-k^1}{5\mu_2} \right] ;$$

$$M_D = \frac{wh^2}{4} \left[- \frac{k(k+3)}{6\mu_1} + \frac{5+20k-k^1}{5\mu_2} \right]$$

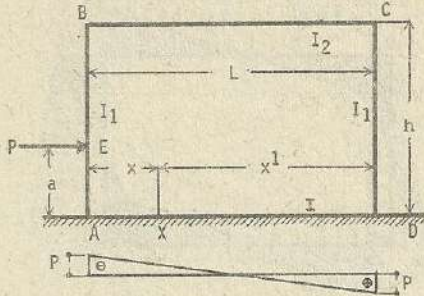
$$M_B = \frac{wh^2}{4} \left[- \frac{k(k+3k^1)}{6\mu_1} + \frac{2(5k+3k^1)}{5\mu_2} \right] ;$$

$$M_C = \frac{wh^2}{4} \left[- \frac{k(k+3k^1)}{6\mu_1} - \frac{2(5k+3k^1)}{5\mu_2} \right]$$

$$M_x = \lambda^1 M_A + \lambda M_D - \eta \frac{wh^2}{2} ;$$

$$p = \frac{3wh^2}{L^2}$$

(357)



$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I}$$

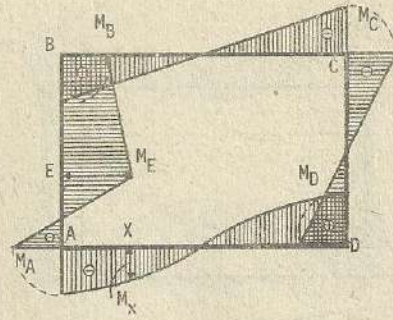
otros datos : (354)

$$C_{AB} = -\frac{Pa(h-a)^2}{h^2} ; \quad C_{BA} = -\frac{Pa^2(h-a)}{h^2}$$

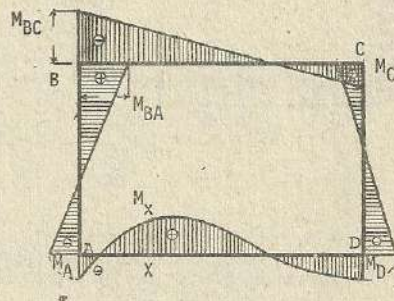
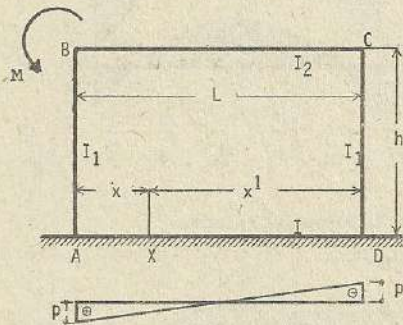
$$M_A = \frac{k(BC_{AB} + C_{BA})}{2\mu_1} + \frac{\delta Pa - 3k(C_{AB} + C_{BA})}{2\mu_2} ; \quad M_D = \frac{k(BC_{AB} + C_{BA})}{2\mu_1} - \frac{\delta Pa - 3k(C_{AB} + C_{BA})}{2\mu_2}$$

$$M_B = \frac{k(k^1 C_{AB} + b^1 C_{BA})}{2\mu_1} + \frac{\delta^1 Pa - 3k(C_{AB} + C_{BA})}{2\mu_2} ; \quad M_C = \frac{k(k^1 C_{AC} + b^1 C_{BA})}{2\mu_1} - \frac{\delta^1 Pa - 3k(C_{AB} + C_{BA})}{2\mu_2}$$

$$M_X = \lambda^1 M_A + \lambda M_D - n Pa \quad p = \frac{6Pa}{L^2}$$



(358)



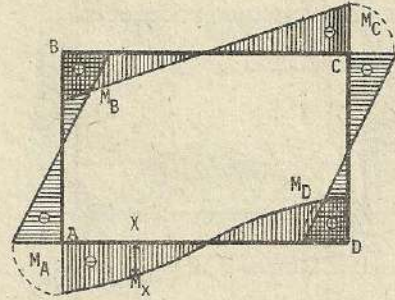
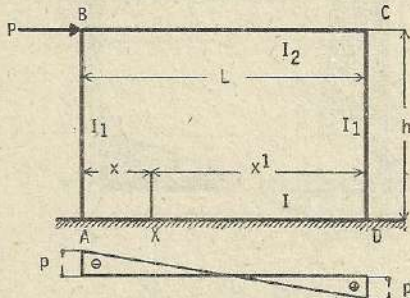
$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos (354)}$$

$$M_A = -\frac{M}{2} \left[\frac{k}{\mu_1} - \frac{5-k^1}{5\mu_2} \right] ; \quad M_D = -\frac{M}{2} \left[\frac{k}{\mu_1} + \frac{5-k^1}{5\mu_2} \right] ; \quad M_{BA} = \frac{M}{2} \left[\frac{\alpha^1}{\mu_1} + \frac{5-k^1}{5\mu_2} \right]$$

$$M_C = \frac{M}{2} \left[\frac{\alpha^1}{\mu_1} - \frac{5-k^1}{5\mu_2} \right] ; \quad M_{BC} = M_{BA} - M \quad ; \quad M_X = \lambda^1 M_A + \lambda M_D + nM$$

$$p = \frac{6M}{L^2}$$

(359)

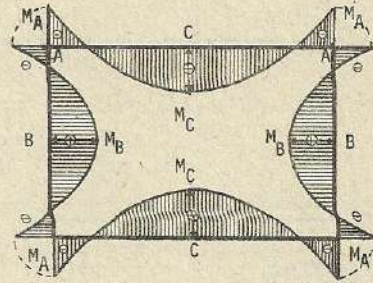
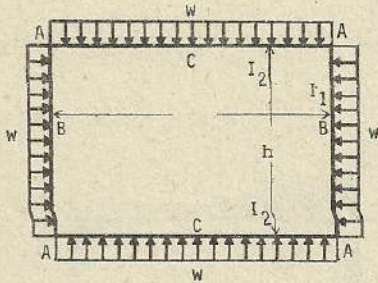


$$k = \frac{I_2 h}{I_1 L} ; \quad k^1 = \frac{I_2}{I} ; \quad \text{otros datos, referencia (354)}$$

$$M_A = -\frac{\delta Ph}{2\mu_2} ; \quad M_B = +\frac{\delta^1 Ph}{2\mu_2} ; \quad M_C = -\frac{\delta^1 Ph}{2\mu_2} ; \quad M_D = +\frac{\delta Ph}{2\mu_2} ; \quad p = \frac{6Ph}{L^2}$$

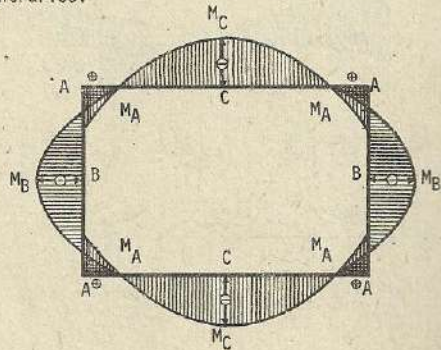
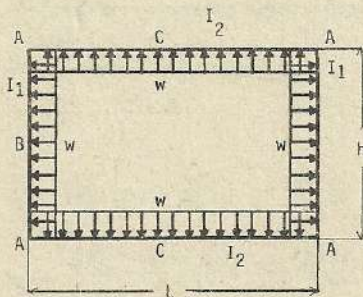
$$M_x = (\lambda - \lambda^1) M_D - \eta Ph$$

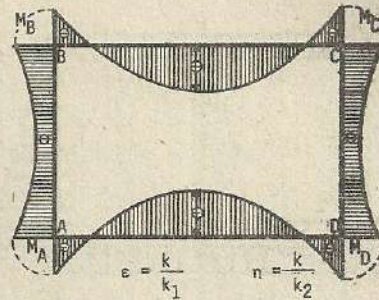
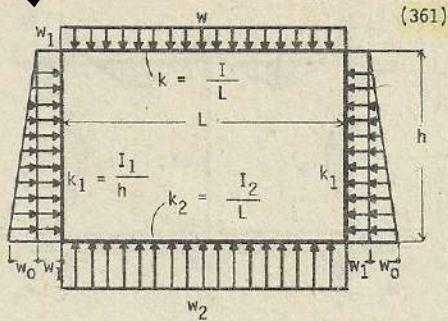
(360)



$$k = \frac{I_2 h}{I_1 L} ; \quad n = \frac{h}{L} ; \quad M_A = -\frac{wL^2}{12} \frac{1+n^2k}{1+k} ; \quad M_B = M_A + \frac{wh^2}{8} ; \quad M_C = M_A + \frac{wL^2}{8}$$

Nota : Se ha expuesto el problema de la acción de la carga (w) externa. En caso donde las fuerzas provengan desde el interior del marco, los esfuerzos de flexión poseen signos contrarios.





$$M_{AD} = \frac{w_2 L^2 (3+2\epsilon) \eta - \epsilon w L^2}{12 \{ (2+\epsilon)\epsilon + (3+2\epsilon)\eta \}} + \frac{w_1 h^2 (3+\epsilon)\epsilon}{12 \{ (2+\epsilon)\epsilon + (3+2\epsilon)\eta \}} + \frac{w_0 h^2 (8+3\epsilon)\epsilon}{60 \{ (2+\epsilon)\epsilon + (3+2\epsilon)\eta \}} = -M_{AB}$$

$$M_{BC} = \frac{\epsilon \eta w_2 L^2 - w L^2 (3\eta + 2\epsilon)}{12 \{ (2+\epsilon)\epsilon + (3+2\epsilon)\eta \}} - \frac{w_1 h^2 (3\eta - \epsilon)\epsilon}{12 \{ (2+\epsilon)\epsilon + (3+2\epsilon)\eta \}} - \frac{w_0 h^2 (7\eta + 2\epsilon)\epsilon}{60 \{ (2+\epsilon)\epsilon + (3+2\epsilon)\eta \}} = -M_{BA}$$

En caso donde $I_1 = I_2$, entonces $\eta = 1$, y si $w_2 = w$, tenemos:

$$M_{AD} = \frac{w L^2}{12(1+\epsilon)} + \frac{w_1 h^2 \epsilon}{12(1+\epsilon)} + \frac{w_0 h^2 (8+3\epsilon)\epsilon}{60(\epsilon^2 + 4\epsilon + 3)} = -M_{AB}$$

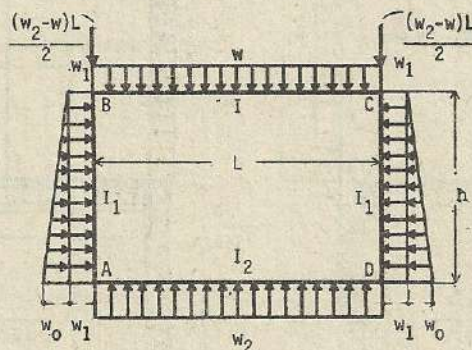
$$M_{BC} = \frac{w L^2}{12(1+\epsilon)} - \frac{w_1 h^2 \epsilon}{12(1+\epsilon)} - \frac{w_0 h^2 (7+2\epsilon)\epsilon}{60(\epsilon^2 + 4\epsilon + 3)} = -M_{BA}$$

Método de solución diferente

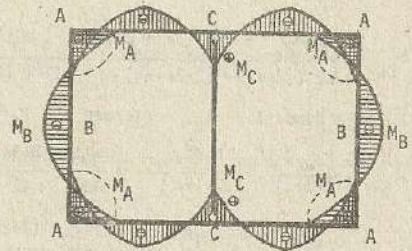
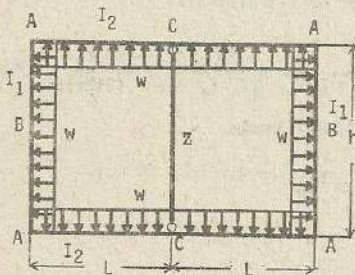
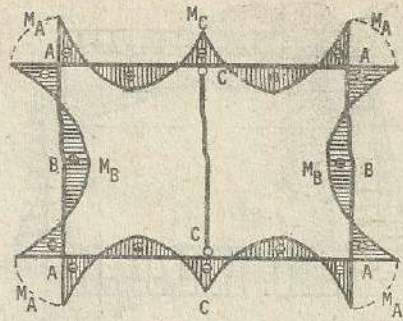
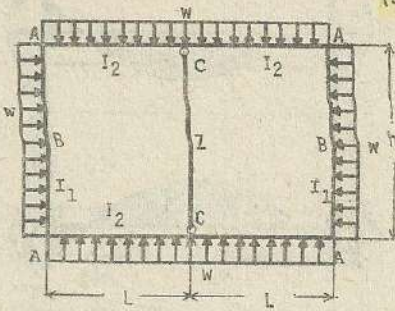
$$k_1 = \frac{h}{L} \frac{I_1}{I_1} ; \quad k_2 = \frac{h}{L} \frac{I_2}{I_1} ; \quad \alpha = (2+k_1) (2+k_2) - 1$$

$$M_B = -\frac{(3+2k_2)}{\alpha} \cdot \frac{w L^2}{12} + \frac{k_1 w_2 L^2}{\alpha} - \frac{(3+k_2) k_1}{\alpha} \cdot \frac{w_1 h^2}{12} - \frac{(7+2k_2) k_1}{\alpha} \cdot \frac{w_0 h^2}{60}$$

$$M_A = \frac{k_2}{\alpha} \cdot \frac{w L^2}{12} - \frac{(3+2k_1)}{\alpha} \cdot \frac{w_2 L^2}{12} - \frac{(3+k_1) k_2}{\alpha} \cdot \frac{w_1 h^2}{12} - \frac{(8+3k_1) k_2}{\alpha} \cdot \frac{w_0 h^2}{60}$$



(362)



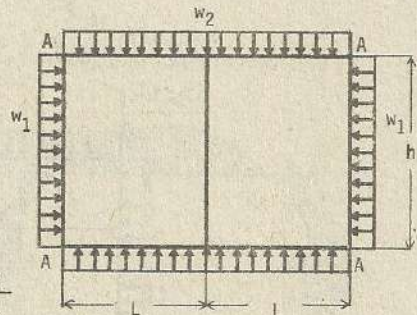
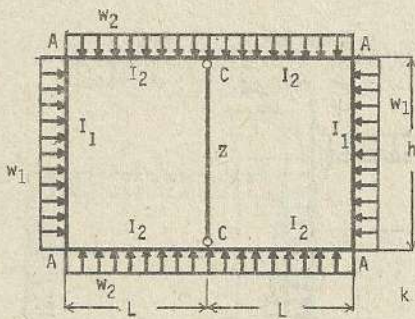
$$k = \frac{I_2 h}{I_1 L} \quad ; \quad n = \frac{h}{L} \quad ;$$

$$M_C = -\frac{wL^2}{12} \cdot \frac{1+(3-n^2)k}{1+2k} \quad ;$$

$$M_A = -\frac{wL^2}{12} \cdot \frac{1+2n^2k}{1+2k} \quad ; \quad M_B = M_A + \frac{wh^2}{8}$$

$$Z = \frac{wL^2}{2} \cdot \frac{2+(5-n^2)k}{1+2k}$$

Nota : Cuando la carga acciona desde la parte externa, el miembro interno trabaja como columna de soporte, en caso contrario se convierte en elemento de amarre, (Z) representa la fuerza en sentido axial.



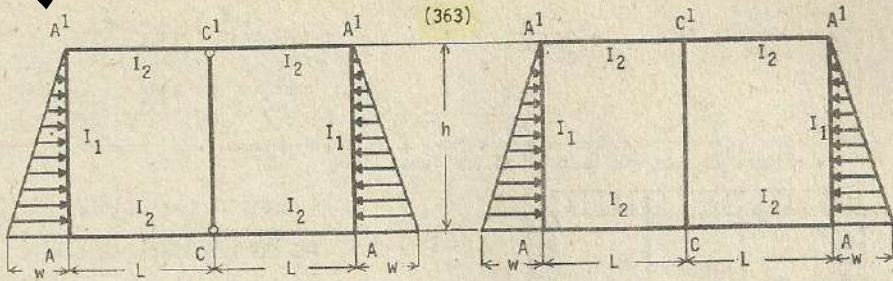
$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{2w_1 h^2 k + w_2 L^2}{12(2k+1)}$$

$$M_C = \frac{w_1 h^2 k - (3k+1) \cdot w_2 L^2}{12(2k+1)}$$

Si $w_1 = w_2$ y $h = L$

entonces : $M_A = M_C = -\frac{wL^2}{12}$



$$k = \frac{I_2 h}{I_1 L}$$

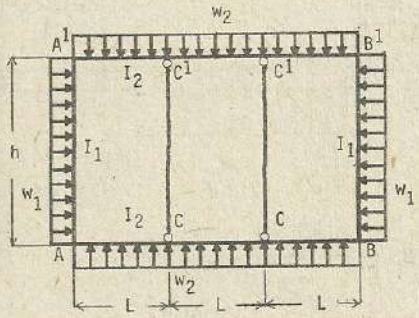
$$M_A = - \frac{wh^2(12k+61)k}{120(2k+1)(k+6)}$$

$$M_C = \frac{wh^2(3k+29)k}{120(2k+1)(k+6)}$$

$$M_{A1} = - \frac{wh^2(8k+59)k}{120(2k+1)(k+6)}$$

$$M_{C1} = \frac{wh^2(7k+31)k}{120(2k+1)(k+6)}$$

(364)

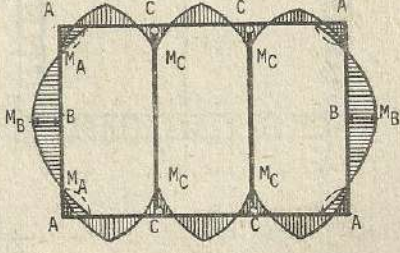
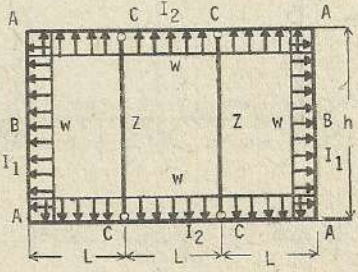
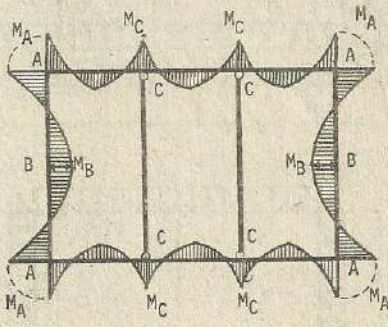
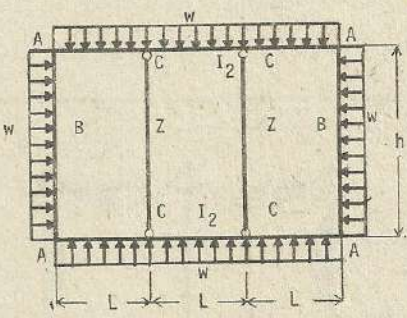


$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = - \frac{2w_2 L^2 + 5w_1 h^2 k}{12(5k+3)}$$

$$M_C = - \frac{3(2k+1)w_2 L^2 - w_1 h^2 k}{12(5k+3)}$$

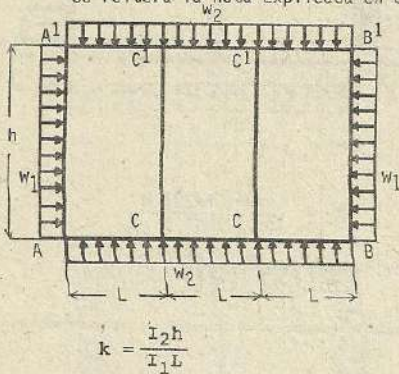
(365)



$$k = \frac{I_2 h}{I_1 L} ; \quad n = \frac{h}{L} ; \quad M_A = - \frac{wL^2}{12} \cdot \frac{3 + 5n^2 k}{3 + 5k}$$

$$M_B = M_A + \frac{wh^2}{8} ; \quad M_C = - \frac{wh^2}{12} \cdot \frac{3 + (6-n^2)k}{3 + 5k} ; \quad Z = \frac{wL^2}{2} \cdot \frac{6 + (11-n^2)k}{3 + 5k}$$

Se reitera la nota explicada en el problema (362)



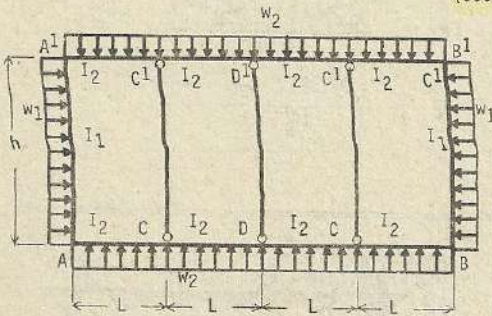
$$M_{AA^1} = - \frac{w_2 L^2 (3k+1) + (5k+2) w_1 h^2 k}{12 (5k^2 + 5k + 1)}$$

$$M_{CA} = - \frac{w_2 L^2 (6k^2 + 6k + 1) - (k+1) w_1 h^2 k}{12 (5k^2 + 5k + 1)}$$

$$M_{CC^1} = \frac{w_2 L^2 k - w_1 h^2 k}{12 (5k^2 + 5k + 1)}$$

$$M_{CC} = - \frac{w_2 L^2 (6k^2 + 5k + 1) - w_1 h^2 k}{12 (5k^2 + 5k + 1)}$$

(366)

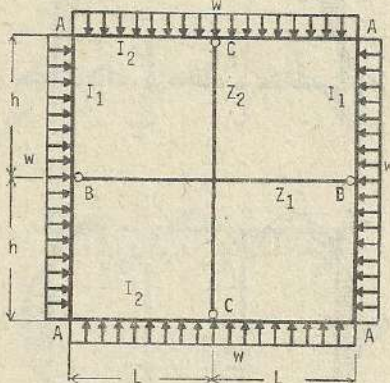


$$k = \frac{I_2 h}{I_1 L} ; \quad M_A = - \frac{4w_2 L + 7w_1 h^2 k}{12 (7k+4)}$$

$$M_C = \frac{2w_1 h^2 k - (9k+4) w_2 L^2}{12 (7k+4)}$$

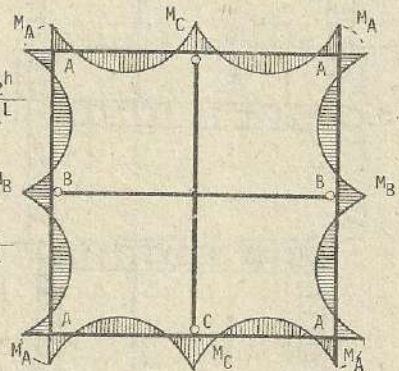
$$M_D = - \frac{w_1 h^2 k + (6k+4) w_2 L^2}{12 (7k+4)}$$

(367)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{h}{L}$$

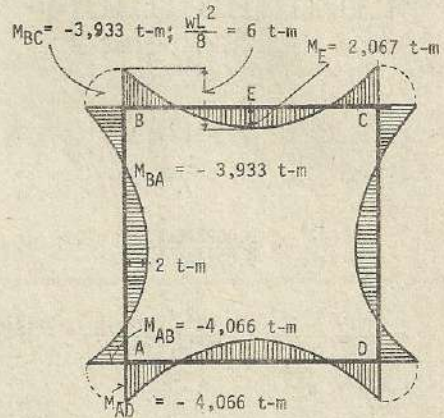
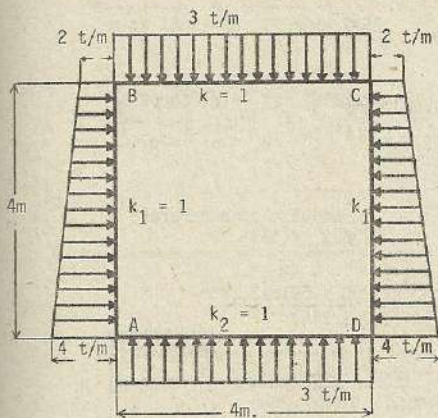


$$M_A = - \frac{wL^2}{12} \cdot \frac{1+n^2 k}{1+k} ; \quad M_B = - \frac{wL^2}{24} \cdot \frac{(3+2k) n^2 - 1}{1+k} ; \quad M_C = \frac{wL^2}{24} \cdot \frac{(2+3k) - n^2 k}{1+k}$$

$$Z_1 = \frac{wL^2}{4} \cdot \frac{(4+5k) - n^2 k}{1+k} ; \quad Z_2 = \frac{wL^2}{4n} \cdot \frac{(5+4k)n^2 - 1}{1+k}$$

Observar la nota del problema (362)

Solución de problema numérico :



Referencia (361)

$$\epsilon = \frac{k}{k_1} = \frac{1}{1} = 1 ; \quad \eta = \frac{k}{k_2} = \frac{1}{1} = 1$$

$$M_{AD} = \frac{w_2 L^2 (3+2\epsilon)\eta - \epsilon w L^2}{12 \{(2+\epsilon)\epsilon + (3+2\epsilon)\eta\}} + \frac{w_1 h^2 (3+\epsilon)\epsilon}{12 \{(2+\epsilon)\epsilon + (3+2\epsilon)\eta\}} + \frac{w_0 h^2 (8+3\epsilon)\epsilon}{60 \{(2+\epsilon)\epsilon + (3+2\epsilon)\eta\}}$$

$$= \frac{3 \times 4 \times 4 (3+2 \times 1) \times 1 - 1 \times 3 \times 4 \times 4}{12 \{(2+1) \times 1 + (3+2 \times 1) \times 1\}} + \frac{2 \times 4 \times 4 (3+1) \times 1}{12 \{(2+1) \times 1 + (3+2 \times 1) \times 1\}} + \frac{2 \times 4 \times 4 (8+3 \times 1) \times 1}{60 \{(2+1) \times 1 + (3+2 \times 1) \times 1\}} = 4,066 \text{ t-m} = -M_{AB}$$

$$M_{BC} = \frac{\epsilon \eta w_2 L^2 - w L^2 (3\eta + 2\epsilon)}{12 \{(2+\epsilon)\epsilon + (3+2\epsilon)\eta\}} - \frac{w_1 h^2 (3\eta + \epsilon)\epsilon}{12 \{(2+\epsilon)\epsilon + (3+2\epsilon)\eta\}} - \frac{w_0 h^2 (7\eta + 2\epsilon)\epsilon}{60 \{(2+\epsilon)\epsilon + (3+2\epsilon)\eta\}}$$

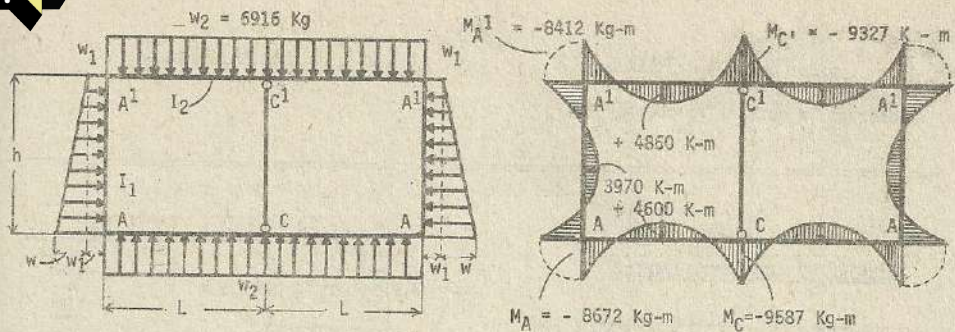
$$= \frac{1 \times 1 \times 3 \times 4 \times 4 - 3 \times 4 \times 4 (3 \times 1 + 2 \times 1)}{12 \{(2+1) \times 1 + (3+2 \times 1) \times 1\}} - \frac{2 \times 4 \times 4 (3 \times 1 + 1) \times 1}{12 \{(2+1) \times 1 + (3+2 \times 1) \times 1\}}$$

$$- \frac{2 \times 4 \times 4 (7 \times 1 + 2 \times 1) \times 1}{60 \{(2+1) \times 1 + (3+2 \times 1) \times 1\}} = -3,933 \text{ t-m} = -M_{BA}$$

$$M_E = \frac{wL^2}{8} - M_{BC} = \frac{3 \times 4 \times 4}{8} - 3,933 = 2,067 \text{ t-m}$$

Debido a que la aplicación del método exacto, para la solución del esfuerzo de momento positivo de los muros extremos, es sumamente fatigoso, se recurre a un sistema abreviado, considerando una carga uniformemente distribuida

$$\frac{1}{2} (2+4) = 3 \text{ t/m} ; \quad \frac{3 \times 4 \times 4}{8} - \frac{1}{2} \cdot (4,066 + 3,933) = 6 - 4 = 2 \text{ t-m.}$$



$$L = 3.985 \text{ m} ; \quad h = 4.620 \text{ m} ; \quad I_2 = 0.003275 \text{ m}^4 ; \quad I_1 = 0.002995 \text{ m}^4$$

$$w_1 = 2574 \text{ Kg.} ; \quad w = 4186 \text{ Kg.} ; \quad w_2 = 6916 \text{ Kg} ; \quad w + w_1 = 6760 \text{ Kg.}$$

$$k = \frac{I_2 h}{I_1 L} = \frac{0.003275 \times 4.620}{0.002995 \times 3.985} = 1.268 \quad \text{Se adoptan como referencia (362) y (363)}$$

$$M_A = - \frac{2w_1 h^2 k + w_2 L^2}{12(1+2k)} - \frac{wh^2(12k+61)k}{120(1+2k)(6+k)} = \frac{2 \times 2574 \times 4.62^2 \times 1.268 + 6916 \times 3.985^2}{12(1+2 \times 1.268)}$$

$$- \frac{4186 \times 4.62^2 (12 \times 1.268 + 61) \times 1.268}{120(1+2 \times 1.268)(6+1.268)} = -8672 \text{ Kg-m}$$

$$M_{A1} = - \frac{2w_1 h^2 k + w_2 L^2}{12(1+2k)} - \frac{wh^2(8k+59)k}{120(1+2k)(6+k)} = - \frac{2 \times 2574 \times 4.62^2 \times 1.268 + 6916 \times 3.985^2}{12(1+2 \times 1.268)}$$

$$- \frac{4186 \times 4.62^2 (8 \times 1.268 + 59) \times 1.268}{120(1+2 \times 1.268)(6+1.268)} = -8412 \text{ Kg-m}$$

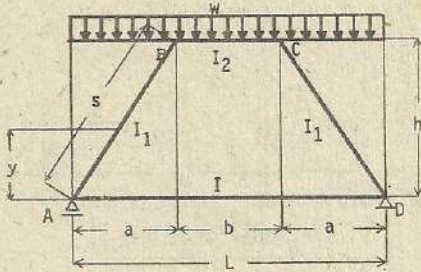
$$M_C = \frac{w_1 h^2 k - (1+3k)w_2 L^2}{12(1+2k)} + \frac{wh^2(5k+29)k}{120(1+2k)(6+k)} = \frac{2574 \times 4.62^2 \times 1.268 - (1+3 \times 1.268) \times 6916 \times 3.985^2}{12(1+2 \times 1.268)}$$

$$+ \frac{4186 \times 4.62^2 (3 \times 1.268 + 29) \times 1.268}{120(1+2 \times 1.268)(6+1.268)} = -9587 \text{ Kg-m}$$

$$M_{C1} = \frac{w_1 h^2 k - (1+3k)w_2 L^2}{12(1+2k)} + \frac{wh^2(7k+31)k}{120(1+2k)(6+k)} = \frac{2574 \times 4.62^2 \times 1.268 - (1+3 \times 1.268) \times 6916 \times 3.985^2}{12(1+2 \times 1.268)}$$

$$+ \frac{4186 \times 4.62^2 (7 \times 1.268 + 31) \times 1.268}{120(1+2 \times 1.268)(6+1.268)} = -9327 \text{ Kg-m}$$

(368)



$$k_1 = \frac{I_2 h}{I_1 b} ; k_2 = \frac{I_2}{I_1 L} ; \alpha = \frac{x}{L}$$

$$\beta = \frac{x^1}{L} ; \lambda = \frac{a}{L}$$

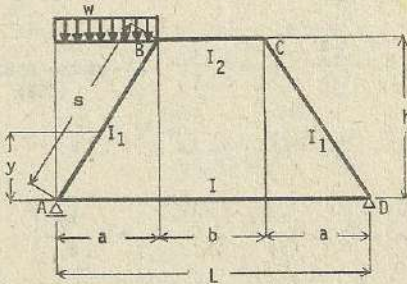
$$\mu = \frac{1}{k_2} (3+2k_1) + 2 + k_1 ; \mu_2 = \frac{1}{1+k_1} \mu$$

$$v = 1 + \frac{k_1}{k_2} + 6k_1 - 4\lambda \left[(1+3k_1) - \lambda(1+2k_1) \right]$$

$$H = \frac{wL}{4} \left[\frac{1}{h} \lambda \left(2 \frac{b}{L} + \lambda \frac{6+5k_1}{3+2k_1} \right) + \frac{1}{\mu_2} \left(\frac{b}{h} \frac{b}{L} \frac{1+k_2}{k_2(1+k_1)} - \lambda^2 \frac{1}{h} \frac{3+k_1}{3+2k_1} \right) \right]$$

$$M_A = M_D = \frac{wL^2}{12} \frac{\left(\frac{b}{L} \right)^2 - \lambda^2 (3+k_1)}{u}$$

(369)



$$k_1 = \frac{I_2 s}{I_1 b} ; k_2 = \frac{I_2}{I_1 L}$$

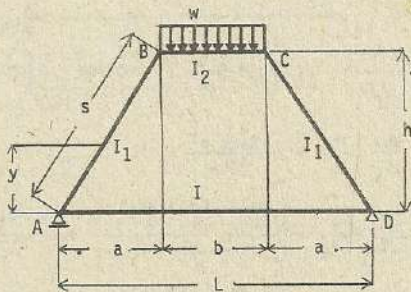
otros datos, referencia (368)

$$H = \frac{wa}{8} \lambda \frac{1}{h} \frac{1}{3+2k_1} \left[6+5k_1 - \frac{3+k_1}{\mu_2} \right]$$

$$M_A = -\frac{wa^2}{4} \left[\frac{3+k_1}{6\mu} + \frac{\frac{b}{L} \{ 1+3k_1 - 2\lambda(1+2k_1) \} + k_1(1-\lambda)}{v} \right]$$

$$M_D = -\frac{wa^2}{4} \left[\frac{3+k_1}{6\mu} - \frac{\frac{b}{L} \{ 1+3k_1 - 2\lambda(1+2k_1) \} + k_1(1-\lambda)}{v} \right]$$

(370)

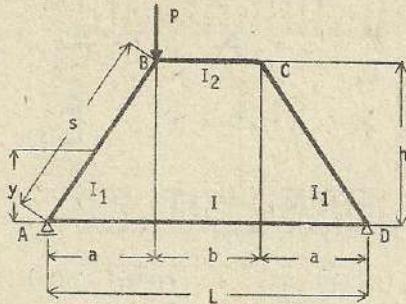


$$k_1 = \frac{I_2 s}{I_1 b} ; k_2 = \frac{I_2}{I_1 L}$$

otros datos (368)

$$H = \frac{wb}{4} \left[2\lambda \frac{L}{h} + \frac{b}{h} \frac{1}{k_2 \mu_2} \frac{1+k_2}{1+k_1} \right]$$

$$M_A = M_D = + \frac{wb^2}{12} \frac{1}{\mu}$$



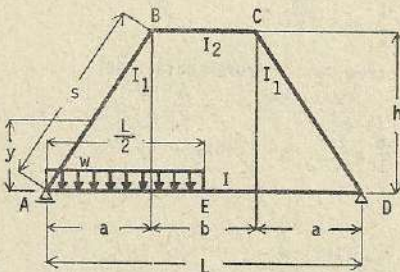
(371)

$$k_1 = \frac{I_2 s}{I_1 b} ; k_2 = \frac{I s}{I_1 L} ; \text{ otros datos (368)}$$

$$H = \frac{P}{2} \cdot \lambda \cdot \frac{L}{h}$$

$$M_A = - \frac{Pa}{2} \cdot \frac{\frac{b}{L} [(1+3k_1) - 2\lambda(1+2k_1)]}{v}$$

$$M = + \frac{Pa}{2} \cdot \frac{\frac{b}{L} [(1+3k_1) - 2\lambda(1+2k_1)]}{v}$$



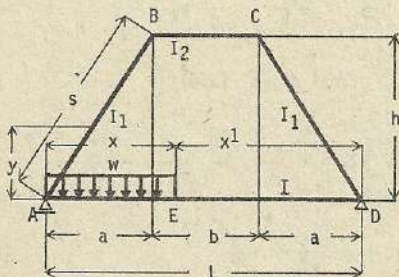
(372)

$$k_1 = \frac{I_2 s}{I_1 b} ; k_2 = \frac{I s}{I_1 L} ; \text{ otros datos (368)}$$

$$H = \frac{wL}{8} \cdot \frac{1}{h} \cdot \frac{1}{k_2 \mu^2}$$

$$M_A = \frac{wL}{16} \cdot \frac{1}{k_2} \left[\frac{\frac{2}{3} \cdot (3+2k_1)}{\mu} + \frac{k_1}{4v} \right]$$

$$M_D = \frac{wL}{16} \cdot \frac{1}{k_2} \left[\frac{\frac{2}{3} \cdot (3+2k_1)}{\mu} - \frac{k_1}{4v} \right]$$



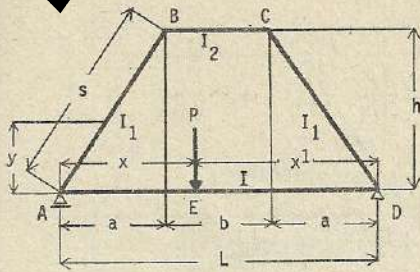
(373)

$$k_1 = \frac{I_2 s}{I_1 b} ; k_2 = \frac{I s}{I_1 L} ; \text{ otros datos (368)}$$

$$H = \frac{wL}{4} \cdot \frac{L}{h} \cdot \frac{(3-2\alpha)a^2}{k_2 \mu^2}$$

$$M_A = \frac{wL^2}{4} \cdot \frac{1}{k_2} a^2 \left[\frac{\frac{1}{3} (3-2\alpha)(3+2k_1)}{\mu} + \frac{\beta k_1}{v} \right]$$

$$M_D = \frac{wL^2}{4} \cdot \frac{1}{k_2} a^2 \left[\frac{\frac{1}{3} (3-2\alpha)(3+2k_1)}{\mu} - \frac{\beta k_1}{v} \right]$$



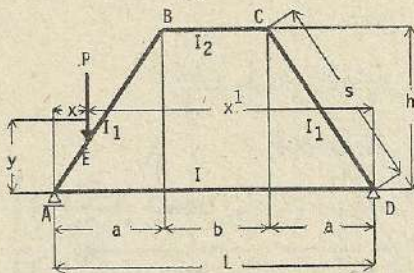
(374)

$$k_1 = \frac{I_2 s}{I_1 b}; \quad k_2 = \frac{I s}{I_1 L}; \quad \text{otros datos (368)}$$

$$H = \frac{3}{2} \cdot P \cdot \frac{L}{h} - \frac{\alpha \beta}{k_2 \nu_2}$$

$$M_A = \frac{PL}{2} - \alpha \beta \cdot \frac{1}{k_2} \cdot \left[\frac{3+2k_1}{\nu} + \frac{(\beta-\alpha) k_1}{\nu} \right]$$

$$M_D = \frac{PL}{2} - \alpha \beta \cdot \frac{1}{k_2} \cdot \left[\frac{3+2k_1}{\nu} - \frac{(\beta-\alpha) k_1}{\nu} \right]$$



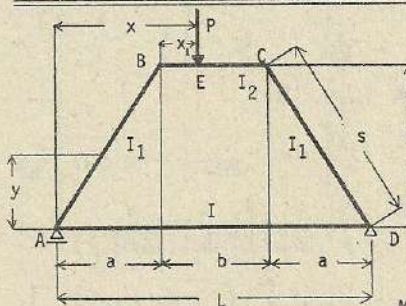
(375)

$$k_1 = \frac{I_2 s}{I_1 b}; \quad k_2 = \frac{I s}{I_1 L}; \quad \text{otros datos (368)}$$

$$H = \frac{3}{2} - \alpha \beta \cdot \frac{L}{h} \left[\frac{1}{3} \left(\frac{x}{a} \right)^2 - \frac{1}{\nu_2} \left\{ \frac{2+k_1}{3+2k_1} - \frac{x}{a} + \left(\frac{x}{a} \right)^2 \frac{1+k_1}{3+2k_1} \right\} \right]$$

$$M_A = - \frac{PL}{2} \cdot \alpha \left[\frac{3+2k_1}{\nu} \left\{ \frac{2+k_1}{3+2k_1} - \frac{x}{a} + \left(\frac{x}{a} \right)^2 \frac{1+k_1}{3+2k_1} \right\} + \frac{1}{\nu} \left\{ \frac{b}{L} \left[(1+3k_1) - 2\lambda(1+2k_1) \right] + k_1 \left[(3-2\lambda) - 3 \frac{x}{a} + 2\lambda \left(\frac{x}{a} \right)^2 \right] \right\} \right]$$

$$M_D = - \frac{PL}{2} \cdot \alpha \left[\frac{3+2k_1}{\nu} \left\{ \frac{2+k_1}{3+2k_1} - \frac{x}{a} + \left(\frac{x}{a} \right)^2 \frac{1+k_1}{3+2k_1} \right\} - \frac{1}{\nu} \left\{ \frac{b}{L} \left[(1+3k_1) - 2\lambda(1+2k_1) \right] + k_1 \left[(3-2\lambda) - 3 \frac{x}{a} + 2\lambda \left(\frac{x}{a} \right)^2 \right] \right\} \right]$$



(376)

$$k_1 = \frac{I_2 s}{I_1 b}; \quad k_2 = \frac{I s}{I_1 L}$$

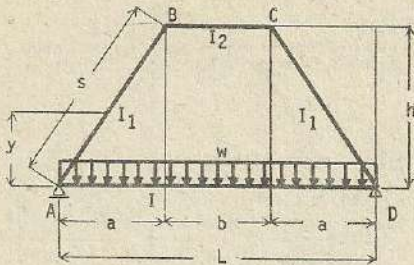
Otros datos (368)

$$H = \frac{P}{2} \left[\lambda \frac{L}{h} + \frac{1+k_2}{1+k_1} \cdot 3 \cdot \frac{b}{h} \cdot \frac{\frac{x_1}{b} (1 - \frac{x_1}{b})}{k_2 \nu_2} \right]$$

$$M_A = \frac{Pb}{2} \left[\frac{\frac{x_1}{b} (1 - \frac{x_1}{b})}{\nu} - \frac{(1-2 \frac{x_1}{b})}{\nu} \left\{ (1-2\lambda) \frac{x_1}{b} (1 - \frac{x_1}{b}) + \lambda(1+3k_1) - 2\lambda^2(1+2k_1) \right\} \right]$$

$$M_D = \frac{Pb}{2} \left[\frac{\frac{x_1}{b} (1 - \frac{x_1}{b})}{\nu} + \frac{(1-2 \frac{x_1}{b})}{\nu} \left\{ (1-2\lambda) \frac{x_1}{b} (1 - \frac{x_1}{b}) + \lambda(1+3k_1) - 2\lambda^2 (1+2k_1) \right\} \right]$$

(377)



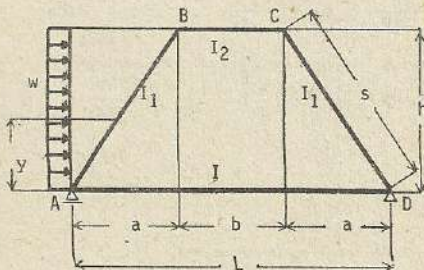
$$k_1 = \frac{I_2 s}{I_1 b} ; \quad k_2 = \frac{I s}{I_1 L}$$

otros datos (368)

$$H = \frac{PL}{4} \cdot \frac{L}{h} \cdot \frac{1}{k_2 \mu_2}$$

$$M_A = M_D = \frac{PL^2}{12} \cdot \frac{3+2\nu}{k_2 \mu}$$

(378)



$$k_1 = \frac{I_2 s}{I_1 b} ; \quad k_2 = \frac{I s}{I_1 L}$$

otros datos (368)

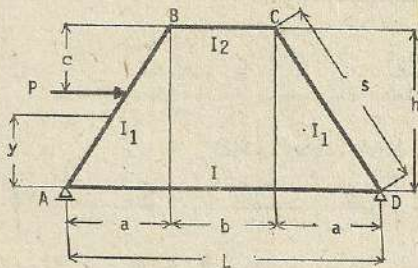
$$H_A = -\frac{wh}{2} \left[1 + \frac{1}{4(3+2k_1)} \left\{ 3(2+k_1) + \frac{3+k_1}{\nu_2} \right\} \right]$$

$$H_D = -\frac{wh}{2} \left[-1 + \frac{1}{4(3+2k_1)} \left\{ 3(2+k_1) + \frac{3+k_1}{\nu_2} \right\} \right]$$

$$M_A = -\frac{wh^2}{4} \left[\frac{3+k_1}{6\nu} + \frac{b}{L} \left\{ (1+3k_1) - 2\lambda(1+2k_1) \right\} + k_1(1-\lambda) \right]$$

$$M_D = -\frac{wh^2}{4} \left[\frac{3+k_1}{6\nu} - \frac{b}{L} \left\{ (1+3k_1) - 2\lambda(1+2k_1) \right\} + k_1(1-\lambda) \right]$$

(379)



$$k_1 = \frac{I_2 s}{I_1 b} ; \quad k_2 = \frac{I s}{I_1 L}$$

otros datos (368)

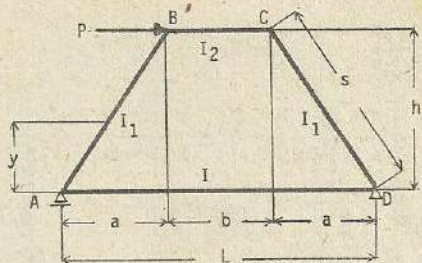
$$H_A = -\frac{P}{2} \left[1 + \frac{c}{h} \frac{1}{3+2k_1} \left\{ k_1 \frac{c}{h} \left(3 - \frac{c}{h} + 3 \right) + 3 \frac{1 - \frac{c}{h} \left(1 + \frac{c}{h} (1+k_1) \right)}{\nu_2} \right\} \right]$$

$$H_D = -\frac{P}{2} \left[-1 + \frac{c}{h} \cdot \frac{1}{3+2k_1} \left\{ k_1 \cdot \frac{c}{h} \left(3 - \frac{c}{h} \right) + 3 + 3 \frac{(1 - \frac{c}{h}) \left(1 + \frac{c}{h} (1+k_1) \right)}{\nu_2} \right\} \right]$$

$$M_A = -\frac{P}{2} (h-c) \left[\frac{c}{h} \frac{1 + \frac{c}{h} (1+k_1)}{\nu} + \frac{b}{L} \left\{ (1+3k_1) - 2\lambda(1+2k_1) \right\} + k_1 \frac{c}{h} \left\{ 3+2\lambda \left(\frac{c}{h} - 2 \right) \right\} \right]$$

$$M_D = -\frac{P}{2} (h-c) \left[\frac{c}{h} \frac{1 + \frac{c}{h} (1+k_1)}{\nu} - \frac{b}{L} \left\{ (1+3k_1) - 2\lambda(1+2k_1) \right\} + k_1 \frac{c}{h} \left\{ 3+2\lambda \left(\frac{c}{h} - 2 \right) \right\} \right]$$

(380)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I s}{I_1 L}$$

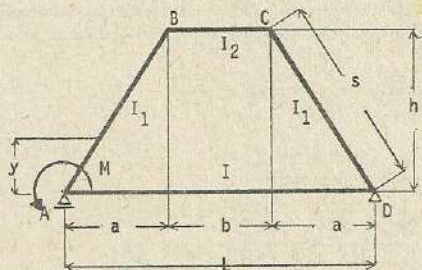
otros datos (368)

$$H_A = -\frac{P}{2} \quad ; \quad H_D = +\frac{P}{2}$$

$$M_A = -\frac{Ph}{2} \cdot \frac{b}{L} \cdot \frac{1}{v} \left\{ (1+3k_1) - 2\lambda (1+2k_1) \right\}$$

$$M_D = +\frac{Ph}{2} \cdot \frac{b}{L} \cdot \frac{1}{v} \left\{ (1+3k_1) - 2\lambda (1+2k_1) \right\}$$

(381)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I s}{I_1 L}$$

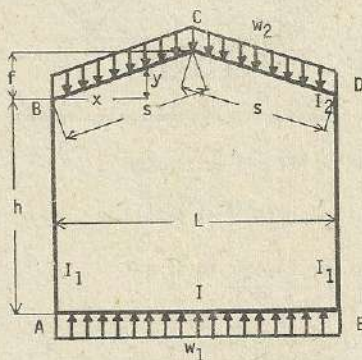
otros datos (368)

$$H = \frac{3}{2} \cdot \frac{M}{h} \cdot \frac{1}{k_2 h^2}$$

$$M_A = -\frac{M}{2} \cdot \frac{1}{k_2} \left\{ \frac{3-2k_1}{u} + \frac{k_1}{v} \right\}$$

$$M_D = -\frac{M}{2} \cdot \frac{1}{k_2} \left\{ \frac{3-2k_1}{u} - \frac{k_1}{v} \right\}$$

(382)



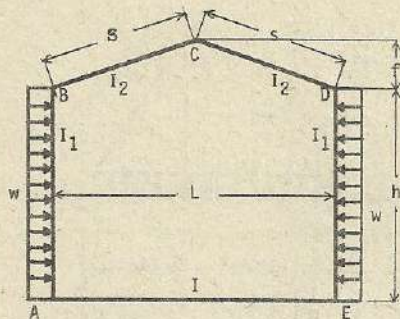
$$k_1 = \frac{I_2 h}{I_1 s} \quad ; \quad k_2 = \frac{I_2 L}{I s} \quad ; \quad \eta = \frac{k_1 h + 2h + f}{2 + 2k_1 + k_2}$$

$$M' = -\frac{L^2 (2w_2 + w_1 k_2)}{12 (2 + 2k_1 + k_2)} \quad M'_A = M' + H' \eta$$

$$M'_B = M' - H' (h - \eta) \quad ; \quad M'_D = M' - H' (h - \eta)$$

$$M'_C = M' - H' (h - \eta + f) + \frac{w_2 L^2}{8} \quad ; \quad M'_E = M' + H' \eta$$

$$H' = \frac{w_2 L^2 \left[(2 + 2k_1 + k_2) 5f + 8 (k_1 h + k_2 h - f) \right] - 4 (k_1 h + 2h + f) k_2 L^2}{16 \left[h^2 (k_1^2 + 2k_1 k_2 + 4k_1 + 6k_2) + 6hf (k_1 + k_2) + f^2 (4k_1 + 2k_2 + 1) \right]}$$



(383)

$$k_1 = \frac{I_2 h}{I_1 s} \quad ; \quad k_2 = \frac{I_2 L}{I s}$$

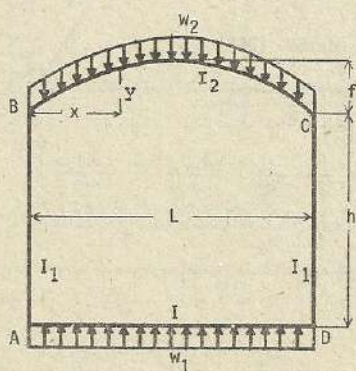
$$n = \frac{k_1 h + 2h + f}{2 + 2k_1 + k_2} \quad ; \quad M' = \frac{wh^2(2k_1 + 3k_2)}{6(2 + 2k_1 + k_2)}$$

$$M_A = M' - H' n - \frac{wh^2}{2} = M_E$$

$$M_B = M' + H'(h-n) = M_D \quad ; \quad M_C = M' + H'(h-n+f)$$

$$H' = - \frac{wh^2 \cdot [k_1 h (2k_1 + 5k_2 - 2) + (4h + 2f)(2k_1 + 3k_2)]}{4 [h_2 (k_1^2 + 2k_1 k_2 + 4k_1 + 6k_2) + 6hf(k_1 + k_2) + f^2(4k_1 + 2k_2 + 1)]}$$

(384)



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2}{I}$$

$$n = \frac{3h + 3k_1 h + 2f}{3(1 + k_2 + 2k_1)}$$

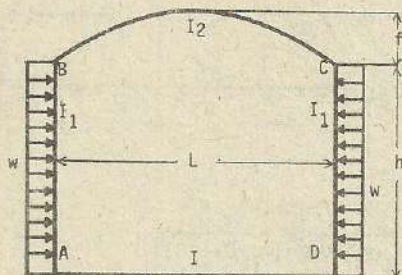
$$M' = - \frac{(w_2 + k_2 w_1) L^2}{12(1 + k_2 + 2k_1)}$$

$$M_A = M_D = - \frac{(w_2 + k_2 w_1) L^2}{12(1 + k_2 + 2k_1)} + H' n$$

$$M_B = M_C = M' - H(h-n)$$

$$H' = \frac{w_2 L^2 [12f(1+k_2+2k_1) + 5(3k_2 h + 3k_1 h - 2f)] - 5w_1 L^2 (3h + 3k_1 h + 2f) k_2}{4 [15h^2 \{ (2+k_1)k_1 + (2k_1+3)k_2 \} + 60hf(k_1+k_2) + 4f^2(1+6k_2+12k_1)]}$$

(385)



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2}{I}$$

$$n' = \frac{3h + 3k_1 h + 2f}{3(1 + k_2 + 2k_1)}$$

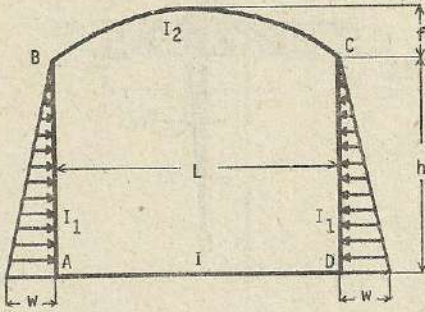
$$M' = \frac{wh^2(2k_1 + 3k_2)}{6(1 + k_2 + 2k_1)}$$

$$M_A = M_D = M' - \frac{wh^2}{2} + H' n$$

$$M_B = M_C = M' - H' n$$

$$H = - \frac{5wh^2 [3k_1 h (3+2k) + 3k_2 h (6+5k_1) + 4f(2k_1+3k_2)]}{4 [15h^2 \{ (2+k_1)k_1 + (2k_2+3)k_2 \} + 60hf(k_2+k_1) + 4f^2(1+6k_2+12k_1)]}$$

(386)



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2}{I}$$

$$n = \frac{3h + 3k_1 h + 2f}{3(1 + k_2 + 2k_1)}$$

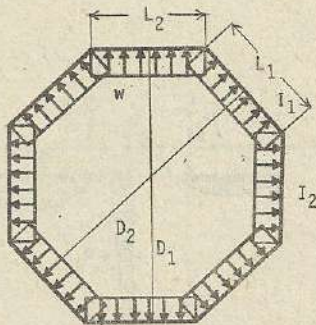
$$M' = \frac{wh^2(k_1 + 2k_2)}{12(1 + k_2 + 2k_1)}$$

$$M_A = M_D = M' - \frac{wh^2}{6} + H'n$$

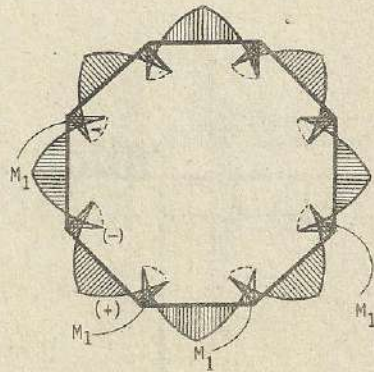
$$M_B = M_C = M' - H'n$$

$$H = - \frac{wh^2 [3k_1 h (3k_1 + 4) + 3k_2 h (9k_1 + 10) + 10f (k_1 + 2k_2)]}{4 [15h^2 \{ (2+k_1) k_1 + (2k_1+3) k_2 \} + 60hf (k_2 + k_1) + 4f^2 (1+6k_2 + 12k_1)]}$$

(387)



$$k = \frac{I_1 L_2}{I_2 L_1}$$



M_1 : Momento de flexión de cada punto extremo

Z_1 : Fuerza de tensión axial del lado L_1

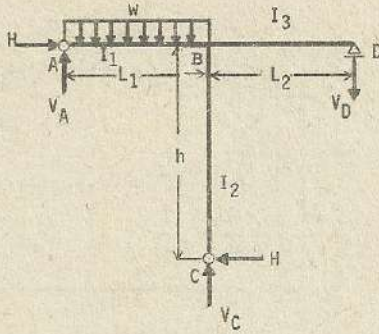
Z_2 : Fuerza de tensión axial del lado L_2

$$M_1 = - \frac{w}{12} \cdot \frac{L_1^2 + kL_2^2}{1+k} \quad ; \quad Z_1 = \frac{wD_1}{2} \quad ; \quad Z_2 = \frac{wD_2}{2}$$

Tanque rectangular $D_1 = L_1 \quad ; \quad D_2 = L_2$

Tanque cuadrado $L_1 = L_2 = L \quad ; \quad M_1 = - \frac{wL^2}{12} \quad ; \quad Z = + \frac{wL}{2}$

$$k = 1$$



(388)

$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2}$$

$$\lambda_2 = \frac{L_2}{L_1}$$

$$\mu = k_1 + k_1 k_2 + k_2$$

$$M_{BD} = -\frac{wL_1^2}{8} \cdot \frac{k_1}{\mu}$$

$$M_{BA} = M_{BC} + M_{BD}$$

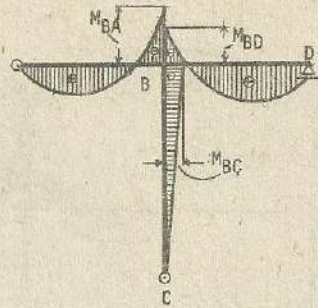
$$M_{BC} = -\frac{wL_1^2}{8} \cdot \frac{k_1 k_2}{\mu}$$

$$H = -\frac{M_{BC}}{h}$$

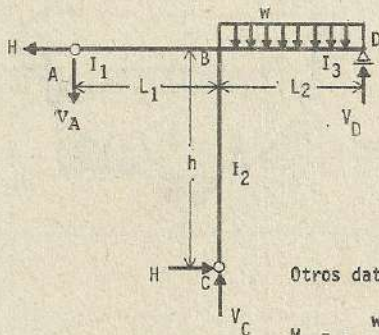
$$V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}$$

$$V_C = \frac{wL_1}{2} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$

$$V_D = -\frac{M_{BC}}{L_2}$$



(389)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

Otros datos, referencia (388)

$$M_{BA} = -\frac{wL_2^2}{8} \cdot \frac{k_2}{\mu}$$

$$M_{BC} = -\frac{wL_2^2}{8} \cdot \frac{k_1 k_2}{\mu}$$

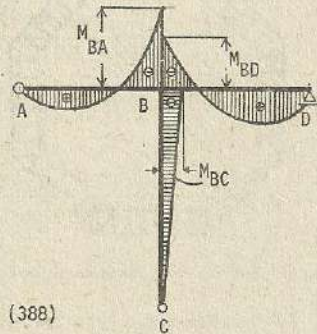
$$M_{BD} = M_{BA} - M_{BC}$$

$$H = \frac{M_{BC}}{h}$$

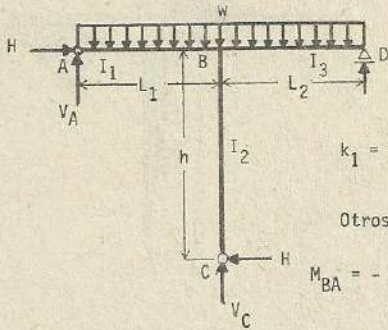
$$V_A = -\frac{M_{BA}}{L_1}$$

$$V_C = -\frac{wL_2}{2} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$

$$V_D = \frac{wL_2}{2} + \frac{M_{BD}}{L_2}$$



(390)



$$k_1 = \frac{I_2 L_1}{I_1 h} ;$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

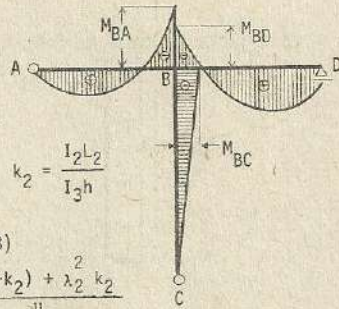
Otros datos (388)

$$M_{BA} = - \frac{w L_1^2}{8} \frac{k_1 (1+k_2) + \lambda_2^2 k_2}{\mu}$$

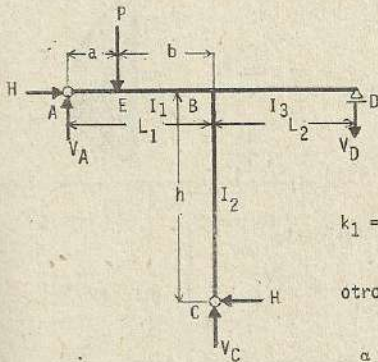
$$M_{BD} = - \frac{w L_1^2}{8} \frac{k_1 + \lambda_2^2 k_2 (1+k_1)}{\mu}$$

$$M_{BC} = M_{BA} - M_{BD} ; H = - \frac{M_{BC}}{h} ; V_A = \frac{w L_1}{2} + \frac{M_{BA}}{L_1} ; V_D = \frac{w L_2}{2} + \frac{M_{BD}}{L_2}$$

$$V_C = \frac{w (L_1 + L_2)}{2} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$



(391)



$$k_1 = \frac{I_2 L_1}{I_1 h} ;$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

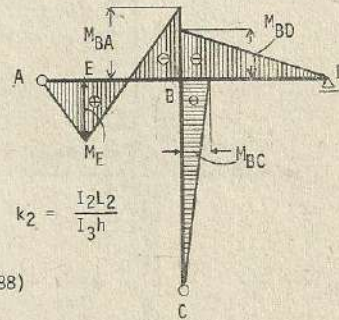
otros datos (388)

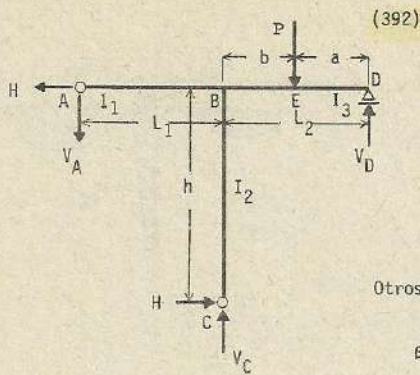
$$\alpha = \frac{a}{L_1}$$

$$M_{BC} = - \frac{pa (1 - \alpha^2)}{2} \cdot \frac{k_1 k_2}{\mu} ; M_{BD} = - \frac{pa (1 - \alpha^2)}{2} \cdot \frac{k_1}{\mu}$$

$$M_{BA} = M_{BC} + M_{BD} ; H = - \frac{M_{BC}}{h} ; V_D = - \frac{M_{BD}}{L_2}$$

$$V_A = \frac{Pb}{L_1} + \frac{M_{BA}}{L_1} ; V_C = \frac{Pa}{L_1} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$





$$k_1 = \frac{I_2 L_1}{I_1 h}$$

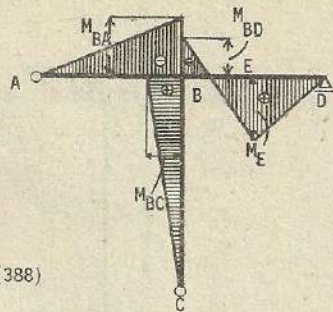
$$k_2 = \frac{I_2 L_2}{I_3 h}$$

Otros datos (388)

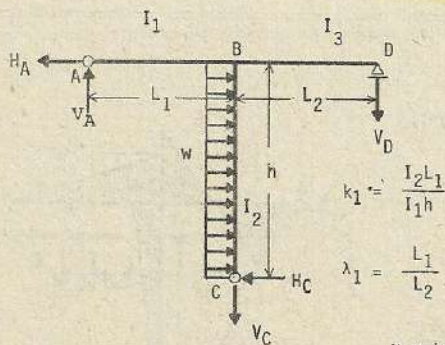
$$\beta = \frac{a}{L_2}$$

$$H = \frac{M_{BC}}{h} ; \quad M_{BA} = -\frac{Pa(1-\beta^2)}{2} \cdot \frac{k_2}{\mu} ; \quad M_{BC} = +\frac{Pa(1-\beta^2)}{2} \cdot \frac{k_1 k_2}{\mu}$$

$$M_{BD} = M_{BA} - M_{BC} ; \quad V_D = \frac{Pb}{L_2} + \frac{M_{BD}}{L_2} ; \quad V_A = -\frac{M_{BA}}{L_1} ; \quad V_C = \frac{Pa}{L_2} - \frac{M_{BD}}{L_2} - \frac{M_{BA}}{L_2}$$



(393)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

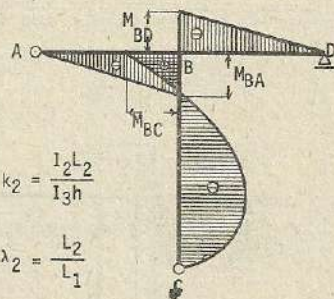
$$\lambda_1 = \frac{L_1}{L_2}$$

$$\lambda_2 = \frac{L_2}{L_1}$$

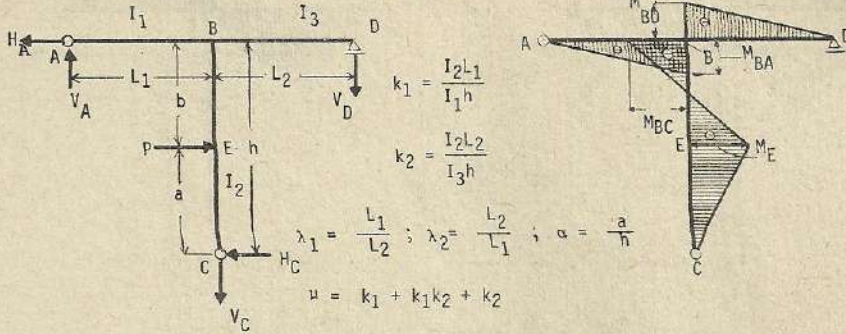
$$\mu = k_1 + k_1 k_2 + k_2$$

$$M_{BA} = +\frac{wL^2}{8} \cdot \frac{k_2}{\mu} ; \quad H_A = \frac{wh}{2} + \frac{M_{BC}}{h} ; \quad V = \frac{M_{BA}}{L_1} ; \quad V_C = V_A - V_D$$

$$M_{BD} = -\frac{wh}{8} \cdot \frac{k_1}{\mu} ; \quad H_C = \frac{wh}{2} - \frac{M_{BC}}{h} ; \quad M_{BC} = M_{BA} - M_{BD} ; \quad V_D = -\frac{M_{BD}}{L_2}$$



(394)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2}; \quad \lambda_2 = \frac{L_2}{L_1}; \quad \alpha = \frac{a}{h}$$

$$\mu = k_1 + k_1 k_2 + k_2$$

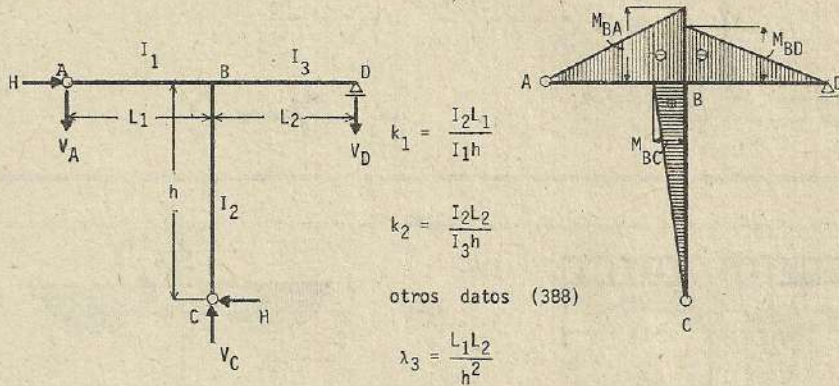
$$M_{BA} = + \frac{Pa(1-\alpha^2)}{2} \cdot \frac{k_2}{\mu}; \quad H_A = P\alpha + \frac{M_{BC}}{h}; \quad V_D = - \frac{M_{BC}}{L_2}$$

$$M_{BD} = - \frac{Pa(1-\alpha^2)}{2} \cdot \frac{k_1}{\mu}; \quad H_C = P(1-\alpha) - \frac{M_{BC}}{h}; \quad V_A = \frac{M_{BA}}{L_1}$$

$$M_{BC} = M_{BA} - M_{BD} \quad ; \quad V_C = V_A - V_D$$

(395)

Esfuerzos debido al cambio de temperatura



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (388)

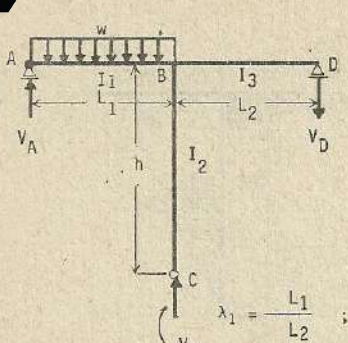
$$\lambda_3 = \frac{L_1 L_2}{h^2}$$

$$H = - \frac{M_{BC}}{h}; \quad M_{BA} = - \frac{3EeI_2 t}{L_2} \cdot \frac{1 + \lambda_2 + \lambda_2 k_2 + \lambda_3 k_2}{\mu}; \quad M_{BC} = M_{BA} - M_{BD}$$

$$M_{BD} = - \frac{3EeI_2 t}{L_2} \cdot \frac{1 + \lambda_2 + k_1 - \lambda_3 k_1}{\mu}; \quad V_A = - \frac{M_{BA}}{L_1}; \quad V_C = - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$

$$V_D = - \frac{M_{BD}}{L_2}$$

(396)



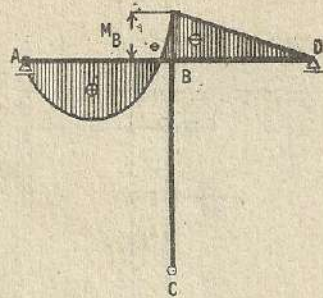
$$k_1 = \frac{I_2 L_1}{I_2 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

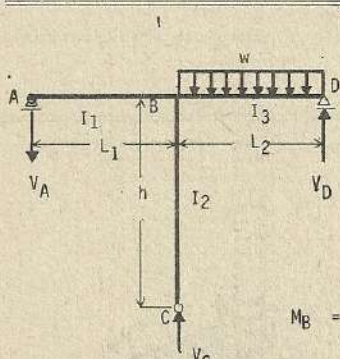
$$\lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1} ; \mu = k_1 + k_2$$

$$M_B = -\frac{w L_1^2}{4} \cdot \frac{k_1}{2\mu} ; V_A = \frac{w L_1}{2} + \frac{M_B}{L_1}$$

$$V_C = \frac{w L_1}{2} - \frac{M_B}{L_1} - \frac{M_B}{L_2} ; V_D = -\frac{M_B}{L_2}$$



(397)



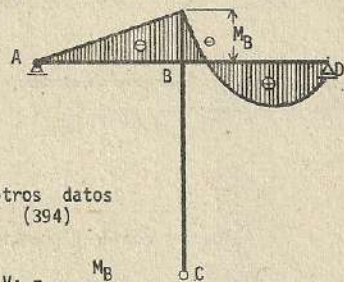
$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

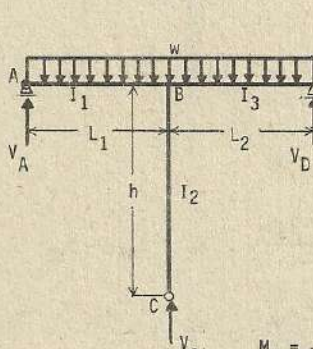
otros datos (394)

$$M_B = -\frac{w L_2^2}{8} \cdot \frac{k_2}{2\mu} ; V_A = -\frac{M_B}{L_1}$$

$$V_C = \frac{w L_2}{2} - \frac{M_B}{L_1} - \frac{M_B}{L_2} ; V_D = \frac{w L_2}{2} + \frac{M_B}{L_2}$$



(398)



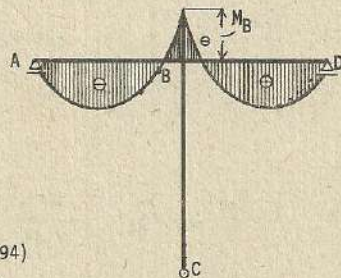
$$k_1 = \frac{I_2 L_1}{I_1 h}$$

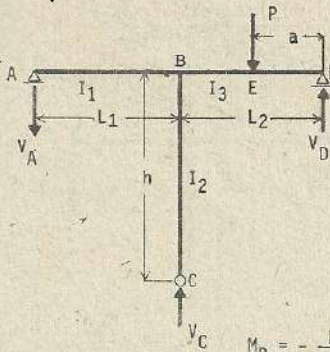
$$k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (394)

$$M_B = -\frac{w L_1^2}{8} \cdot \frac{k_1 + \lambda_2^2 k_2}{\mu}$$

$$V_A = \frac{w L_1}{2} + \frac{M_B}{L_1} ; V_C = \frac{w (L_1 + L_2)}{2} - \frac{M_B}{L_1} - \frac{M_B}{L_2} ; V_D = \frac{w L_2}{2} + \frac{M_B}{L_2}$$





(399)

$$k_1 = \frac{I_2 L_1}{I_1 h} ;$$

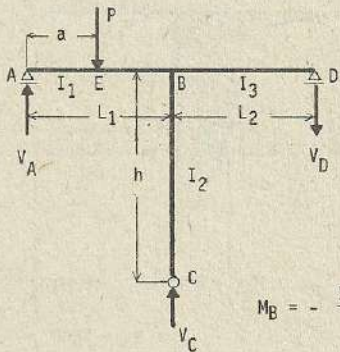
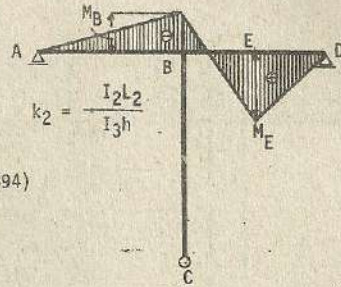
$$k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (394)

$$\alpha = \frac{a}{L_1}$$

$$M_B = - \frac{Pa (1 - \alpha^2)}{2} \cdot \frac{k_1}{\mu} ; \quad V_A = P (1 - \alpha) + \frac{M_B}{L_1}$$

$$V_C = P \alpha - \frac{M_B}{L_1} - \frac{M_B}{L_2} ; \quad V_D = - \frac{M_B}{L_2}$$



(400)

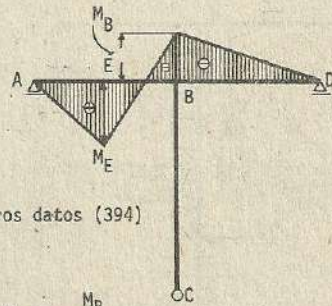
$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

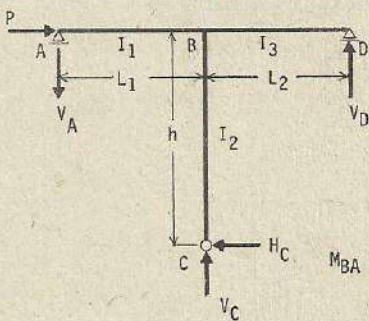
$$\beta = \frac{a}{L_2}$$

$$M_B = - \frac{Pa (1 - \beta^2)}{2} \cdot \frac{k_2}{\mu} ; \quad V_A = - \frac{M_B}{L_1}$$

$$V_C = P \beta - \frac{M_B}{L_1} - \frac{M_B}{L_2} ; \quad V_D = P (1 - \beta) + \frac{M_B}{L_2}$$



otros datos (394)



(401)

$$k_1 = \frac{I_2 L_1}{I_1 h} ;$$

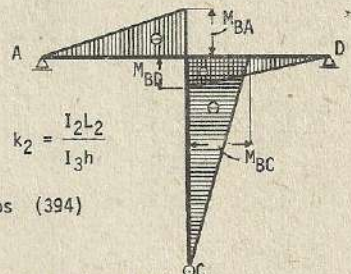
$$k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (394)

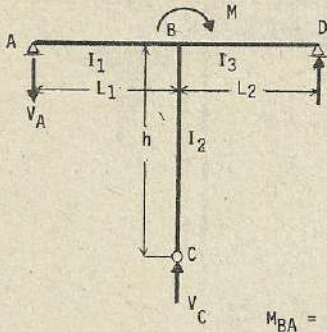
$$M_{BA} = - Ph \frac{k_2}{\mu} ; \quad M_{BC} = - Ph = M_{BA} - M_{BD}$$

$$M_{BD} = - Ph \frac{k_1}{\mu} ; \quad V_C = V_A - V_D ; \quad V_D = \frac{M_{BD}}{L_2}$$

$$H_C = P$$



(402)



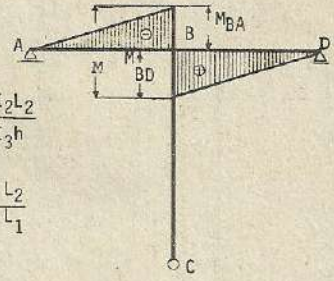
$$k_1 = \frac{I_2 L_1}{I_1 h} ; k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1}$$

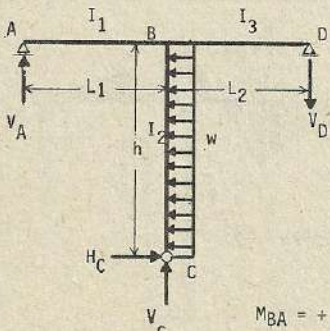
$$\mu = k_1 + k_2$$

$$M_{BA} = -M \frac{k_2}{\mu} ; M + M_{BA} - M_{BD} = 0 ; V_A = -\frac{M_{BA}}{L_1}$$

$$M_{BD} = +M \frac{k_1}{\mu} ; V = \frac{M_{BD}}{L_2} ; V_C = V_A - V_D$$



(403)



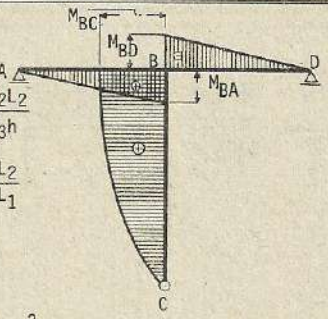
$$k_1 = \frac{I_2 L_1}{I_1 h} ; k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1}$$

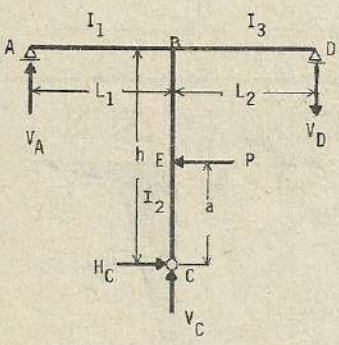
$$\mu = k_1 + k_2$$

$$M_{BA} = +\frac{wh^2}{2} \cdot \frac{k_2}{\mu} ; M_{BC} = +\frac{wh^2}{2} = -M_{BA} - M_{BD} ; H_C = wh$$

$$M_{BD} = -\frac{wh^2}{2} \cdot \frac{k_1}{\mu} ; V_A = \frac{M_{BA}}{L_1} ; V_D = -\frac{M_{BD}}{L_2} ; V_C = V_A - V_D$$



(404)



$$k_1 = \frac{I_2 L_1}{I_1 h} ; k_2 = \frac{I_2 L_2}{I_3 h}$$

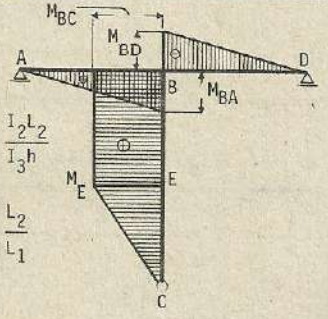
$$\lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1}$$

$$\nu = k_1 + k_2$$

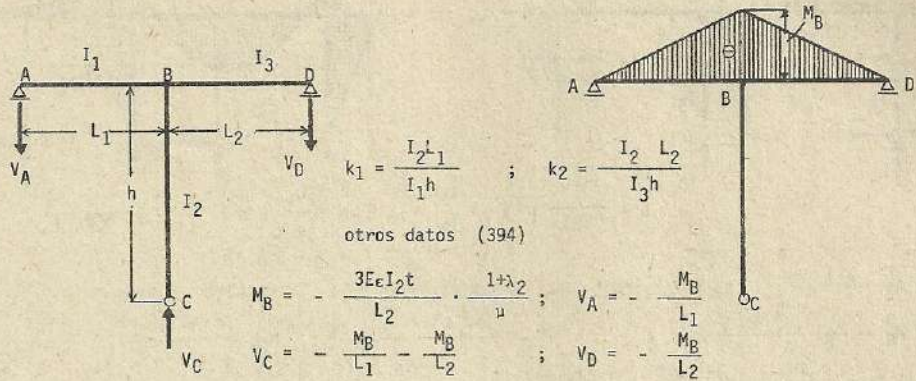
$$M_{BA} = +Pa \frac{k_2}{\nu} ; M_{BC} = +Pa = M_{BA} - M_{BD} ; H_C = P$$

$$M_{BD} = -Pa \frac{k_1}{\nu} ; V_A = \frac{M_{BA}}{L_1} ; V_D = -\frac{M_{BD}}{L_2}$$

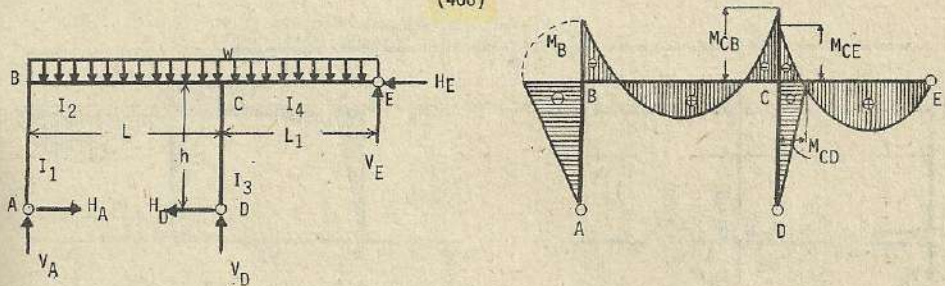
$$V_C = V_D - V_A$$



(405) : Esfuerzos por cambio de temperatura



(406)



$$k = \frac{I_2 h}{I_1 L} ; k_1 = \frac{I_2 h}{I_3 L} ; k_2 = \frac{I_2 L_1}{I_4 L} ; \lambda = \frac{L}{L_1} ; \lambda_1 = \frac{L_1}{L}$$

$$s_1 = k + k k_1 + k_1 ; s_2 = k + k k_2 + k_2 ; s = k_1 + k_1 k_2 + k_2 ; \mu = s (3 + 4k) + k_1 k_2$$

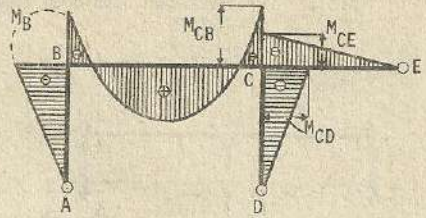
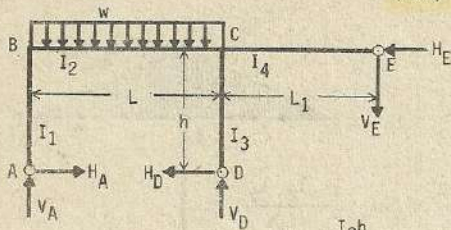
$$M_B = - \frac{w L^2}{4} \cdot \frac{s + k_1 k_2 (1 - \lambda_1^2)}{\mu} ; M_{CB} = - \frac{w L^2}{4} \cdot \frac{(1 + 2k)(k_1 + k_2) + 2 \lambda_1^2 k_1 k_2 (1 + k)}{\mu}$$

$$M_{CD} = - \frac{w L^2 k_2}{8} \cdot \frac{2(1 + 2k) - \lambda_1^2 (3 + 4k)}{\mu} ; M_{CE} = - \frac{w L^2}{8} \cdot \frac{2k_1 (1 + 2k) + \lambda_1^2 k_2 (3 + 4s_1)}{\mu}$$

$$H_D = - \frac{M_{CD}}{h} ; H_A = - \frac{M_B}{h} ; H_E = H_A - H_D ; V_A = \frac{w L}{2} + \frac{M_{CB} - M_B}{L}$$

$$V_D = \frac{w (L + L_1)}{2} + \frac{M_B - M_{CB}}{L} - \frac{M_{CE}}{L_1} ; V_E = \frac{w L_1}{2} + \frac{M_{CE}}{L}$$

(407)



$$k = \frac{I_2 h}{I_1 L}; \quad k_1 = \frac{I_2 h}{I_3 L}; \quad k_2 = \frac{I_2 L_1}{I_4 L}; \quad \lambda = \frac{L}{L_1}; \quad \lambda_1 = \frac{L_1}{L}$$

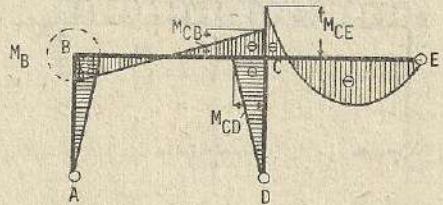
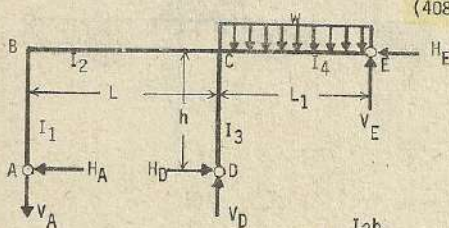
$$s_1 = k + k k_1 + k_1; \quad s_2 = k + k_2 k + k_2; \quad s = k_1 + k_2 k_1 + k_2; \quad \mu = s(3+4k) + k_1 k_2$$

$$M_B = -\frac{wL^2}{4} \cdot \frac{s + k_1 k_2}{\mu}; \quad M_{CD} = -\frac{wL^2}{4} \cdot \frac{k_2 \cdot 1 + 2k}{\mu}; \quad M_{CB} = -\frac{wL^2(k_1 + k_2)}{4} \cdot \frac{1+2k}{\mu}$$

$$M_{CE} = -\frac{wL^2 k_1}{4} \cdot \frac{1+2k}{\mu}; \quad H_D = -\frac{M_{CD}}{h}; \quad H_A = -\frac{M_B}{h}; \quad H_E = -\frac{M_B}{h} - \frac{M_{CD}}{h}$$

$$V_A = \frac{wL}{2} + \frac{M_{CB} - M_B}{L}; \quad V_D = \frac{wL}{2} + \frac{M_B - M_{CB}}{L} - \frac{M_{CE}}{L_1}; \quad V_E = -\frac{M_{CE}}{L_1}$$

(408)



$$k = \frac{I_2 h}{I_1 L}; \quad k_1 = \frac{I_2 h}{I_3 L}; \quad k_2 = \frac{I_2 L_1}{I_4 L}$$

$$\lambda = \frac{L}{L_1}; \quad \lambda_1 = \frac{L_1}{L}; \quad s_1 = k + k k_1 + k_1$$

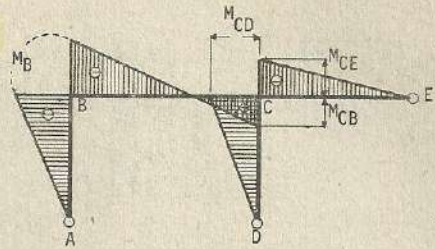
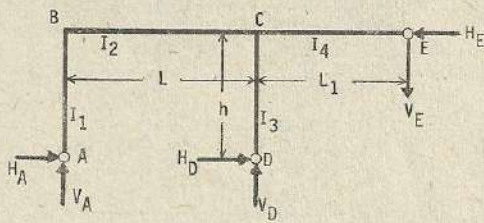
$$s_2 = k + k k_2 + k_2; \quad s = k_1 + k_1 k_2 + k_2; \quad \mu = s(3+4k) + k_1 k_2$$

$$M_B = +\frac{wL^2 k_1 k_2}{4\mu}; \quad M_{CB} = -\frac{wL^2 k_1 k_2}{2} \cdot \frac{1+k}{\mu}; \quad M_{CD} = +\frac{wL^2 \cdot k_2}{8} \cdot \frac{3+4k}{\mu}$$

$$M_{CE} = -\frac{wL^2 k_2}{8} \cdot \frac{3+4s_1}{\mu}; \quad H_D = \frac{M_{CD}}{h}; \quad H_A = +\frac{M_B}{h}; \quad H_E = +\frac{M_{CD} - M_B}{h}$$

$$V_A = \frac{M_B - M_{CB}}{L}; \quad V_D = \frac{wL_1}{2} + \frac{M_B - M_{CB}}{L} - \frac{M_{CE}}{L_1}; \quad V_E = \frac{wL_1}{2} + \frac{M_{CE}}{L_1}$$

Esfuerzos por cambio de temperatura : (409)



$$k = \frac{I_2 h}{I_1 L} ; k_1 = \frac{I_2 h}{I_3 L} ; k_2 = \frac{I_2 L_1}{I_4 L} ; \lambda = \frac{L}{L_1} ; \lambda_1 = \frac{L_1}{L} ; \alpha = \frac{h^2}{L^2}$$

$$s_1 = k + k_1 + k_1 ; s_2 = 1 + k k_2 + k_2 ; s = k_1 + k_1 k_2 + k_2 ; u = s(3+4k) + k_1 k_2$$

$$M_B = - \frac{6EI_2 \epsilon t L_1}{hL} \cdot \frac{k_2 + 2(\lambda+1)s - \alpha k_1}{u} ; M_{CB} = \frac{6EI_2 \epsilon t L_1}{hL} \cdot \frac{2(k_2 - \alpha k_1)(1+k) + (\lambda+1)(k_1 + k_3)}{u}$$

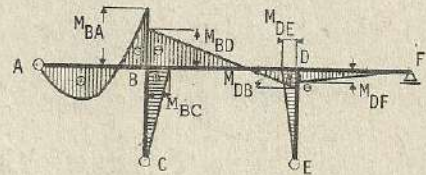
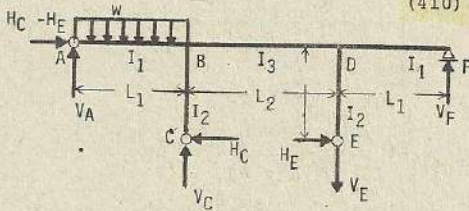
$$M_{CE} = - \frac{3EI_2 \epsilon t L_1}{hL} \cdot \frac{3 + 4k - 2(\lambda+1)k_1 + \alpha(3+4s_1)}{u} ; M_{CD} = M_{CB} - M_{CE}$$

$$H_A = - \frac{M_B}{h} ; H_D = \frac{M_{CD}}{h} ; H_E = H_A + H_D ; V_A = \frac{M_{CB} - M_B}{L} ; V_D = V_E - V_A$$

Nota : En caso de descenso de temperatura los signos de los esfuerzos de momentos son contrarios a las presentadas

$$V_E = - \frac{M_{CE}}{L_1}$$

(410)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; k_2 = \frac{I_1 h}{I_2 L_1} ; u_1 = 3 + 2k_1 + 3k_2$$

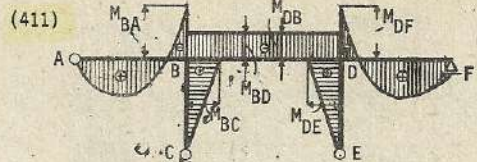
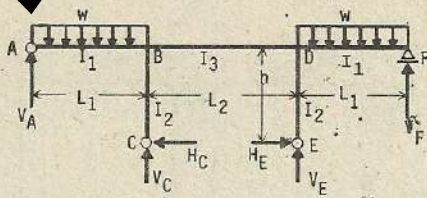
$$u_2 = 1 + 2k_1 + k_2 ; \lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = - \frac{wL_1^2}{16} \left[\frac{1+2k_1}{u_2} + \frac{3+2k_1}{u_1} \right] ; M_{DF} = + \frac{wL_1^2}{16} \left[\frac{1+2k_1}{u_2} - \frac{3+2k_1}{u_1} \right]$$

$$M_{BD} = - \frac{wL_1^2}{8} \left[\frac{1}{u_2} + \frac{1}{u_1} \right] ; M_{DB} = + \frac{wL_1^2}{8} \left[\frac{1}{u_2} - \frac{1}{u_1} \right]$$

$$M_{BC} = M_{BD} - M_{BA} ; M_{DE} = M_{DB} - M_{DF} ; V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} ; V_F = \frac{M_{DF}}{L_1}$$

$$V_C = \frac{wL_1}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1} ; H_C = \frac{M_{BC}}{h} ; H_E = \frac{M_{DE}}{h}$$

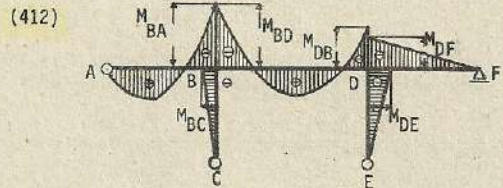
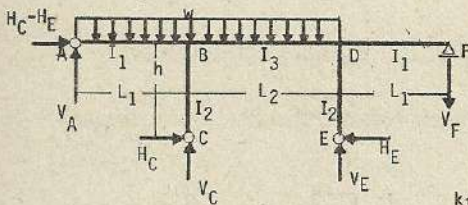


$$k_1 = \frac{I_3 h}{I_2 L_2} ; k_2 = \frac{I_1 h}{I_2 L_1} ; \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 ; \lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1} ; M_{BA} = M_{DF} = -\frac{wL_1^2}{8} \cdot \frac{3 + 2k_1}{\mu_1}$$

$$M_{BC} = M_{DE} = +\frac{3wL_1^2}{8\mu_1} ; M_{BD} = M_{DB} = -\frac{wL_1^2}{4} \cdot \frac{k_1}{\mu_1} ; H_C = H_E = H = \frac{M_{BC}}{h}$$

$$V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} ; V_C = V_E = \frac{wL_1}{2} - \frac{M_{BA}}{L_1}$$



$$k_1 = \frac{I_3 h}{I_2 L_2} ; k_2 = \frac{I_1 h}{I_2 L_1} ; \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 ; \lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = -\frac{wL_1^2}{16} \left[\frac{3+2k_1+4\lambda_2^2 k_1}{\mu_1} + \frac{1+2k_1}{\mu_2} \right] ; M_{DF} = -\frac{wL_1^2}{16} \left[\frac{3+2k_1+4\lambda_2^2 k_2}{\mu_1} - \frac{1+2k_1}{\mu_2} \right]$$

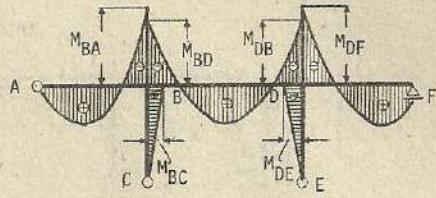
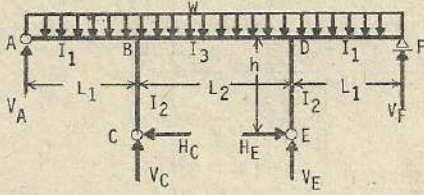
$$M_{BD} = -\frac{wL_1^2}{8} \left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} + \frac{k_1}{\mu_2} \right] ; M_{DB} = \frac{wL_1^2}{8} \left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} - \frac{k_1}{\mu_2} \right]$$

$$M_{BC} = M_{BD} - M_{BA} ; M_{DE} = M_{DB} - M_{DF} ; H_C = -\frac{M_{BC}}{h} ; H_E = -\frac{M_{DE}}{h}$$

$$V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} ; V_C = \frac{w(L_1 + L_2)}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; V_F = -\frac{M_{DF}}{L_1}$$

$$V_E = \frac{wL_2}{2} - \frac{M_{DF}}{L_1} - \frac{M_{DB} - M_{BD}}{L_2}$$

(413)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; \quad k = \frac{I_1 h}{I_2 L_1} ; \quad \mu_1 = 2 + 2k_1 + 3k_2$$

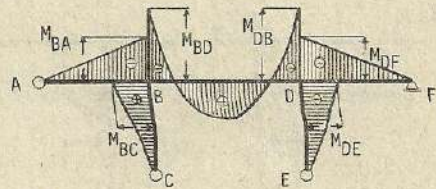
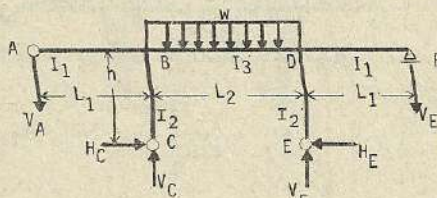
$$\mu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} ; \quad \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = M_{DF} = -\frac{wL_1^2}{8} \cdot \frac{3+2k_1+2\lambda_2^2 k_2}{\mu_1} ; \quad M_{BC} = M_{DE} = +\frac{wL_1^2}{8} \cdot \frac{3-2\lambda_2^2}{\mu_1}$$

$$M_{BD} = M_{DB} = -\frac{wL_1^2}{4} \cdot \frac{k_1 + \lambda_2^2(1+k_2)}{\mu_1} ; \quad H_C = H_E = H = \frac{M_{BC}}{h}$$

$$V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} ; \quad V_C = V_E = \frac{w(L_1+L_2)}{2} - \frac{M_{BA}}{L_1}$$

(414)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

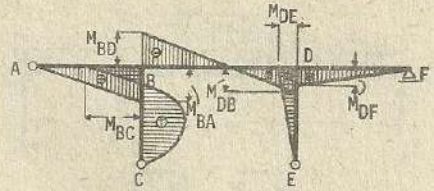
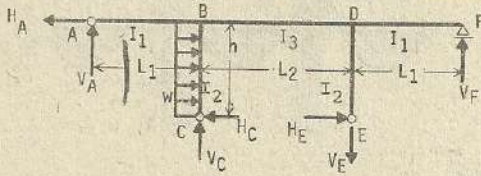
$$\mu_1 = 3 + 2k_1 + 3k_2 ; \quad \mu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} ; \quad \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = M_{DF} = -\frac{wL_2^2 k_2}{4\mu_1} ; \quad M_{BC} = M_{DE} = -\frac{wL_2^2}{4\mu_1}$$

$$M_{BD} = M_{DB} = -\frac{wL_2^2 (L+k_2)}{4\mu_1} ; \quad H_C = H_E = H = -\frac{M_{BC}}{h} ; \quad V_A = V_F = -\frac{M_{BA}}{L_1}$$

$$V_C = V_E = \frac{wL_2}{2} - \frac{M_{BA}}{L_1}$$

(415)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} ; \quad \lambda_2 = \frac{L_2}{L_1}$$

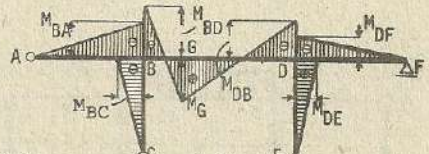
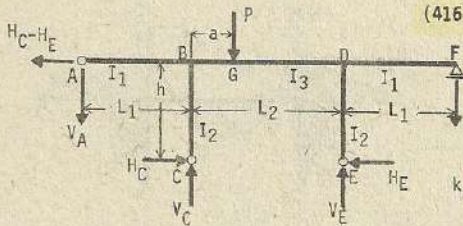
$$M_{BA} = + \frac{wh^2 k_2}{16} \left[\frac{3}{\mu_1} + \frac{1}{\mu_2} \right] ; \quad M_{DF} = + \frac{wh^2 k_2}{16} \left[\frac{3}{\mu_1} - \frac{1}{\mu_1} \right] ; \quad M_{BD} = - \frac{wh^2 k_1}{8} \left[\frac{1}{\mu_2} + \frac{1}{\mu_1} \right]$$

$$M_{DB} = + \frac{wh^2 k_1}{8} \left[\frac{1}{\mu_2} - \frac{1}{\mu_1} \right] ; \quad M_{BC} = M_{BD} - M_{BA} ; \quad M_{DE} = M_{DB} - M_{DF}$$

$$H_A = \frac{wh}{2} + \frac{M_{DE} - M_{BC}}{h} ; \quad H_C = \frac{wh}{2} + \frac{M_{BC}}{h} ; \quad H_E = \frac{M_{DE}}{h} ; \quad V_A = \frac{M_{BA}}{L_1}$$

$$V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; \quad V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DB}}{L_1} ; \quad V_F = \frac{M_{DF}}{L_1}$$

(416)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} ; \quad \lambda_2 = \frac{L_2}{L_1} ; \quad \alpha = \frac{a}{L_2}$$

$$M_{BA} = - \frac{Pa(1-\alpha)k_2}{2} \left[\frac{3}{\mu_1} + \frac{1-2\alpha}{\mu_2} \right] ; \quad M_{DF} = - \frac{Pa(1-\alpha)k_2}{2} \left[\frac{3}{\mu_1} - \frac{1-2\alpha}{\mu_2} \right]$$

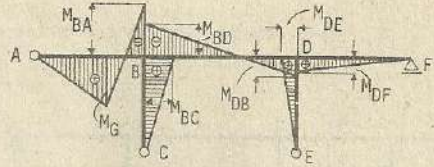
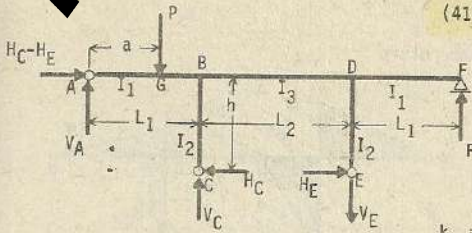
$$M_{BC} = - \frac{Pa(1-\alpha)}{2} \left[\frac{3}{\mu_1} + \frac{1-2\alpha}{\mu_2} \right] ; \quad M_{DE} = - \frac{Pa(1-\alpha)}{2} \left[\frac{3}{\mu_1} - \frac{1-2\alpha}{\mu_2} \right]$$

$$M_{BD} = M_{BA} + M_{BC} ; \quad M_{DB} = M_{DF} + M_{DE} ; \quad H_C = - \frac{M_{BC}}{h} ; \quad H_E = - \frac{M_{DE}}{h}$$

$$V_A = - \frac{M_{BA}}{L_1} ; \quad V_C = P(1-\alpha) - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; \quad V_F = - \frac{M_{DF}}{L_1}$$

$$V_E = P\alpha + \frac{M_{BD} - M_{DB}}{L_2} - \frac{M_{DF}}{L_1}$$

(417)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \nu_1 = 3 + 2k_1 + 3k_2$$

$$\nu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} ; \quad \lambda_2 = \frac{L_2}{L_1} ; \quad \alpha = \frac{a}{L_1}$$

$$M_{BA} = - \frac{Pa (1 - \alpha^2)}{4} \left[\frac{1+2k_1}{\nu_2} + \frac{3+2k_1}{\nu_1} \right] ; \quad M_{DF} = + \frac{Pa (1 - \alpha^2)}{4} \left[\frac{1+2k_1}{\nu_2} - \frac{3+2k_1}{\nu_1} \right]$$

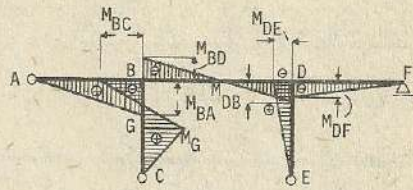
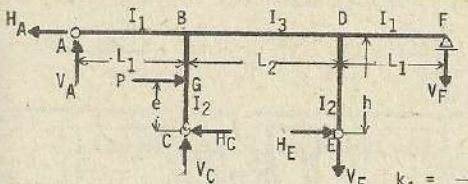
$$M_{BD} = - \frac{Pa (1 - \alpha^2) k_1}{2} \left[\frac{1}{\nu_2} + \frac{1}{\nu_1} \right] ; \quad M_{DB} = + \frac{Pa (1 - \alpha^2) k_1}{2} \left[\frac{1}{\nu_2} - \frac{1}{\nu_1} \right]$$

$$M_{BC} = M_{BD} - M_{BA} ; \quad M_{DE} = M_{DB} - M_{DF} ; \quad H_C = \frac{M_{BC}}{h} ; \quad H_E = \frac{M_{DE}}{h}$$

$$V_A = P (1 - \alpha) + \frac{M_{BA}}{L_1} ; \quad V_C = P \alpha - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; \quad V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1}$$

$$V_F = \frac{M_{DF}}{L_1}$$

(418)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \nu_1 = 3 + 2k_1 + 3k_2$$

$$\nu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} ; \quad \lambda_2 = \frac{L_2}{L_1} ; \quad \beta = \frac{e}{h}$$

$$M_{BA} = + \frac{Pe (1 - \beta^2) k_2}{4} \left[\frac{3}{\nu_1} + \frac{1}{\nu_2} \right] ; \quad M_{DF} = + \frac{Pe (1 - \beta^2) k_2}{4} \left[\frac{3}{\nu_1} - \frac{1}{\nu_2} \right]$$

$$M_{BD} = - \frac{Pe (1 - \beta^2) k_1}{2} \left[\frac{1}{\nu_2} + \frac{1}{\nu_1} \right] ; \quad M_{DB} = + \frac{Pe (1 - \beta^2) k_1}{2} \left[\frac{1}{\nu_2} - \frac{1}{\nu_1} \right]$$

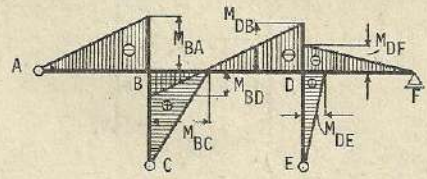
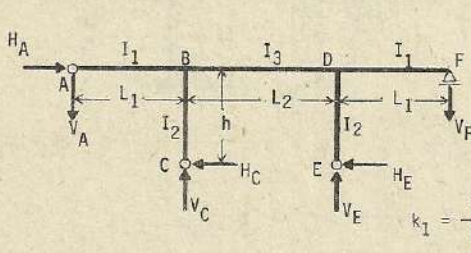
$$M_{BC} = M_{BD} - M_{BA} ; \quad M_{DE} = M_{DB} - M_{DF} ; \quad H_A = P \beta + \frac{M_{DE} - M_{BC}}{h} ; \quad H_E = \frac{M_{DE}}{h}$$

$$H_C = P (1 - \beta) + \frac{M_{BC}}{h} ; \quad V_A = \frac{M_{BA}}{L_1} ; \quad V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; \quad V_F = \frac{M_{DF}}{L_1}$$

$$V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1}$$

(419)

Esfuerzos por efectos del cambio de temperatura



$$k_1 = \frac{I_3 h}{I_2 L_2} ; k_2 = \frac{I_1 h}{I_2 L_1} ; \nu_1 = 3 + 2k_1 + 3k_2$$

$$\nu_2 = 1 + 2k_1 + k_2 ; \lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1} ; \alpha_1 = \frac{h^2}{L_1^2}$$

$$M_{BA} = - \frac{3EI_2 \epsilon t L_1 k_2}{2h^2} \left[\frac{2 \alpha_1 (3+2k_1) - 3 \lambda_2}{\nu_1} + \frac{2 + \lambda_2}{\nu_2} \right]$$

$$M_{DF} = - \frac{3EI_2 \epsilon t L_1 k_2}{2h^2} \left[\frac{2 \alpha_1 (3+2k_1) - 3 \lambda_2}{\nu_1} - \frac{2 + \lambda_2}{\nu_2} \right]$$

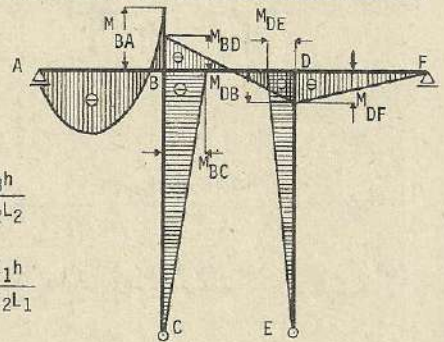
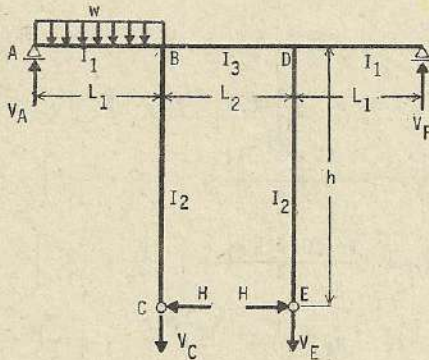
$$M_{BD} = + \frac{3EI_2 \epsilon t L_1 k_1}{h^2} \left[\frac{2 + \lambda_2}{\nu_2} - \frac{2 \alpha_1 k_2 + \lambda_2}{\nu_1} \right]$$

$$M_{DB} = - \frac{3EI_2 \epsilon t L_1 k_1}{h^2} \left[\frac{2 + \lambda_2}{\nu_2} + \frac{2 \alpha_1 k_2 + \lambda_2}{\nu_1} \right] ; M_{BC} = M_{BD} - M_{BA} ; M_{DE} = M_{DB} - M_{DF}$$

$$H_A = H_C + H_E ; H_C = \frac{M_{BC}}{h} ; H_E = - \frac{M_{DE}}{h} ; V_A = - \frac{M_{BA}}{L_1}$$

$$V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} ; V_F = - \frac{M_{DF}}{L_1} ; V_E = - \frac{M_{DF}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

(420)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\nu_1 = 3 + 2k_1 + 3k_2$$

$$\nu_2 = 1 + 2k_1 + k_2 ; \lambda_1 = \frac{L_1}{L_2} ; \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = + \frac{wL_1^2}{4} \cdot \frac{1}{4} \left[- \frac{3+2k_1}{\nu_1} - \frac{2k_1+1}{\nu_2} \right]$$

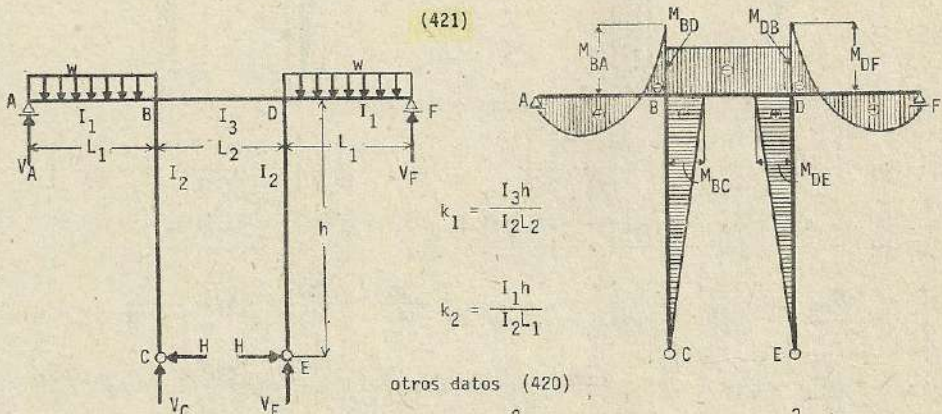
$$M_{DF} = + \frac{wL_1^2}{4} \cdot \frac{1}{4} \left[- \frac{3+2k_1}{\nu_1} + \frac{2k_1+1}{\nu_2} \right] ; M_{BD} = + \frac{wL_1^2}{4} \cdot \frac{k_1}{2} \left[- \frac{1}{\nu_1} - \frac{1}{\nu_2} \right]$$

$$M_{DB} = + \frac{wL_1^2}{4} \cdot \frac{k_1}{2} \left[-\frac{1}{\mu_1} + \frac{1}{\mu_2} \right]; \quad M_{BC} = M_{DE} = \frac{3}{4\mu_1} \cdot \frac{wL_1^2}{4}; \quad H = \frac{M_{BC}}{h}$$

$$V_A = \frac{w}{2} - \frac{M_{BA}}{L_1}; \quad V_C = \frac{w}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2}$$

$$V_E = \frac{M_{DB} - M_{BD}}{L_2} - \frac{M_{DF}}{L_1}; \quad V_F = \frac{M_{DF}}{L_1}$$

(421)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

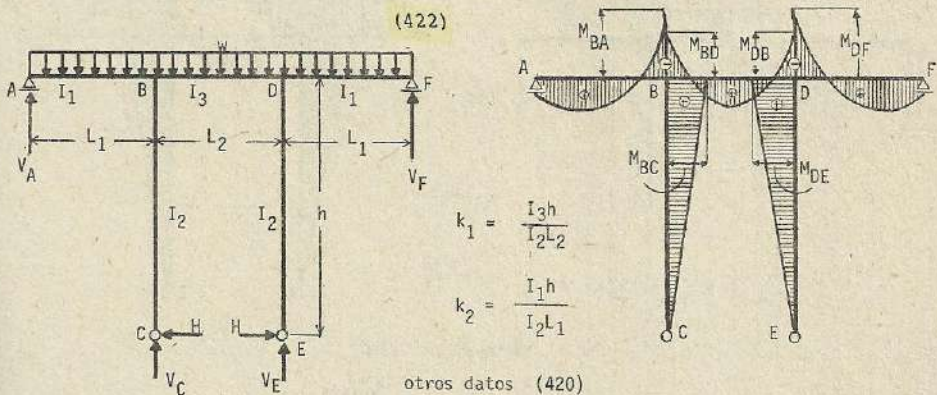
$$k_2 = \frac{I_1 h}{I_2 L_1}$$

otros datos (420)

$$M_{BA} = M_{DF} = - \frac{wL_1^2}{8} \cdot \frac{3 + 2k_1}{\mu_1}; \quad M_{BD} = - \frac{wL_1^2}{4} \cdot \frac{k_1}{\mu_1}$$

$$M_{BC} = \frac{3wL_1^2}{8\mu_1}; \quad V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}; \quad V_C = V_E = \frac{wL_1}{2} - \frac{M_{BA}}{L_1}$$

(422)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

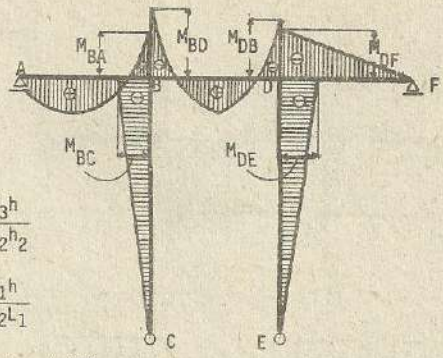
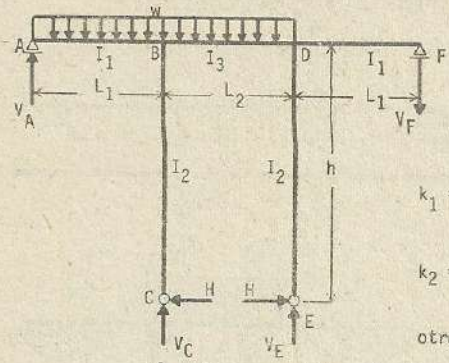
otros datos (420)

$$M_{BA} = M_{DF} = - \frac{wL_1^2}{8} \cdot \frac{3 + 2k_1 + 2\lambda_2 k_2}{\mu_1}$$

$$M_{BC} = M_{DE} = \frac{wL_1^2}{8} \cdot \frac{3 - 2\lambda_2^2}{\mu_1}; \quad M_{BD} = M_{DB} = - \frac{wL_1^2}{4} \cdot \frac{k_1 + \lambda_2^2 (1 + k_2)}{\mu_1}$$

$$V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}; \quad V_C = V_E = \frac{w(L_1 + L_2)}{2} - \frac{M_{BA}}{L_1}; \quad H = \frac{M_{BC}}{h}$$

(423)



$$k_1 = \frac{I_3 h}{I_2 h^2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

otros datos (420)

$$M_{BA} = -\frac{wL_1^2}{16} \left[\frac{3+2k_1+4\lambda_2^2 k_2}{\mu_1} + \frac{2k_1}{\mu_2} \right]$$

$$M_{DF} = -\frac{wL_1^2}{16} \left[\frac{3+2k_1+4\lambda_2^2 k_2}{\mu_1} - \frac{2k_1}{\mu_2} \right]$$

$$M_{BD} = -\frac{wL_1^2}{8} \left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} + \frac{k_1}{\mu_2} \right]$$

$$M_{DB} = -\frac{wL_1^2}{8} \left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} - \frac{k_1}{\mu_2} \right]$$

$$M_{BC} = M_{DB} = -\frac{wL_1^2}{16} \cdot \frac{4\lambda_2^2-3}{\mu_1}$$

$$V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}$$

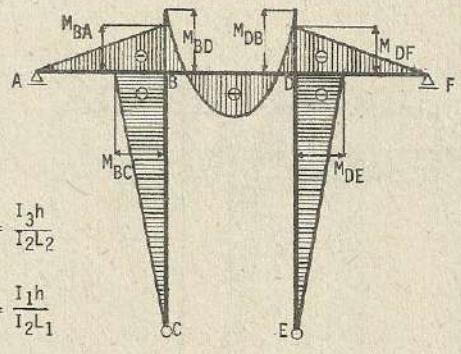
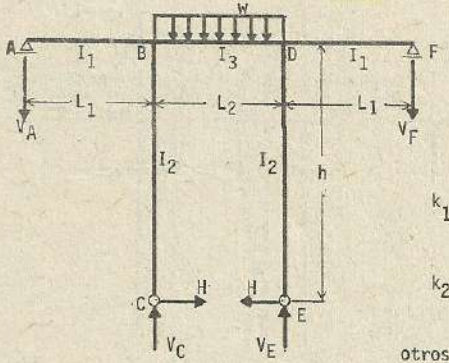
$$V_F = -\frac{M_{DF}}{L_1}$$

$$V_C = \frac{w(L_1+L_2)}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2}$$

$$V_E = \frac{wL_2}{2} - \frac{M_{DF}}{L_1} - \frac{M_{DB} - M_{BD}}{L_2}$$

$$H = \frac{M_{BC}}{h}$$

(424)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

otros datos (420)

$$M_{BA} = M_{DF} = -\frac{wL_2^2 k_2}{4\mu_1}$$

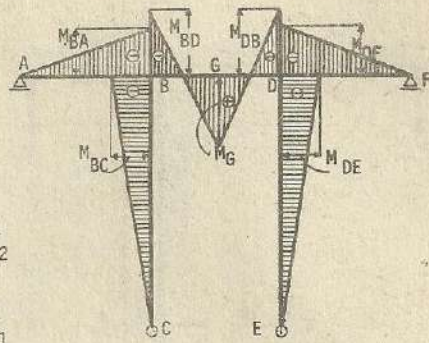
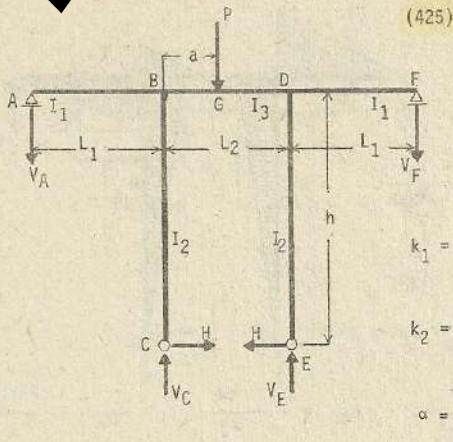
$$M_{BD} = M_{DB} = -\frac{wL_2^2(1+k_2)}{4\mu_1}$$

$$M_{BC} = M_{DE} = -\frac{wL_2^2}{4\mu_1}$$

$$V_A = V_F = -\frac{M_{BA}}{L_1}$$

$$V_C = \frac{wL_2}{2} + \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2}$$

$$V_E = \frac{wL_2}{2} + \frac{M_{BD} - M_{DB} - M_{DF}}{L_1}$$



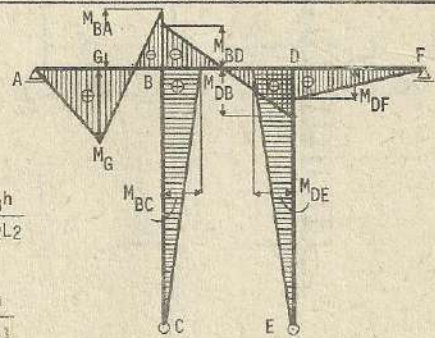
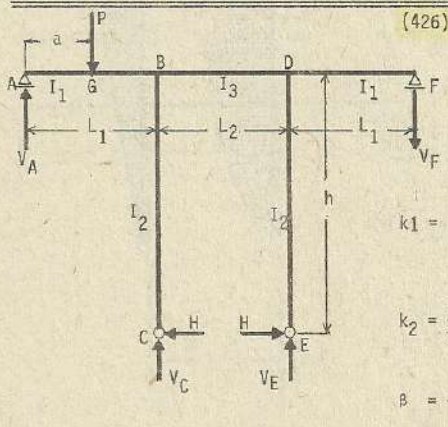
otros datos (420)

$$M_{BA} = -\frac{Pa(1-\alpha)k_2}{2} \left[\frac{3}{\nu_1} + \frac{1-2\alpha}{\nu_2} \right]; \quad M_{DF} = \frac{Pa(1-\alpha)k_2}{2} \left[\frac{3}{\nu_1} - \frac{1-2\alpha}{\nu_2} \right]$$

$$M_{BD} = -\frac{Pa(1-\alpha)}{2} \left[\frac{3(1+k_2)}{\nu_1} + \frac{(1-2\alpha)k_2}{\nu_2} \right]; \quad M_{DB} = -\frac{Pa(1-\alpha)}{2} \left[\frac{3(1+k_2)}{\nu_1} - \frac{(1-2\alpha)k_2}{\nu_2} \right]$$

$$M_{BC} = M_{DE} = -\frac{3Pa(1-\alpha)}{2\nu_1}; \quad V_C = P(1-\alpha) - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2}; \quad V_A = -\frac{M_{BA}}{L_1}$$

$$V_E = P\alpha + \frac{M_{BD} - M_{DB}}{L_2} + \frac{M_{DF}}{L_1}; \quad V_F = -\frac{M_{DF}}{L_1}; \quad H = -\frac{M_{BC}}{h}$$



otros datos (420)

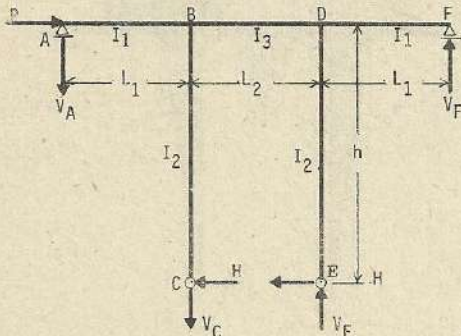
$$M_{BA} = -\frac{Pa(1-\beta^2)}{4} \left[\frac{2k_1+1}{\nu_2} + \frac{3+2k_1}{\nu_1} \right]; \quad M_{DF} = +\frac{pa(1-\beta^2)}{4} \left[\frac{2k_1+1}{\nu_2} + \frac{3+2k_1}{\nu_1} \right]$$

$$M_{BD} = -\frac{pa(1-\beta^2)k_1}{2} \left[\frac{1}{\nu_2} + \frac{1}{\nu_1} \right]; \quad M_{DB} = +\frac{Pa(1-\beta^2)k_1}{2} \left[\frac{1}{\nu_2} - \frac{1}{\nu_1} \right]$$

$$M_{BC} = M_{DE} = +\frac{3Pa(1-\beta^2)}{4\nu_1}; \quad V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1}; \quad V_A = P(1-\beta) + \frac{M_{BA}}{L_1}$$

$$V_C = P\beta - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2}; \quad V_F = \frac{M_{DF}}{L_1}; \quad H = \frac{M_{BC}}{h}$$

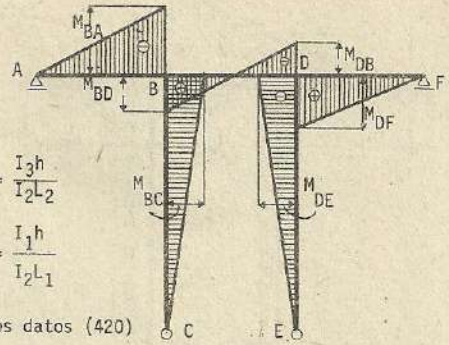
(427)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

otros datos (420)

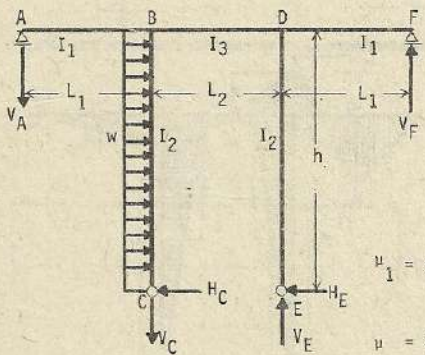


$$M_{BA} = -\frac{Phk_2}{2\mu_2} ; M_{DF} = +\frac{Phk_2}{\mu_2} ; M_{BD} = +\frac{Phk_1}{\mu_2}$$

$$M_{DB} = -\frac{Phk_1}{\mu_2} ; M_{BC} = +\frac{Ph}{2} ; M_{DE} = -\frac{Ph}{2} ; H = \frac{P}{2} ;$$

$$V_A = V_F = \frac{M_{DF}}{L_1} ; V_C = V_E = \frac{M_{BA}}{L_1} + \frac{2M_{BD}}{L_2}$$

(428)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

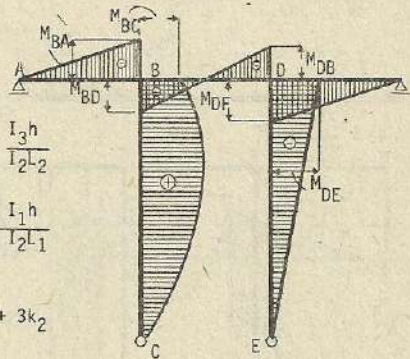
$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu = 2k_1 + k_2$$

$$\lambda_1 = \frac{L_1}{L_2} ;$$

$$\lambda_1 = \frac{L_2}{L_1}$$



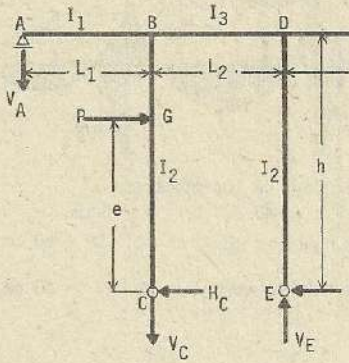
$$M_{BA} = -\frac{wh^2 k_2}{16} \left[\frac{4}{\mu_2} - \frac{3}{\mu_1} \right] ; M_{DF} = +\frac{wh^2 k_2}{16} \left[\frac{4}{\mu_2} + \frac{3}{\mu_1} \right] ; M_{BC} = M_{BD} - M_{BA}$$

$$M_{BD} = +\frac{wh^2 k_1}{8} \left[\frac{4}{\mu_2} - \frac{1}{\mu_1} \right] ; M_{DB} = -\frac{wh^2 k_1}{8} \left[\frac{4}{\mu_2} + \frac{1}{\mu_1} \right] ; M_{DE} = M_{DB} - M_{DF}$$

$$H_E = -\frac{M_{DE}}{h} ; H_C = wh - H_E ; V_A = -\frac{M_{BA}}{L_1} ; V_C = +\frac{M_{BA}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

$$V_F = \frac{M_{DF}}{L_1} ; V_E = -\frac{M_{DF}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

(429)



$$k_1 = \frac{I_3 h}{I_2 L_2} ; k_2 = \frac{I_1 h}{I_2 L_1}$$

$$u_1 = 3 + 2k_1 + 3k_2$$

$$u_2 = 2k_1 + k_2$$

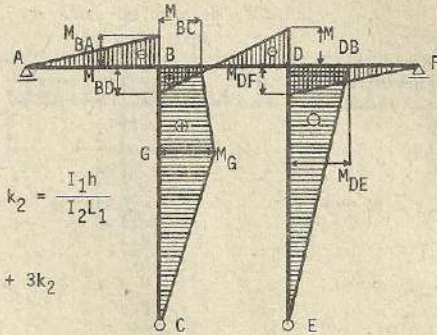
$$\lambda_1 = -\frac{L_1}{L_2} ; \lambda_2 = -\frac{L_2}{L_1} ; \beta = -\frac{e}{h}$$

$$M_{BA} = -\frac{Pek_2}{4} \left[\frac{2}{u_2} - \frac{3(1-\beta^2)}{u_1} \right] ; M_{DF} = +\frac{Pek_2}{4} \left[\frac{2}{u_2} + \frac{3(1-\beta^2)}{u_1} \right]$$

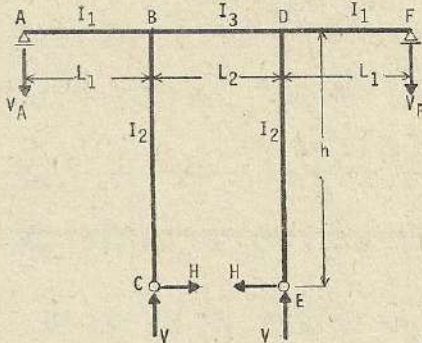
$$M_{BD} = +\frac{Pek_1}{2} \left[\frac{2}{u_2} - \frac{1-\beta^2}{u_1} \right] ; M_{DB} = -\frac{Pek_1}{2} \left[\frac{2}{u_2} + \frac{1-\beta^2}{u_1} \right]$$

$$M_{BC} = M_{BD} - M_{BA} ; M_{DE} = M_{DB} - M_{DF} ; H_E = -\frac{M_{DE}}{h} ; H_C = P - H_E ; V_A = -\frac{M_{BA}}{L_1}$$

$$V_C = \frac{M_{BD}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2} ; V_F = \frac{M_{DF}}{L_1} ; V_E = -\frac{M_{DF}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$



Esfuerzos debido a cambio de temperatura (430)



$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

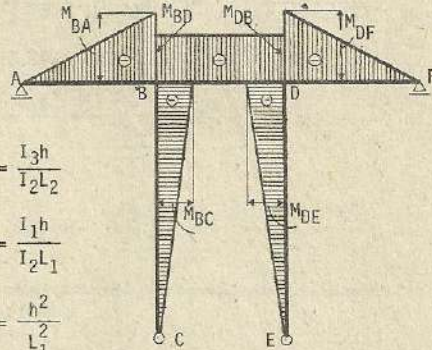
$$\lambda_3 = \frac{h^2}{L_1^2}$$

otros datos (420)

$$M_{BA} = M_{DF} = -\frac{3E\epsilon I_2 t L_1 k_2}{2h^2} \cdot \frac{2\lambda_3(3+2k_1) - 3\lambda_1}{u_1}$$

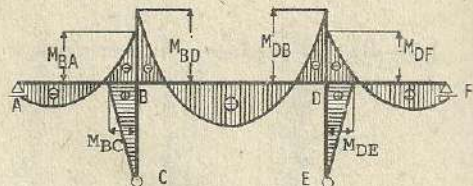
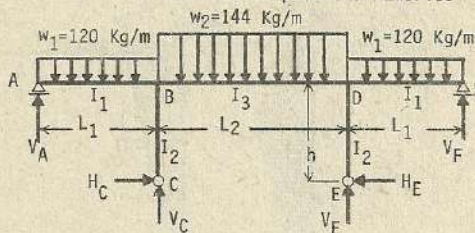
$$M_{BD} = M_{DB} = -\frac{3E\epsilon I_2 t L_1 k_1}{h^2} \cdot \frac{2\lambda_3 k_2 + \lambda_1}{u_1}$$

$$M_{BC} = M_{DE} = M_{BD} - M_{BA} ; V = -\frac{M_{BA}}{L_1} ; H = \frac{M_{BC}}{h}$$



Nota : En caso de descenso de temperatura, los esfuerzos tendrán signos contrarios a lo expuesto.

Solución de problema numérico



Altura útil (d_1) de los miembros

AB, DF = 40 cm.

(d_2) de los miembros BC, DE = 50 cm.

(d_3) del miembro BD = 50 cm.

$$\frac{I_1}{I_3} = \left(\frac{d_1}{d_3}\right)^3 = 0.512 \quad ; \quad \frac{I_3}{I_2} = \left(\frac{d_3}{d_2}\right)^3 = 1$$

$$k_1 = \frac{I_3 h}{I_2 L_2} = 1 \times \frac{5}{10} = 0.5 \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_2} = 0.512 \times \frac{5}{6} = 0.43$$

$$M_{BA} = M_{DF} = \frac{-w_1 L_1^2 (3+2k_1) - 2w_2 L_2^2 k_2}{8(3+2k_1+3k_2)} = \frac{-120 \times 36(3+2 \times 0.5) - 2 \times 144 \times 100 \times 0.43}{8(3+2 \times 0.5+3 \times 0.43)} = -700.9 \text{ Kg} \cdot \text{m}$$

$$M_{BC} = M_{DE} = \frac{3w_1 L_1^2}{8 \mu_1} - \frac{w_2 L_2^2}{4 \mu_1} = \frac{3 \times 120 \times 36}{8 \mu_1} - \frac{2 \times 144 \times 100}{8 \mu_1} = \frac{3 \times 120 \times 36 - 2 \times 144 \times 100}{8(3+2 \times 0.5+3 \times 0.43)} = -374.3 \text{ Kg} \cdot \text{m}$$

$$M_{BD} = M_{DB} = -700.9 - 374.3 = -1075.2 \text{ Kg} \cdot \text{m}$$

$$V_A = \frac{1}{L_1} \left[\frac{w_1 L_1^2}{2} - (M_A + M_{BA}) \right] = \frac{1}{6} \left[\frac{120 \times 36}{2} - (0 + 700.9) \right] = 243.2 \text{ Kg}$$

$$V_C = w_1 L_1 - V_A + \frac{w_2 L_2}{2} = 720 - 243.2 + 720 = 1196.8 \text{ Kg}$$

$$H_C = H_E = -\frac{M_{BC}}{h} = +\frac{374.3}{5} = 74.9 \text{ Kg}$$

(431)

$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L} \quad ; \quad k_3 = \frac{I_2 L_1}{I_4 L} \quad ; \quad m = k_2 + k_2 k_3 + k_3$$

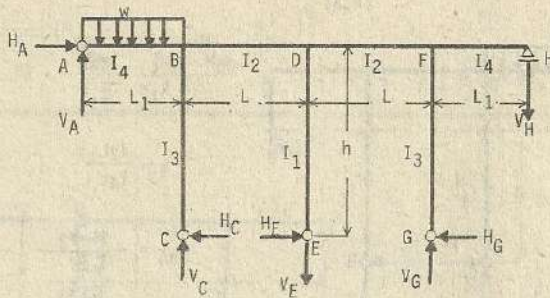
$$\mu_1 = 3m + k_2 k_3 \quad ; \quad \mu_2 = \mu_1 + 8k_1 m \quad ; \quad \lambda_1 = \frac{L_1}{L} \quad ; \quad \lambda_2 = \frac{L}{L_1}$$

$$M_{BA} = M_{BD} - M_{BC} \quad ; \quad M_{FH} = M_{FD} - M_{FG} \quad ; \quad V_H = -\frac{M_{FH}}{L_1} \quad ; \quad H_C = \frac{M_{BC}}{h}$$

$$H_C = \frac{M_{DE}}{h} \quad ; \quad H_{DE} = -\frac{M_{FG}}{h} \quad ; \quad M_{BC} = +\frac{w L_1^2 k_3}{16} \left[\frac{3+8k_1}{\mu_2} + \frac{3}{\mu_1} \right]$$

$$M_{FG} = -\frac{w L_1^2 k_3}{16} \left[\frac{3+8k_1}{\mu_2} - \frac{3}{\mu_1} \right] \quad ; \quad M_{BD} = +\frac{w L_1^2 k_2 k_3}{4} \left[\frac{1}{\mu_1} + \frac{2k_1}{\mu_2} \right]$$

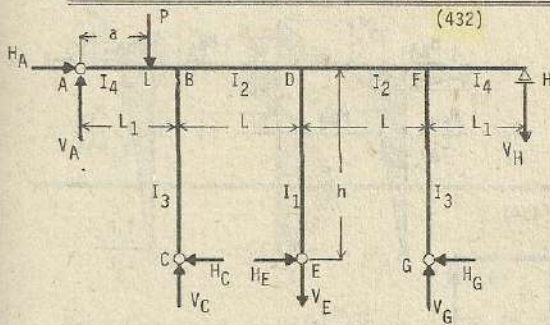
$$M_{FD} = -\frac{w L_1^2 k_2 k_3}{4} \left[\frac{1}{\mu_1} - \frac{2k_1}{\mu_2} \right] \quad ; \quad M_{DB} = +\frac{w L_1^2 k_2 k_3}{8} \left[\frac{1}{\mu_1} + \frac{1}{\mu_2} \right]$$



$$M_{DF} = + \frac{wL_1^2 k_2 k_3}{8} \left[\frac{1}{\mu_1} - \frac{1}{\mu_2} \right]; \quad M_{DE} = - \frac{wL_1^2 k_2 k_3}{4}; \quad V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}$$

$$V_C = \frac{wL_1}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}; \quad V_G = - \frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

$$V_E = \frac{1}{L} (M_{DB} - M_{BD} + M_{DF} - M_{FD}); \quad H_A = H_C + H_G - H_E$$



$$k_1 = \frac{I_2 h}{I_1 L}; \quad k_2 = \frac{I_2 h}{I_3 L}$$

$$k_3 = \frac{I_2 L_1}{I_4 L}; \quad \text{; otros datos (431)}$$

$$\alpha = \frac{a}{L_1}; \quad V_A = P(1-\alpha) + \frac{M_{BA}}{L_1}$$

$$V_C = P\alpha - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}$$

$$V_E = \frac{1}{L} (M_{DB} - M_{BD} + M_{DF} - M_{FD}); \quad V_G = - \frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

$$M_{BC} = + \frac{Pa(1-\alpha^2)k_3}{4} \left[\frac{3+8k_1}{\mu_2} + \frac{3}{\mu_1} \right]; \quad M_{FG} = - \frac{Pa(1-\alpha^2)k_3}{4} \left[\frac{3+8k_1}{\mu_2} - \frac{3}{\mu_1} \right]$$

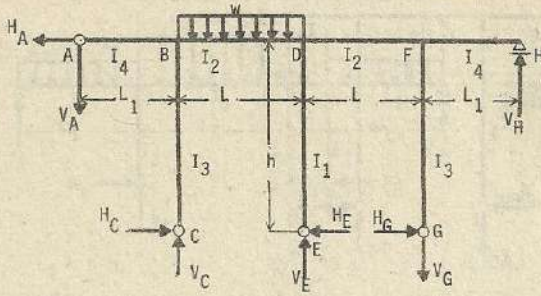
$$M_{BD} = - Pa(1-\alpha^2)k_2 k_3 \left[\frac{1}{\mu_1} + \frac{1+2k_1}{\mu_2} \right]; \quad M_{FD} = - Pa(1-\alpha^2)k_2 k_3 \left[\frac{1}{\mu_1} - \frac{1+2k_1}{\mu_2} \right]$$

$$M_{BA} = M_{BD} - M_{BC}; \quad M_{FH} = M_{FD} - M_{FG}; \quad M_{DB} = + \frac{Pa(1-\alpha^2)k_2 k_3}{2} \left[\frac{1}{\mu_1} + \frac{1}{\mu_2} \right]$$

$$M_{DF} = + \frac{Pa(1-\alpha^2)k_2 k_3}{2} \left[\frac{1}{\mu_1} - \frac{1}{\mu_2} \right]; \quad M_{DE} = + \frac{Pa(1-\alpha^2)k_2 k_3}{\mu_2}$$

$$H_C = \frac{M_{BC}}{L}; \quad H_E = \frac{M_{DE}}{h}; \quad H_A = H_C + H_G - H_E; \quad H_G = - \frac{H_{FG}}{h}; \quad V_H = - \frac{M_{FH}}{L_1}$$

(433)



$$k_1 = \frac{I_2 h}{I_1 L} ; \quad k_2 = \frac{I_2 h}{I_3 L}$$

$$k_3 = \frac{I_2 L_1}{I_4 L} ; \quad \text{otros datos (431)}$$

$$M_{BA} = - \frac{wL^2 k_2}{8} \left[\frac{1+4k_1}{\mu_2} + \frac{1}{\mu_1} \right]$$

$$M_{FH} = + \frac{wL^2 k_2}{8} \left[\frac{1+4k_1}{\mu_2} - \frac{1}{\mu_1} \right]$$

$$M_{BC} = - \frac{wL^2 k_3}{8} \left[\frac{1+4k_1}{\mu_2} + \frac{1}{\mu_1} \right]$$

$$M_{FG} = + \frac{wL^2 k_3}{8} \left[\frac{1+4k_1}{\mu_2} - \frac{1}{\mu_1} \right]$$

$$M_{DB} = - \frac{wL^2 (m+k_2 k_3)}{8} \left[\frac{1}{\mu_1} + \frac{1}{\mu_2} \right]$$

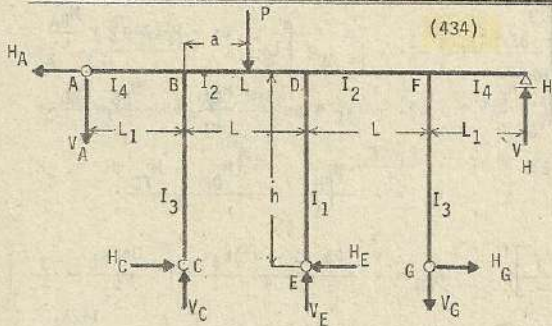
$$M_{DF} = - \frac{wL^2 (m+k_2 k_3)}{8} \left[\frac{1}{\mu_1} - \frac{1}{\mu_2} \right]$$

$$M_{DE} = - \frac{wL^2}{4} \cdot \frac{m+k_2 k_3}{\mu_2} ; \quad M_{BD} = M_{BA} + M_{BC} ; \quad M_{FD} = M_{FH} + M_{FG} ; \quad H = - \frac{M_{BC}}{h}$$

$$H_E = - \frac{M_{DE}}{h} ; \quad H_G = \frac{M_{FG}}{h} ; \quad V_A = - \frac{M_{BA}}{L_1} ; \quad V_C = \frac{wL}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}$$

$$V_E = \frac{wL}{2} + \frac{M_{BD} - M_{DB} + M_{FD}}{L} ; \quad V_G = \frac{M_{FH}}{L_1} + \frac{M_{FD} - M_{DF}}{L} ; \quad V_H = \frac{M_{FH}}{L_1}$$

(434)



$$k_1 = \frac{I_2 h}{I_1 L} ; \quad k_2 = \frac{I_2 h}{I_3 L}$$

$$k_3 = \frac{I_2 L_1}{I_4 L} ; \quad \text{otros datos (431)}$$

$$\beta = \frac{a}{L}$$

$$M_{BD} = M_{BA} + M_{BC} ; \quad M_{FD} = M_{FH} + M_{FG}$$

$$M_{DE} = M_{DB} - M_{DF}$$

$$M_{BA} = - \frac{Pa(1-\beta)k_2}{2} \left[\frac{(1-\beta)(3+4k_1) + 4k_1}{\mu_2} + \frac{3(1-\beta)}{\mu_1} \right] ; \quad H_C = - \frac{M_{BC}}{h}$$

$$M_{FH} = + \frac{Pa(1-\beta)k_2}{2} \left[\frac{(1-\beta)(3+4k_1) + 4k_1}{\mu_2} - \frac{3(1-\beta)}{\mu_1} \right] ; \quad H_E = - \frac{M_{DE}}{h}$$

$$M_{BC} = - \frac{Pa(1+\beta)k_3}{2} \left[\frac{(1-\beta)(3+4k_1) + 4k_1}{\mu_2} + \frac{3(1-\beta)}{\mu_1} \right] ; \quad H_G = \frac{M_{FG}}{h}$$

$$M_{FG} = + \frac{Pa(1+\beta)k_3}{2} \left[\frac{(1-\beta)(3+4k_1) + 4k_1}{\mu_2} - \frac{3(1-\beta)}{\mu_1} \right]$$

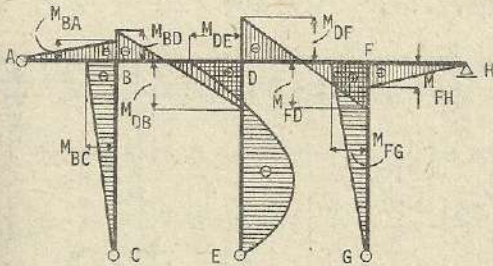
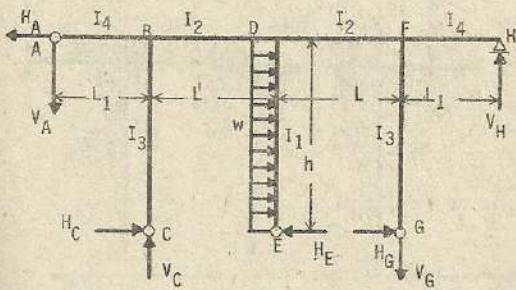
$$M_{DF} = - \frac{Pa(1-\beta)}{2} \left[2(1+\beta)m - (2-\beta)(k_2+k_3) \right] \cdot \left[\frac{1}{\mu_1} + \frac{1}{\mu_2} \right] ; \quad V_A = - \frac{M_{BA}}{L_1}$$

$$M_{DB} = M_{DF}$$

$$V_E = P\beta + \frac{1}{L} \left[(M_{BD} - M_{DB} - M_{DF} + M_{FD}) \right] ; \quad V_F = \frac{M_{FH}}{L_1} ; \quad V_G = \frac{M_{FH}}{L_1} + \frac{M_{FD} - M_{DF}}{L}$$

$$V_C = P(1 - \beta) - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L} ; \quad H_A = H_C + H_G - H_E$$

(435)



$$V_E = 0 ; \quad V_A = V_H = \frac{M_{FH}}{L_1} ; \quad V_C = V_G = \frac{M_{FH}}{L_1} + \frac{M_{DB} - M_{FD}}{L}$$

$$k_1 = \frac{I_2 h}{I_1 L} ; \quad k_2 = \frac{I_2 h}{I_3 L} ; \quad k_3 = \frac{I_2 L_1}{I_4 L}$$

$$m = k_2 + k_2 k_3 + k_3 ; \quad \mu_1 = 3m + k_2 k_3$$

$$\mu_2 = \mu_1 + 8k_1 m$$

$$n = 3 + 8k_1 + 4k_2 (1 + 2k_1)$$

$$\lambda_1 = \frac{L_1}{L} ; \quad \lambda_2 = \frac{L}{L_1}$$

$$M_{BA} = -\frac{wh^2}{4} \cdot \frac{k_1 k_2}{\mu_2} ; \quad M_{FH} = +\frac{wh^2}{4} \cdot \frac{k_1 k_2}{\mu_2}$$

$$M_{BC} = -\frac{wh^2}{4} \cdot \frac{k_1 k_3}{\mu_2} ; \quad M_{FG} = +\frac{wh^2}{4} \cdot \frac{k_1 k_3}{\mu_2}$$

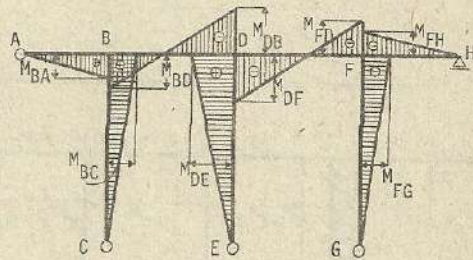
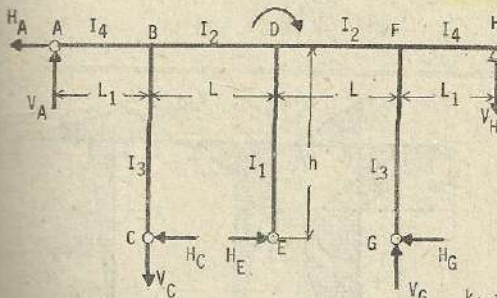
$$M_{BD} = M_{BA} + M_{BC} ; \quad M_{FD} = M_{FH} + M_{FG}$$

$$M_{DB} = +\frac{wh^2}{2} \cdot \frac{k_1 m}{\mu_2} ; \quad M_{DF} = -\frac{wh^2}{2} \cdot \frac{k_1 m}{\mu_2}$$

$$M_{DE} = +wh^2 \cdot \frac{k_1 m}{2} = 2M_{DB} ; \quad H_C = H_G = \frac{M_{FG}}{h}$$

$$H_E = \frac{wh}{2} - \frac{M_{DE}}{h} ; \quad H_A = \frac{wh}{2} + \frac{M_{DE}}{h} + 2H_C$$

(436)



$$k_1 = \frac{I_2 h}{I_1 L} ; \quad k_2 = \frac{I_2 h}{I_3 L} ; \quad k_3 = \frac{I_2 L_1}{I_4 L}$$

$$m = k_2 + k_2 k_3 + k_3 ; \quad \mu_1 = 3m + k_2 k_3 ; \quad \mu_2 = \mu_1 + 8k_1 m ; \quad \lambda_1 = \frac{L_1}{L} ; \quad \lambda_2 = \frac{L}{L_1}$$

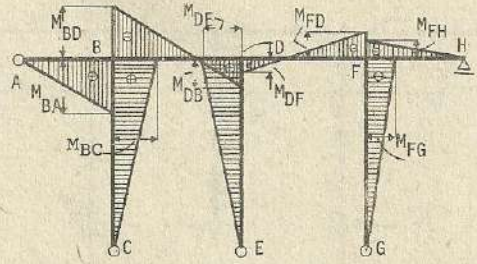
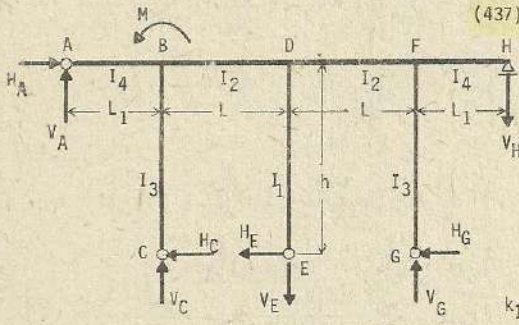
$$n = 3 + 8k_1 + 4k_2 (1 + 2k_1)$$

$$M_{BA} = +\frac{2Mk_1 k_2}{\mu_2} ; \quad M_{FH} = -\frac{2Mk_1 k_2}{\mu_2} ; \quad M_{BC} = +\frac{2Mk_1 k_3}{\mu_2} ; \quad M_{FG} = -\frac{2Mk_1 k_3}{\mu_2}$$

$$M_{BD} = M_{BA} + M_{BC} ; \quad M_{FD} = M_{FH} + M_{FG} ; \quad M_{DB} = -\frac{4Mk_1 m}{\mu_2} ; \quad M_{DF} = +\frac{4Mk_1 m}{\mu_2}$$

$$M_{DE} = +M \cdot \frac{\nu_1}{\nu_2} \quad ; \quad M + M_{DB} - M_{DE} - M_{DF} = 0 \quad ; \quad H_C = H_G = \frac{M_{BC}}{h} \quad ; \quad H_E = \frac{M_{DE}}{h}$$

$$H_A = H_E - 2H_C \quad ; \quad V_A = V_H = \frac{M_{BA}}{L_1} \quad ; \quad V_C = V_G = \frac{M_{BA}}{L_1} + \frac{M_{BD} - M_{DB}}{L} \quad ; \quad V_E = 0$$



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L} \quad ; \quad k_3 = \frac{I_2 L_1}{I_4 L}$$

$$m = k_2 + k_2 k_3 + k_3 \quad ; \quad \nu_1 = 3m + k_2 k_3 \quad ; \quad \nu_2 = \nu_1 + 8k_1 m \quad ; \quad \lambda_1 = \frac{L_1}{L} \quad ; \quad \lambda_2 = \frac{L}{L_1}$$

$$n = 3 + 8k_1 + 4k_2(1 + 2k_1)$$

$$M_{BA} = + \frac{Mk_2}{2} \left[\frac{3 + 8k_1}{\nu_2} + \frac{3}{\nu_1} \right] \quad ; \quad M_{FH} = - \frac{Mk_2}{2} \left[\frac{3 + 8k_1}{\nu_2} - \frac{3}{\nu_1} \right] \quad ; \quad M_{BC} = + \frac{Mk_3}{2} \left[\frac{3 + 8k_1}{\nu_2} + \frac{3}{\nu_1} \right]$$

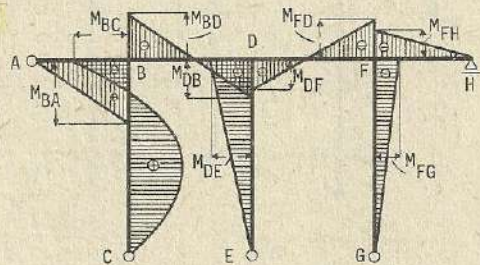
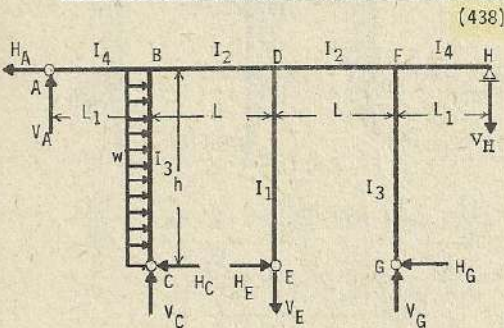
$$M_{BD} = - 2Mk_2 k_3 \left[\frac{1}{\nu_1} + \frac{1 + 2k_1}{\nu_2} \right] \quad ; \quad M_{FD} = - 2Mk_2 k_3 \left[\frac{1}{\nu_1} - \frac{1 + 2k_1}{\nu_2} \right] \quad ; \quad M_{DB} = + Mk_2 k_3 \left[\frac{1}{\nu_1} + \frac{1}{\nu_2} \right]$$

$$M_{BA} + M_{BC} - M_{BD} - M = 0 \quad ; \quad M_{FG} = - \frac{Mk_3}{2} \left[\frac{3 + 8k_1}{\nu_2} - \frac{3}{\nu_1} \right] \quad ; \quad M_{DF} = + Mk_2 k_3 \left[\frac{1}{\nu_1} - \frac{1}{\nu_2} \right]$$

$$M_{DE} = + \frac{2Mk_2 k_3}{\nu_2} = M_{DB} - M_{DF} \quad ; \quad H_C = \frac{M_{BC}}{h} \quad ; \quad H_E = \frac{M_{DE}}{h} \quad ; \quad H_G = - \frac{M_{FG}}{h}$$

$$H_A = H_C + H_G - H_E \quad ; \quad V_A = \frac{M_{BA}}{L_1} \quad ; \quad V_H = - \frac{M_{FH}}{L_1} \quad ; \quad V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}$$

$$V_G = - \frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L} \quad ; \quad V_E = \frac{M_{DB} - M_{BD} + M_{DF} - M_{FD}}{L}$$



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L} \quad ; \quad k_3 = \frac{I_2 L_1}{I_4 L}$$

$$m = k_2 + k_2 k_3 + k_3 \quad ; \quad \nu_1 = 3m + k_2 k_3 \quad ; \quad \nu_2 = \nu_1 + 8k_1 m = \lambda_1 = \frac{L_1}{L} \quad ; \quad \lambda_1 = \frac{L}{L_1}$$

$$n = 3 + 8k_1 + 4k_2(1 + 2k_1)$$

$$M_{BA} = + \frac{wh^2k_2}{16} \left[\frac{3+8k_1}{u_2} + \frac{3}{u_1} \right]; \quad M_{FH} = - \frac{wh^2k_2}{16} \left[\frac{3+8k_1}{u_2} - \frac{3}{u_1} \right]; \quad M_{BC} = M_{BD} - M_{BA}$$

$$M_{BD} = - \frac{wh^2k_2k_3}{4} \left[\frac{1}{u_1} + \frac{1+2k_1}{u_2} \right]; \quad M_{FD} = - \frac{wh^2k_2k_3}{4} \left[\frac{1}{u_1} - \frac{1+2k_1}{u_2} \right]; \quad M_{FG} = M_{FD} - M_{FH}$$

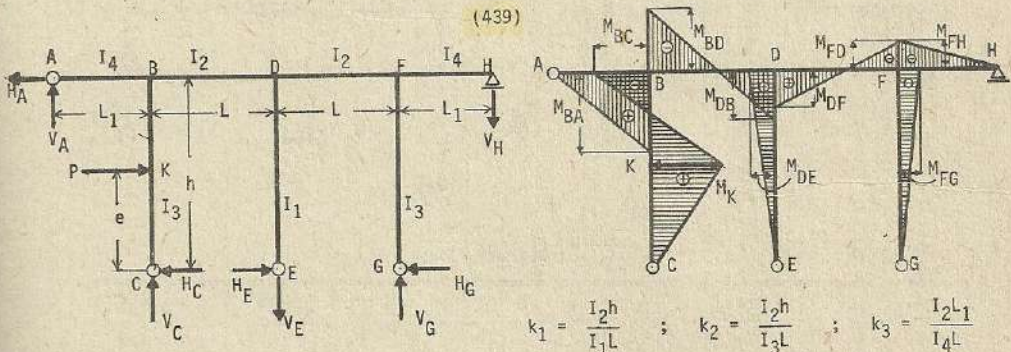
$$M_{DB} = + \frac{wh^2k_2k_3}{8} \left[\frac{1}{u_1} + \frac{1}{u_2} \right]; \quad M_{DF} = + \frac{wh^2k_2k_3}{8} \left[\frac{1}{u_1} - \frac{1}{u_2} \right]$$

$$M_{DE} = + \frac{wh^2k_2k_3}{4u_2} = M_{DB} - M_{DF}; \quad H_C = \frac{wh}{2} + \frac{M_{BC}}{h}; \quad H_E = \frac{M_{DE}}{h}; \quad H_G = - \frac{M_{FG}}{h}$$

$$H_A = \frac{wh}{2} - \frac{M_{BC}}{h} + H_E - H_G; \quad V_A = \frac{M_{BA}}{L_1}; \quad V_E = \frac{M_{DB} - M_{BD} + M_{DF} - M_{FD}}{L}$$

$$V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}; \quad V_D = - \frac{M_{FH}}{L} + \frac{M_{DF} - M_{FD}}{L}; \quad V_G = - \frac{M_{FH}}{L_1}$$

(439)



$$k_1 = \frac{I_2h}{I_1L}; \quad k_2 = \frac{I_2h}{I_3L}; \quad k_3 = \frac{I_2L_1}{I_4L}$$

$$m = k_2 + k_2k_3 + k_3$$

$$u_1 = 3m + k_2k_3; \quad u_2 = u_1 + 8k_1m; \quad n = 3 + 8k_1 + 4k_2(1+2k_1); \quad \lambda_1 = \frac{L_1}{L}$$

$$M_{BA} = + \frac{Pe(1-\beta^2)k_2}{4} \left[\frac{3+8k_1}{u_2} + \frac{3}{u_1} \right]; \quad \beta = \frac{e}{h}; \quad \lambda_2 = \frac{L}{L_1}$$

$$M_{FH} = - \frac{Pe(1-\beta^2)k_2}{4} \left[\frac{3+8k_1}{u_2} - \frac{3}{u_1} \right]; \quad M_{BD} = -Pe(1-\beta^2)k_2k_3 \left[\frac{1}{u_1} + \frac{1+2k_1}{u_2} \right]$$

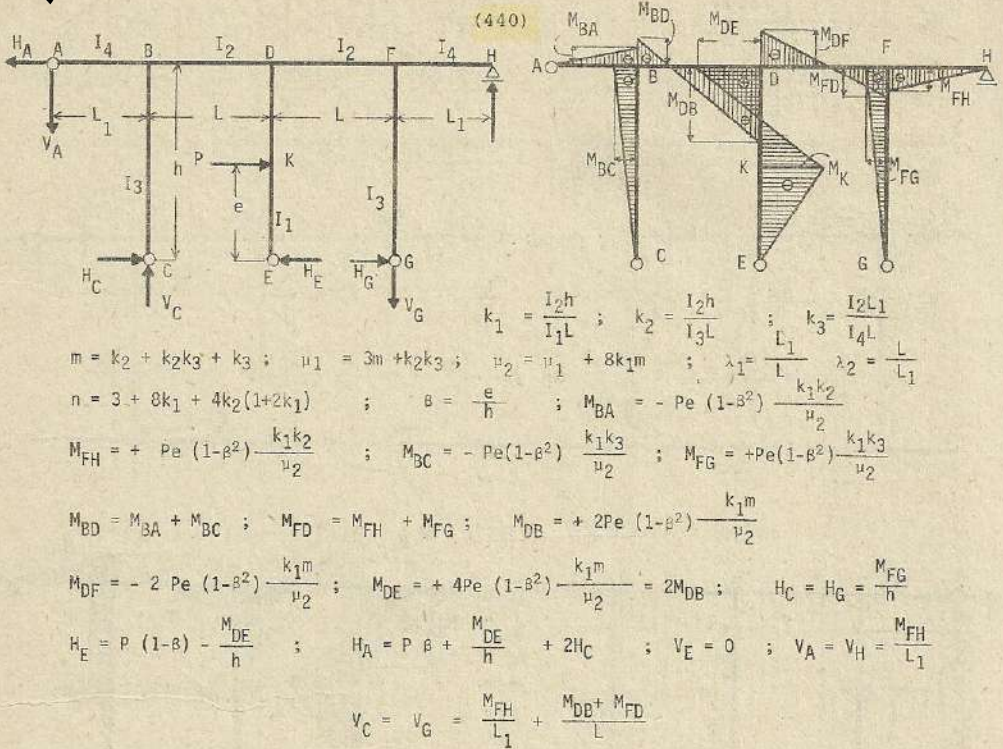
$$M_{FD} = -Pe(1-\beta^2)k_2k_3 \left[\frac{1}{u_1} - \frac{1+2k_1}{u_2} \right]; \quad M_{BC} = M_{BD} - M_{BA}; \quad M_{FG} = M_{FD} - M_{FH}$$

$$M_{DB} = + \frac{Pe(1-\beta^2)k_2k_3}{2} \left[\frac{1}{u_1} + \frac{1}{u_2} \right]; \quad M_{DF} = + \frac{Pe(1-\beta^2)k_2k_3}{2} \left[\frac{1}{u_1} - \frac{1}{u_2} \right]$$

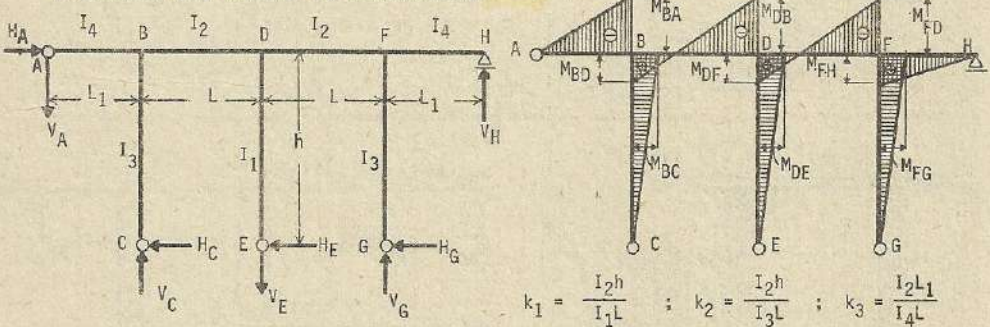
$$M_{DE} = + \frac{Pe(1-\beta^2)k_2k_3}{u_2} = M_{DB} - M_{DF}; \quad H_E = \frac{M_{DE}}{h}; \quad H_A = P\beta - \frac{M_{BC}}{h} + H_E - H_G$$

$$H_C = P(1-\beta) + \frac{M_{BC}}{h}; \quad H_G = - \frac{M_{FG}}{h}; \quad V_A = \frac{M_{BA}}{L_1}; \quad V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} + M_{BD}}{L}$$

$$V_H = - \frac{M_{FH}}{L_1}; \quad V_E = \frac{M_{DB} - M_{BD} + M_{DF} - M_{FD}}{L}; \quad V_G = - \frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$



Esfuerzos por cambio de temperatura (441)



$$\lambda_3 = \frac{h^2}{L_1^2}$$

otros datos (431)

$$M_{BA} = -\frac{3E\epsilon I_2 t L_1}{hL} \left[\frac{(\lambda_2 + 1)(3 + 8k_1 - 2k_2)}{\mu_2} - \frac{3\lambda_2 - \lambda_3(3 + 4k_2)}{\mu_1} \right]$$

$$M_{FH} = +\frac{3E\epsilon I_2 t L_1}{hL} \left[\frac{(\lambda_2 + 1)(3 + 8k_1 - 2k_2)}{\mu_2} + \frac{3\lambda_2 - \lambda_3(3 + 4k_2)}{\mu_1} \right]$$

$$M_{BD} = +\frac{6E\epsilon I_2 t L_1}{hL} \left[\frac{(1 + \lambda_2)(k_3(3 + 4k_1) + k_2)}{\mu_2} - \frac{2(\lambda_3 k_2 + \lambda_2 k_3)}{\mu_1} \right]$$

$$M_{FD} = -\frac{6E\epsilon I_2 t L_1}{hL} \left[\frac{(1+\lambda_2) \left[\frac{k_3(3+4k_1)+k_2}{\mu_2} \right] + \frac{2(\lambda_3 k_2 + \lambda_2 k_3)}{\mu_1}}{\mu_2} \right] ; M_{BC} = M_{BD} - M_{BA}$$

$$M_{DB} = -\frac{6E\epsilon I_2 t L_1}{hL} \left[\frac{(1+\lambda_2) (2m+k_3)}{\mu_2} - \frac{\lambda_3 k_2 + \lambda_2 k_3}{\mu_1} \right] ; M_{FG} = M_{FD} - M_{FB}$$

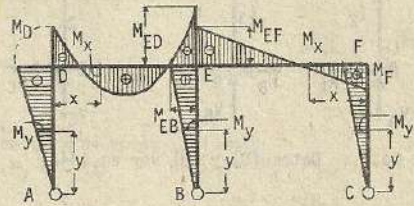
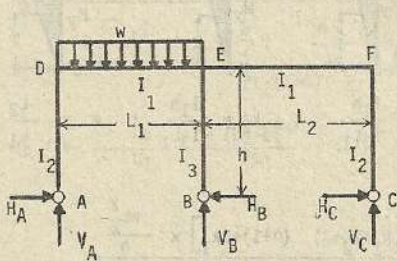
$$M_{DF} = +\frac{6E\epsilon I_2 t L_1}{hL} \left[\frac{(1+\lambda_2) (2m+k_3)}{\mu_2} - \frac{\lambda_3 k_2 + \lambda_2 k_3}{\mu_1} \right] ; M_{DE} = M_{DB} - M_{DF}$$

$$H_C = \frac{M_{BC}}{h} ; H_E = -\frac{M_{DE}}{h} ; H_G = -\frac{M_{FG}}{h} ; H_A = H_C + H_E + H_G ; V_A = -\frac{M_{BA}}{L_1}$$

$$V_H = \frac{H_{FH}}{L} ; V_C = -\frac{M_{BA}}{L} + \frac{M_{DB} - M_{BD}}{L} ; V_G = -\frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

$$V_E = \frac{-M_{BD} + M_{DB} + M_{DF} - M_{FD}}{L} ; \text{Nota: En caso de descenso de temperatura los signos de los esfuerzos serán contrarios a los presentados.}$$

(442)



$$k_1 = \frac{I_1 h}{I_2 L_1} ; k_2 = \frac{I_1 h}{I_3 L_1} ; \alpha = \frac{L_2}{L_1}$$

$$D = k_2 \left(3\alpha + \frac{2}{3} k_2 + \frac{2}{3} \alpha k_2 \right) + (k_1 + k_2 + 3) \left[\frac{3}{2} \alpha^2 - \frac{2}{3} (1+\alpha) (k_1 + k_2 + 3\alpha) \right] + \frac{3}{2} (k_1 + k_2 + 3\alpha)$$

$$H_A = \frac{-\frac{1}{24} \left[(k_1 + k_2 + 3\alpha) (1+\alpha) + 3\alpha k_1 \right] \times \frac{wL_1^2}{h}}{D} ; H_C = \frac{\frac{1}{24} (k_2 + \alpha k_2 - 3\alpha k_1) \times \frac{wL_1^2}{h}}{D}$$

$$H_B = \frac{-\frac{1}{24} \left[(k_1 + 3\alpha) (1+\alpha) + 6\alpha k_1 \right] \times \frac{wL_1^2}{h}}{D} ; V_A = \frac{3 \left[\left(\frac{1}{4} + \frac{\alpha}{3} \right) wL_1^2 - H_A h + H_C h \right]}{2(L_1 + L_2)}$$

$$V_B = w \frac{L_1}{L_2} \left(\frac{L_1}{2} + L_2 \right) - V_A \frac{(L_1 + L_2)}{L_2} ; V_C = V_A \frac{L_1}{L_2} - \frac{wL_1^2}{2L_2}$$

Momentos de flexión en las columnas : AD $\rightarrow M_y = -H_A y$; BE $\rightarrow M_y = H_B y$

CF $\rightarrow M_y = H_C y$

luego, $M_{DA} = -H_A h$; $M_{EB} \text{ (columna)} = H_B h$; $H_{FC} = H_C h$

Momentos de flexión en las vigas.

$$DE \rightarrow M_x = -H_A h + V_A x - \frac{wx^2}{2}$$

$$EF \rightarrow M_x = H_C h + V_A \frac{L_1}{L_2} x - \frac{wL_1^2}{2L_2} x$$

luego,

$$M_{ED} = -H_A h + V_A L_1 - \frac{wL_1^2}{2} ; M_{EF} = H_C h + V_A L_1 - \frac{wL_1^2}{2}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$

$$M = 12h \left[(\alpha+1)(4k^2+3\alpha) + 2k(\alpha^2+5\alpha+1) \right]$$

$$H_A = \frac{\left[k(5\alpha+2) + 3\alpha(\alpha+1) \right] wL_1^2}{M} ;$$

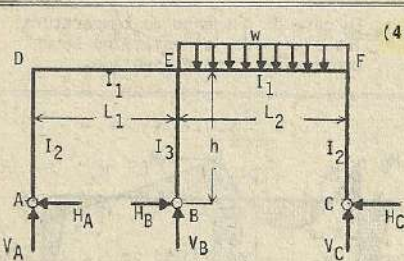
$$H_C = \frac{-k(1-2\alpha)wL_1^2}{M}$$

$$H_B = \frac{\left[k(7\alpha+1) + 3\alpha(\alpha+1) \right] wL_1^2}{M} ;$$

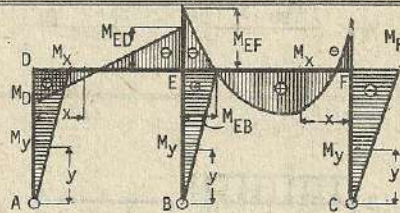
$$V_A = \frac{3 \left[\left(\frac{1}{4} + \frac{\alpha}{3} \right) wL_1^2 - H_A h + H_C h \alpha \right]}{2(L_1 + L_2)}$$

$$V_B = w \frac{L_1}{L_2} \left(\frac{L_1}{2} + L_2 \right) - V_A \frac{(L_1 + L_2)}{L_2} ;$$

$$V_C = V_A \frac{L_1}{L_2} - \frac{wL_1^2}{2L_1}$$



(443)



$$k_1 = \frac{I_1 h}{I_2 L_1} ; k_2 = \frac{I_1 h}{I_3 L_1} ; \alpha = \frac{L_2}{L_1}$$

Nota : Datos (D) y (M) ver en (442)

$$H_A = \frac{\frac{\alpha}{24} (k_2 - 3k_1 + \alpha k_2) \times \frac{wL_2^2}{h}}{D} ; H_C = \frac{-\frac{\alpha}{24} [(k_1 + k_2 + 3)(\alpha + 1) + 3k_1] \times \frac{wL_2^2}{h}}{D}$$

$$H_B = \frac{-\frac{\alpha}{24} [(k_1 + 3)(\alpha + 1) + 6k_1] \times \frac{wL_2^2}{h}}{D}$$

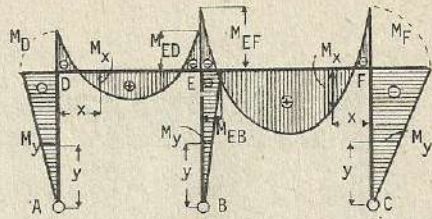
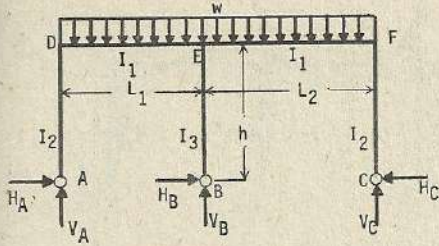
$$V_A = V_C \frac{L_2}{L_1} - \frac{wL_2^2}{2L_1}$$

$$V_C = \frac{3 \left[wL_2^2 \left(\frac{1}{3} + \frac{\alpha}{4} \right) - H_A h + H_C h \alpha \right]}{2L_2(1 + \alpha)}$$

$$V_B = \frac{wL_2^2 \left(\frac{L_2}{2} + L_1 \right)}{L_1} - V_C \frac{L_1 + L_2}{L_1}$$

Momentos de flexión en las columnas, $AD \rightarrow M_y = H_A y$; $BE \rightarrow M_y = -H_B y$ $CF \rightarrow M_y = -H_C y$ Luego, $M_{DA} = H_A h$; $M_{EB} = -H_B h$; $M_{FC} = -H_C h$ Momentos de flexión en las vigas, $DE \rightarrow M_x = H_A h + V_A x$ $EF \rightarrow M_x = -H_C h + V_C x - \frac{wx^2}{2}$ Luego, $M_{ED} = H_A h + V_C L_2 - \frac{wL_2^2}{2}$ $M_{EF} = -H_C h + V_C L_2 - \frac{wL_2^2}{2}$ Si $I_2 = I_3$, entonces $k_1 = k_2 = k$ $H_A = \frac{\alpha k (2-\alpha) wL_2^2}{M}$; $H_B = \frac{\alpha [k(\alpha+7) + 3(\alpha+1)]}{M}$ $H_C = \frac{\alpha [k(2\alpha+5) + 3(\alpha+1)] wL_2^2}{M}$; $V_C = \frac{3 \left[wL_2^2 \left(\frac{1}{3} + \frac{\alpha}{4} \right) - H_A h + H_C h \alpha \right]}{2L_2(1 + \alpha)}$ $V_B = \frac{wL_2^2 \left(\frac{L_2}{2} + L_1 \right)}{L_1} - V_C \frac{(L_1 + L_2)}{L_1}$; $V_A = V_C \frac{L_2}{L_1} - \frac{wL_2^2}{2L_1}$

(444)



$$k_1 = \frac{I_1 h}{I_2 L_1} ; \quad k_2 = \frac{I_1 h}{I_2 L_2} ; \quad \alpha = \frac{L_2}{L_1}$$

otros datos (442)

$$H_A = \frac{-\frac{1}{24h} [(k_1+k_2+3\alpha)(1+\alpha) + 3\alpha k_1] wL_1^2 - \frac{\alpha}{24h} (k_2-3k_1+\alpha k_2) wL_2^2}{D}$$

$$H_C = \frac{-\frac{\alpha}{24h} [(k_1+k_2+3)(1+\alpha) + 3k_1] wL_2^2 - \frac{1}{24h} (k_2+\alpha k_2 - 3\alpha k_1) wL_1^2}{D}$$

$$H_B = \frac{-\frac{\alpha}{24} [(k_1+3)(\alpha+1)+6k_1] \frac{wL_2^2}{h} - \frac{1}{24} [(k_1+3\alpha)(1+\alpha)+6\alpha k_1] \frac{wL_1^2}{h}}{D}$$

$$V_C = \frac{3 \left[wL_2^2 \left(\frac{1}{3} + \frac{\alpha}{4} \right) - \frac{wL_1^2}{12} - H_A h + H_C h \alpha \right]}{2L_2(1+\alpha)} ; \quad V_B = -\frac{V_C(L_1+L_2)}{L_1} + \frac{w(L_1+L_2)^2}{2L_1}$$

$$V_A = V_C \frac{L_2}{L_1} - \frac{w(L_2-L_1)^2}{2L_1} ;$$

Momentos de flexión en las columnas

AD $\rightarrow M_y = -H_A h$; BE $\rightarrow M_y = -H_B h$

CF $\rightarrow M_y = -H_C h$

luego, $M_{DA} = -H_A h$; $M_{EB} = -H_B h$; $M_{FC} = -H_C h$

Momentos de flexión en las vigas,

DE $\rightarrow M_x = -H_A h + V_A x - \frac{wx^2}{2}$

EF $\rightarrow M_x = -H_C h + V_C x - \frac{wx^2}{2}$

luego, $M_{ED} = -H_A h + V_A L_1 - \frac{wL_1^2}{2}$

$M_{EF} = -H_C h + V_C L_2 - \frac{wL_2^2}{2}$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$

$$H_A = \frac{[k(5\alpha+2) + 3\alpha(\alpha+1)] wL_1^2}{M} - \alpha k(2-\alpha) wL_2^2$$

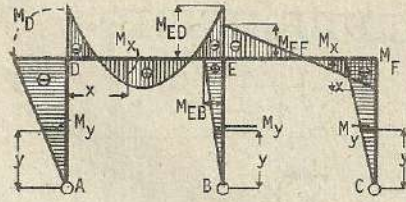
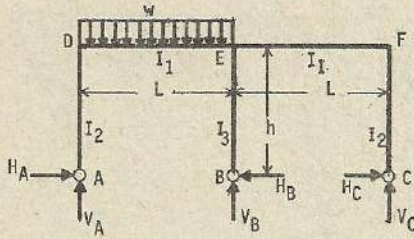
$$H_C = \frac{\alpha [k(2\alpha+5) + 3(\alpha+1)] wL_2^2 - k(2\alpha-1) wL_1^2}{M}$$

$$H_B = \frac{\alpha [k(\alpha+7)+3(\alpha+1)] wL_2^2 - [k(7\alpha+1)+3\alpha(\alpha+1)] wL_1^2}{M}$$

$$V_C = \frac{3 \left[wL_2^2 \left(\frac{1}{3} + \frac{\alpha}{4} \right) - \frac{wL_1^2}{12} - H_A h + H_C h \alpha \right]}{2L_2(1+\alpha)}$$

$$V_B = \frac{V_C(L_1+L_2)}{L_1} + \frac{w(L_1+L_2)^2}{2L_1} ; \quad V_A = V_C \frac{L_2}{L_1} - \frac{w(L_2-L_1)^2}{2L_1}$$

(445)



$$k_1 = \frac{L_1 h}{I_2 L} \quad ; \quad k_2 = \frac{L_1 h}{I_3 L}$$

$$E = (4k_1^2 + 8k_1 k_2 + 15k_1 + 6k_2 + 9) \quad ; \quad N = 24 \cdot (4k_1 + 3) \cdot (k_1 + 1)$$

$$H_A = \frac{(5k_1 + 2k_2 + 6)}{8E} \cdot \frac{wL^2}{h} \quad ; \quad H_B = \frac{(4k_1 + 3)}{4E} \cdot \frac{wL^2}{h} \quad ; \quad H_C = \frac{(3k_1 - 2k_2)}{8E} \cdot \frac{wL^2}{h}$$

$$V_A = \frac{(7k_1^2 + 14k_1 k_2 + 27k_1 + 12k_2 + 18)}{4E} \cdot wL \quad ; \quad V_B = \frac{(5k_1^2 + 10k_1 k_2 + 18k_1 + 6k_2 + 9)}{2E} \cdot wL$$

$$V_C = -\frac{(k_1 + 2k_2 + 3)k_1}{4E} \cdot wL \quad ; \quad M_D = -\frac{(5k_1 + 2k_2 + 6)}{8E} \cdot wL^2 \quad ; \quad M_{EB} = \frac{(4k_1 + 3)}{4E} \cdot wL^2$$

$$M_F = \frac{(3k_1 - 2k_2)}{8E} \cdot wL^2 \quad ; \quad M_{EF} = -\frac{(2k_1 + 4k_1 k_2 + 3k_1 + 2k_2)}{8E} \cdot wL^2$$

$$M_{ED} = -\frac{(2k_1^2 + 4k_1 k_2 + 11k_1 + 2k_2 + 6)}{8E} \cdot wL^2$$

Esfuerzo de flexión en las columnas, $AD \rightarrow M_y = -H_A y$

$BE \rightarrow M_y = -H_B y$

$CF \rightarrow M_y = H_C y$

Momentos de flexión en las vigas,

$DE \leftarrow M_x = -H_A h + V_A x - \frac{w x^2}{2}$

$EF \rightarrow M_x = H_C h + V_C x$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$

$$H_A = \frac{(7k+6)}{N} \cdot \frac{wL^2}{h} \quad ; \quad H_B = \frac{1}{12(k+1)} \cdot \frac{wL^2}{h} \quad ; \quad H_C = \frac{k}{N} \cdot \frac{wL^2}{h} \quad ; \quad V_A = \frac{7k+6}{4(4k+3)} \cdot wL$$

$$V_B = \frac{5k+3}{2(4k+3)} \cdot wL \quad ; \quad V_C = -\frac{k}{4(4k+3)} \cdot wL \quad ; \quad M_D = \frac{7k+6}{N} \cdot wL^2 \quad ; \quad M_{EB} = \frac{1}{12(k+1)} \cdot wL^2$$

$$M_{ED} = \frac{-6k^2 + 13k + 6}{N} \cdot wL^2 \quad ; \quad M_{EF} = -\frac{(6k+5)}{N} \cdot wL^2 \quad ; \quad M_F = \frac{k}{N} \cdot wL^2$$

$$k_1 = \frac{I_1 h}{I_2 L} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L} \quad ; \quad \text{otros datos (445)}$$

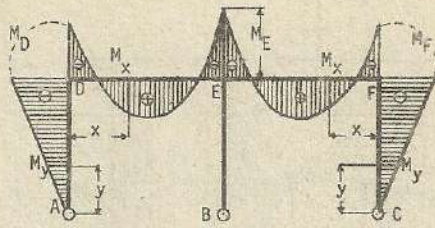
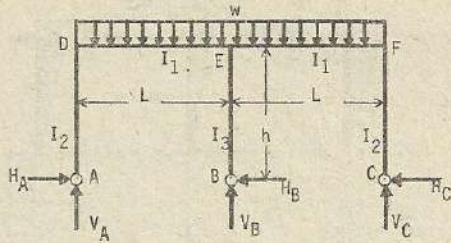
$$H_A = H_C = \frac{(k_1 + 2k_2 + 3)}{4E} \cdot \frac{wL^2}{h} \quad ; \quad H_B = 0 \quad ; \quad V_B = \frac{(5k_1^2 + 10k_1 k_2 + 18k_1 + 6k_2 + 9)}{E} \cdot wL$$

$$V_A = V_C = \frac{(3k_1^2 + 6k_1 k_2 + 12k_1 + 6k_2 + 9)}{2E} \cdot wL \quad ; \quad M_D = M_F = -\frac{(k_1 + 2k_2 + 3)}{4E} \cdot wL^2$$

$$M_{EB} = 0 \quad ; \quad M_{ED} = M_{EF} = -\frac{(2k_1 + 4k_1 k_2 + 7k_1 + 2k_2 + 3)}{4E} \cdot wL^2$$

Momentos por flexión en las columnas, AD y $CF \rightarrow M_y = -H_A y$

$BE \rightarrow M_y = 0$



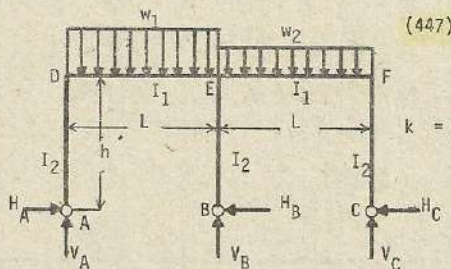
Esfuerzos de momentos en las vigas, DE y EF

$$M_x = -H_A h + V_A x - \frac{w x^2}{2}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$, donde

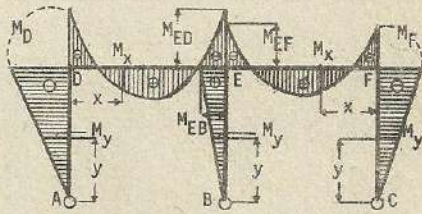
$$H_A = H_C = \frac{wL^2}{4(4k+3)h} ; \quad H_B = 0 ; \quad V_A = V_C = \frac{3(k+1)}{2(4k+3)}wL ; \quad V_B = \frac{(5k+3)}{(4k+3)} \cdot wL$$

$$M_D = M_F = -\frac{wL^2}{4(4k+3)} ; \quad M_{EB} = 0 ; \quad M_{ED} = M_{EF} = -\frac{2k+1}{4(4k+3)} \cdot wL^2$$



(447)

$$k = \frac{I_1 h}{I_2 L}$$



$$N = 24(4k+3)(k+1)$$

$$H_A = \frac{(7k+6)w_1 - kw_2}{N} \cdot \frac{L^2}{h} ; \quad H_B = \frac{(w_1 - w_2)}{12(k+1)} \cdot \frac{L^2}{h} ; \quad H_C = \frac{(7k+6)w_2 - kw_1}{N} \cdot \frac{L^2}{h}$$

$$V_A = \frac{(7k+6)w_1 - kw_2}{4(4k+3)} \cdot L ; \quad V_B = \frac{(5k+3)(w_1 + w_2)}{2(4k+3)} \cdot L ; \quad V_C = \frac{(7k+6)w_2 - kw_1}{4(4k+3)} \cdot L$$

$$M_D = -\frac{(7k+6)w_1 - kw_2}{N} \cdot L^2 ; \quad M_{EB} = \frac{(w_1 + w_2)L^2}{12(k+1)} ; \quad M_F = -\frac{(7k+6)w_2 - kw_1}{N} \cdot L^2$$

$$M_{ED} = -\frac{w_1(6k+13k+6) + w_2k(6k+5)}{N} \cdot L^2 ; \quad M_{EF} = -\frac{w_2(6k^2+13k+6) + w_1k(6k+5)}{N} \cdot L^2$$

Momentos por flexión en las columnas,

$$AD \rightarrow M_y = -H_A y$$

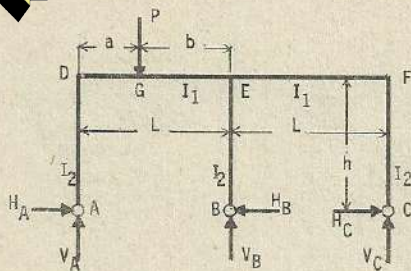
$$BE \rightarrow M_y = +H_B y$$

$$CF \rightarrow M_y = -H_C y$$

Momentos por flexión en las vigas :

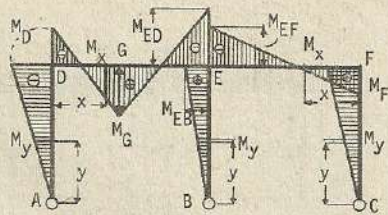
$$DE \rightarrow M_x = -H_A x + V_A x - \frac{w_1 x^2}{2}$$

$$EF \rightarrow M_x = -H_C x + V_C x - \frac{w_2 x^2}{2}$$



(448)

$$k = \frac{I_1 h}{I_2 L}$$



$$H_A = \frac{a(4k+3) + b(10k+9)}{4(k+1)(4k+3)} \cdot \frac{Pab}{hL^2} ; H_B = \frac{1}{2(k+1)} \cdot \frac{Pab}{hL^2}$$

$$H_C = \frac{a(4k+3) - b(2k+3)}{4(k+1)(4k+3)} \cdot \frac{Pab}{hL^2} ; V_A = \frac{[2k(3ab+2b^2+2L^2) + 3(3ab+b^2+L^2)]}{2(4k+3)L^3} \cdot P$$

$$V_B = \frac{[2k(2L^3-ab^2-2b^2L)+3(L^3-2ab^2-b^2L)]}{(4k+3)L^3} \cdot P ; V_C = \frac{a[3(b-a) - 2k(b+2a)]}{2(4k+3)L^3} P$$

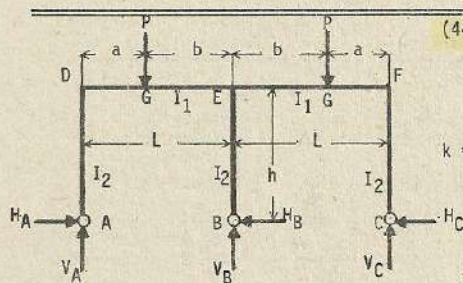
$$M_D = -\frac{a(4k+3) + b(10k+9)}{4(k+1)(4k+3)} \cdot \frac{Pab}{L^2} ; M_E = \frac{Pab}{2(k+1)L^2} ; M_F = \frac{a(4k+3) - b(2k+3)}{4(k+1)(4k+3)} \cdot \frac{Pab}{L^2}$$

$$M_G = \frac{[4k^2(3ab+2b^2+2L^2) + 2k(12ab+4b^2+L^2) + 3(4ab+L^2)]}{4(k+1)(4k+3)L^3} \cdot Pab$$

Momentos por flexión en las columnas , AD $\rightarrow M_y = -H_A y$; BE $\rightarrow M_y = H_B y$
CF $\rightarrow M_y = H_C y$

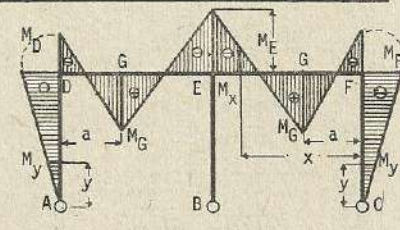
Momentos de flexión en las vigas : DE $\rightarrow M_x = -H_A h + V_A x$ $x \leq a$
 $M_x = -H_A h + V_A x - P(x-a)$ $x \geq a$

$$EF \rightarrow M_x = H_C h + V_C x$$



(449)

$$k = \frac{I_1 h}{I_2 L}$$



$$H_A = H_C = \frac{3ab^2 p}{(4k+3)hL^2} ; H_B = 0$$

$$V_A = V_C = \frac{2k(a+2L) + 3(2a+L)}{(4k+3)} \cdot \frac{Pb^2}{L^3} ; V_B = \frac{4k(2L^3-ab^2-2b^2L) + 6(L^3-3ab^2-b^2L)}{(4k+3)} \cdot \frac{P}{L^3}$$

$$M_D = M_F = -\frac{3ab^2}{(4k+3)} \cdot \frac{P}{L^2} ; M_E (\text{VIGA}) = -\frac{2k(a+L) + 3a}{(4k+3)} \cdot \frac{Pab}{L^2}$$

$$M_G = \frac{2k(a+2L) + 6a}{(4k+3)} \cdot \frac{Pab^2}{L^2} ; \text{Momentos de flexión en las columnas AD y CF}$$

$$M_y = -H_A y = -H_C y$$

Esfuerzos de flexión en las vigas DE y EF :

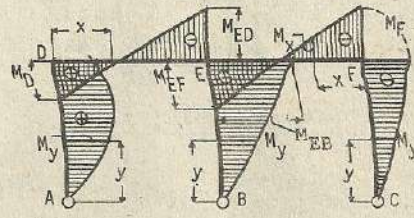
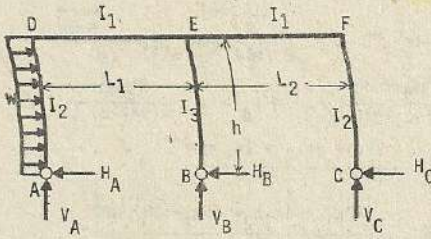
$$M_x = -H_A h + V_A x = -H_C h + V_C x \quad x \geq a$$

$$M_x = H_A h + V_A x - P(x-a)$$

$$= -H_C h + V_C x - P(x-a)$$

$$x \geq a$$

(450)



$$k_1 = \frac{I_1 h}{I_2 L_1} ; k_2 = \frac{I_1 h}{I_3 L_1} ; \alpha = \frac{L_2}{L_1}$$

$$F = (1+\alpha)(9\alpha+3\alpha k_2+3k_2+8k_1 k_2+4k_1^2) + 3k_1(1+8\alpha+\alpha^2)$$

$$P = 4k^2(1+\alpha) + 2k(1+5\alpha+\alpha^2) + 3\alpha(1+\alpha) ; R = (k_1+2k_2+3)(4k_1+3)$$

$$H_A = \frac{wh [4(1+\alpha)(3\alpha k_2+3k_2+8k_1 k_2+3k_1^2) + 3k_1(4+24\alpha+3\alpha^2) + 12(\alpha+k_1)(3\alpha+\alpha k_2+k_2) + 18\alpha(\alpha+2)]}{8F}$$

$$H_B = \frac{wh [18(\alpha+1)\alpha+20k_1^2+12k_1+51\alpha k_1+\alpha k_1(15\alpha+20k_1)]}{8F} ; H_C = \frac{wh [18\alpha+33\alpha k_1+4(3+5k_1)(1+\alpha)k_2]}{8F}$$

$$V_A = \frac{wh^2 [6\alpha^2(3+k_1+k_2) + \alpha(6k_2+15k_1+k_1 k_2+8k_1^2) + 3k_1(3k_2-k_1)]}{4L_1 F}$$

$$V_B = \frac{wh^2 [k_1(9\alpha+7k_1+3-3\alpha^2-4\alpha k_1) + k_1 k_2(7\alpha-1) + (1-\alpha)(9\alpha+3k_2+3\alpha k_2)]}{2L_1 F}$$

$$V_C = \frac{wh^2 [k_1(33\alpha+6+11k_1) + k_1(6+6\alpha) + k_1 k_2(7+15\alpha) + 18\alpha]}{4L_1 F}$$

$$M_D = \frac{wh^2 [12(\alpha+k_1)(3\alpha+\alpha k_2+k_2) - 3\alpha k_1(\alpha+8) - 4k_1^2(1+\alpha) - 18\alpha^2]}{8F}$$

$$M_{ED} = \frac{wh^2 [18\alpha^2+\alpha k_1(18+15\alpha+20k_1) + 6k_1 k_2 - 2k_1^2 - 10\alpha k_1 k_2]}{8F}$$

$$M_{EF} = \frac{wh^2 [33\alpha k_1+10\alpha k_1 k_2 - 6k_1 k_2 + 18\alpha+12k_1 + 22k_1^2]}{8F}$$

$$M_{EB} = \frac{wh^2 [18(1+\alpha)\alpha + 20k_1^2+12k_1+51\alpha k_1+\alpha k_1(15\alpha+20k_1)]}{8F}$$

$$M_F = \frac{wh^2 [18\alpha+33\alpha k_1+4(3+5k_1)(1+\alpha)k_2]}{8F} ; \text{ Si } I_2 = I_3, \text{ entonces } k_2 = k_1 = k, \text{ donde :}$$

$$H_A = \frac{wh [56(1+\alpha)k^2 + (24+144\alpha+33\alpha^2)k + 18(2+3\alpha)\alpha]}{24P}$$

$$H_B = \frac{wh [20(1+\alpha)k^2 + (12+51\alpha+15\alpha^2)k + 18(1+\alpha)\alpha]}{24P}$$

$$H_C = \frac{wh [20(1+\alpha)k^2 + (12+45\alpha)k + 18\alpha]}{24P} ; V_A = \frac{wh^2 [(2+3\alpha)k^2 + (7+4\alpha)\alpha k + 6\alpha^2]}{4L_1 P}$$

$$V_B = \frac{wh^2 [(6+3\alpha)k^2 + (4+9\alpha-4\alpha^2)k + 9(1-\alpha)\alpha]}{6L_1 P} ; V_C = \frac{wh^2 [(6+5\alpha)k^2 + (4+13\alpha)k + 6\alpha]}{4L_1 P}$$

$$M_D = \frac{wh^2 [8(1+\alpha)k^2 + 3(8+3\alpha)\alpha k + 18\alpha^2]}{24P} ; M_{ED} = \frac{wh^2 [(4+10\alpha)k^2 + (18+15\alpha)\alpha k + 18\alpha^2]}{24P}$$

$$M_{EF} = \frac{wh^2 [(16+10\alpha)k^2 + (12+33\alpha)k + 18\alpha]}{24P}; \quad M_F = - \frac{wh^2 [20(1+\alpha)k^2 + (12+45\alpha)k + 18\alpha]}{24P}$$

$$M_{EB} = - \frac{wh^2 [20(1+\alpha)k^2 + (12+51\alpha + 15\alpha^2)k + 18(1+\alpha)\alpha]}{24P}$$

Si $L_1 = L_2 = L$, entonces $k_1 = \frac{I_1 h}{I_2 L}$; $k_2 = \frac{I_1 h}{I_3 L}$, luego

$$H_A = \frac{(24k_1^2 + 88k_1 k_2 + 129k_1 + 72k_2 + 90)}{16R} wh; \quad H_B = \frac{(5k_1 + 6)}{8(k_1 + 2k_2 + 3)} wh$$

$$H_C = \frac{(40k_1 k_2 + 24k_2 + 33k_1 + 18)}{16R} wh; \quad V_A = - \frac{(5k_1 + 6)wh^2}{3(4k_1 + 3)L}; \quad V_B = - \frac{3k_1 wh^2}{4(4k_1 + 3)L}$$

$$V_C = \frac{(11k_1 + 6)wh^2}{8(4k_1 + 3)L}; \quad M_D = \frac{(-8k_1^2 + 24k_1 k_2 + 9k_1 + 24k_2 + 18)wh^2}{16R}$$

$$M_{ED} = - \frac{(18k_1^2 - 4k_1 k_2 + 33k_1 + 18)wh^2}{16R}; \quad M_{EF} = \frac{(22k_1^2 + 4k_1 k_2 + 45k_1 + 18)wh^2}{16R}$$

$$M_{EB} = - \frac{(5k_1 + 6)}{8(k_1 + 2k_2 + 3)} wh^2; \quad M_F = - \frac{(40k_1 k_2 + 24k_2 + 33k_1 + 18)wh^2}{16R}$$

Si $L_1 = L_2 = L$, $I_1 = I_2 = I_3$, o sea cuando $k_1 = k_2 = k$

$$H_A = \frac{(112k^2 + 201k + 90)}{48(k+1)(4k+3)} wh; \quad H_B = \frac{(5k+6)}{24(k+1)} wh; \quad H_C = \frac{(40k^2 + 57k + 18)}{48(k+1)(4k+3)} wh$$

$$V_A = - \frac{(5k+6)wh^2}{8(4k+3)L}; \quad V_B = - \frac{3kwh^2}{4(4k+3)L}; \quad V_C = \frac{(11k+6)wh^2}{8(4k+3)L}$$

$$M_D = \frac{(16k^2 + 33k + 18)wh^2}{48(1+k)(4k+3)}; \quad M_{ED} = - \frac{(14k^2 + 33k + 18)wh^2}{48(k+1)(4k+3)}; \quad M_{EF} = \frac{(26k^2 + 45k + 18)wh^2}{48(k+1)(4k+3)}$$

$$M_{EB} = - \frac{(5k+6)wh^2}{24(k+1)}; \quad M_F = - \frac{(40k^2 + 57k + 18)wh^2}{48(k+1)(4k+3)}$$

Esfuerzos de momentos por flexión de las columnas.

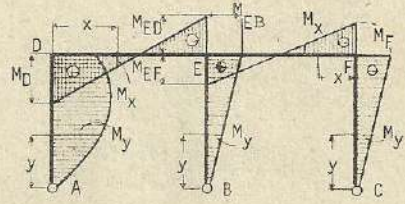
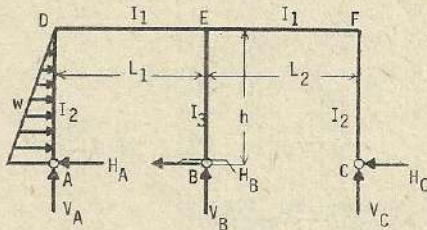
$$AD \rightarrow M_y = H_{Ay} - \frac{w x^2}{2}; \quad BE \rightarrow M_y = - H_{By}; \quad CF \rightarrow M_y = - H_{Cy}$$

Momentos en las vigas :

$$DE \rightarrow M_x = H_A h - \frac{wh^2}{2} + V_A x$$

$$EF \rightarrow M_x = - H_C h + V_C x$$

(451)



$$k_1 = \frac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{L_2}{L_1}$$

otros datos (450)

$$H_A = W \frac{[44(1+\alpha)k_1^2 + (40+332\alpha+33\alpha^2)k_1 + 124(1+\alpha)k_1 k_2 + 20(2+5\alpha+3\alpha^2)k_2 + 30(4+5\alpha)\alpha]}{20F}$$

$$H_B = W \frac{[36(\alpha+1)k_1^2 + (27\alpha^2+87\alpha+20)k_1 + 30\alpha(\alpha+1)]}{20F} \quad ; \quad H_C = W \frac{[61\alpha k_1 + 36(1+\alpha)k_1 k_2 + 20(1+\alpha)k_2 + 30\alpha]}{20F}$$

$$V_A = - \frac{Wh [(40\alpha-21)k_1^2 + \alpha(57+30\alpha)k_1 + (39-\alpha)k_1 k_2 + 30(1+\alpha)\alpha k_2 + 90\alpha^2]}{30L_1 F}$$

$$V_B = - \frac{Wh [(82-40\alpha)k_1^2 + (30+126\alpha-30\alpha^2)k_1 + (82\alpha+2)k_1 k_2 + 30(\alpha+1)(1-\alpha)k_2 + 90(1-\alpha)]}{30L_1 F}$$

$$V_C = \frac{Wh [61k_1^2 + (183\alpha+30)k_1 + (81\alpha+41)k_1 k_2 + 30(1+\alpha)k_2 + 90\alpha^2]}{30L_1 F}$$

$$M_D = Wh \frac{[-28(1+\alpha)k_1^2 + (36-21\alpha)\alpha k_1 + 52(1+\alpha)k_1 k_2 + 60(1+\alpha)\alpha k_2 + 90\alpha^2]}{60F}$$

$$M_{ED} = - \frac{Wh [(108\alpha-14)k_1^2 + \alpha(81\alpha+78)k_1 + (26-54\alpha)k_1 k_2 + 90\alpha^2]}{60F}$$

$$M_{EF} = \frac{Wh [122k_1^2 + (83\alpha+60)k_1 + (54\alpha-26)k_1 k_2 + 90\alpha]}{60F}$$

$$M_{EB} = \frac{Wh [36(1+\alpha)k_1^2 + (27\alpha^2+87\alpha+20)k_1 + 30\alpha(1+\alpha)]}{20F}$$

$$M_F = - \frac{Wh [61\alpha k_1 + 36(1+\alpha)k_1 k_2 + 20(1+\alpha)k_2 + 30\alpha]}{20F}$$

Si $L_2 = L_3$, entonces $k_1 = k_2 = k$, luego

$$H_A = \frac{W [168(1+\alpha)k^2 + (80+432\alpha+93\alpha^2)k + 30(4+5\alpha)\alpha]}{60P}$$

$$H_B = \frac{W [36(1+\alpha)k^2 + (20+87\alpha+27\alpha^2)k + 30(1+\alpha)\alpha]}{60P} \quad ; \quad H_C = \frac{W [36(1+\alpha)k^2 + (20+81\alpha)k + 30\alpha]}{60P}$$

$$V_A = - \frac{Wh [(13+6)k^2 + (20\alpha+29)\alpha k + 30\alpha]}{30L_1 P} \quad ; \quad V_B = - \frac{Wh [7(2+\alpha)k^2 + (10+21\alpha-10\alpha^2)k + 15(1-\alpha)]}{15L_1 P}$$

$$V_C = \frac{Wh [(27\alpha+34)k^2 + (71\alpha+20)k + 30\alpha]}{30L_1 P} \quad ; \quad M_D = \frac{Wh [8(1+\alpha)k^2 + (32+13\alpha)k + 30\alpha^2]}{60P}$$

$$M_{ED} = - \frac{Wh [(4+18\alpha)k^2 + \alpha(27\alpha+26)k + 30\alpha^2]}{60P} \quad ; \quad M_{EF} = - \frac{Wh [(32+18\alpha)k^2 + (20+61\alpha)k + 30\alpha]}{60P}$$

$$M_{EB} = \frac{Wh [(36+36\alpha)k^2 + (27\alpha^2+87\alpha+20)k + 30\alpha(\alpha+1)]}{60P} \quad ; \quad M_F = - \frac{Wh [36(1+\alpha)k^2 + (20+81\alpha)k + 30\alpha]}{60P}$$

Si $L_1 = L_2 = L$, entonces $k_1 = \frac{I_1 h}{I_2 L}$; $k_2 = \frac{I_1 h}{I_3 L}$

$$H_A = W \frac{(88k_1^2 + 248k_1 k_2 + 405k_1 + 200k_2 + 270)}{40R} ; H_B = W \frac{(9k_1 + 10)}{20(k_1 + 2k_2 + 3)}$$

$$H_C = W \frac{(72k_1 k_2 + 40k_2 + 61k_1 + 30)}{40R} ; V_A = -\frac{Wh(19k_1 + 30)}{60L(4k_1 + 3)} ; V_B = -\frac{7Whk_1}{10L(4k_1 + 3)}$$

$$V_C = \frac{Wh(61k_1 + 30)}{60L(4k_1 + 3)} ; M_D = Wh \frac{(-56k_1 + 104k_1 k_2 + 15k_1 + 120k_2 + 90)}{120R}$$

$$M_{ED} = -Wh \frac{(94k_1^2 - 28k_1 k_2 + 159k_1 + 90)}{120R} ; M_{EF} = Wh \frac{(122k_1^2 + 28k_1 k_2 + 243k_1 + 90)}{120R}$$

$$M_{EB} = -Wh \frac{(9k_1 + 10)}{20(k_1 + 2k_2 + 3)} ; M_F = -Wh \frac{72k_2 k_1 + 40k_2 + 61k_1 + 30}{40R}$$

Si $L_1 = L_2 = L$, $I_3 = I_2$, entonces $k_1 = k_2 = k$ donde : $H_A = W \frac{(336k^2 + 605k + 270)}{120(k+1)(4k+3)}$

$$H_B = W \frac{(9k+10)}{60(k+1)} ; H_C = W \frac{(72k^2 + 101k + 30)}{120(k+1)(4k+3)} ; V_A = -\frac{Wh(19k+30)}{60L(4k+3)}$$

$$V_B = -\frac{7kWh}{10L(4k+3)} ; V_C = \frac{Wh(61k+30)}{60L(4k+3)} ; M_D = Wh \frac{(16k^2 + 45k + 30)}{120(k+1)(4k+3)}$$

$$M_{ED} = -Wh \frac{(22k^2 + 53k + 30)}{120(k+1)(4k+3)} ; M_{EF} = Wh \frac{(50k^2 + 81k + 30)}{120(k+1)(4k+3)} ; M_{EB} = -Wh \frac{(9k+10)}{60(k+1)}$$

$$M_F = Wh \frac{(50k^2 + 81k + 30)}{120(k+1)(4k+3)}$$

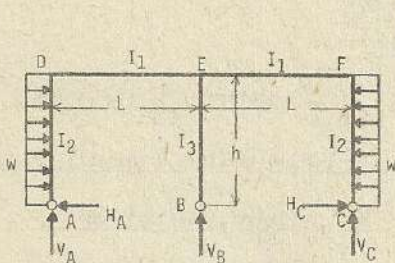
Esfuerzos de momentos por flexión en las columnas :

$$AD \rightarrow M_y = H_A y - \frac{W}{h^2} (hy^2 - \frac{y^3}{3}) ; BE \rightarrow M_y = -H_B y ; CF \rightarrow M_y = -H_C y$$

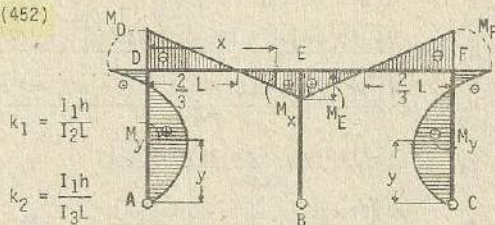
Esfuerzos de momentos por flexión en las vigas :

$$DE \rightarrow M_x = H_A h - \frac{2}{3} Wh + V_A x$$

$$EF \rightarrow M_x = -H_C h + V_C x$$



(452)



$$H_A = H_C = \frac{3(k_1 + 1)}{2(4k_1 + 3)} wh ; H_B = 0$$

$$V_A = V_C = \frac{3k_1}{4(4k_1 + 3)} \frac{wh^2}{2} ; V_B = \frac{3k_1}{2(4k_1 + 3)} \frac{wh^2}{2} ; M_D = M_F = -\frac{k_1}{2(4k_1 + 3)} wh^2$$

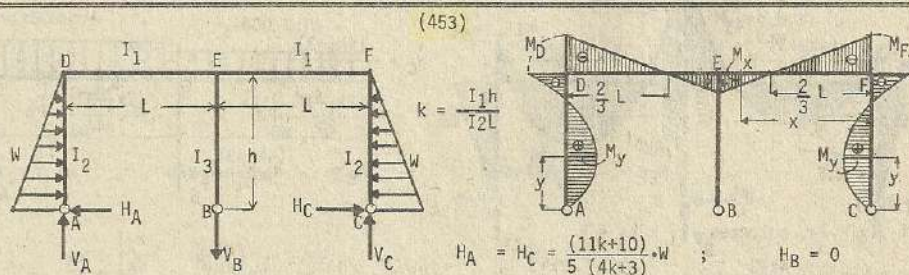
$$M_{ED} = M_{EF} = \frac{k_1}{4(4k_1 + 1)} wh^2 ; M_{EB} = 0$$

Momentos por flexión en las columnas AD y CF :

$$M_y = H_A y - \frac{w y^2}{2} = H_C y - \frac{w y^2}{2}$$

Momentos por flexión en las vigas : DE y FE :

$$M_x = H_A h - \frac{w h^2}{2} + V_A x = H_C h - \frac{w h^2}{2} + V_C x$$



$$V_A = V_C = \frac{7k}{10(4k+3)} W ; \quad V_B = -\frac{7k}{5(4k+3)} W ; \quad M_D = M_F = -\frac{7k}{15(4k+3)} W h$$

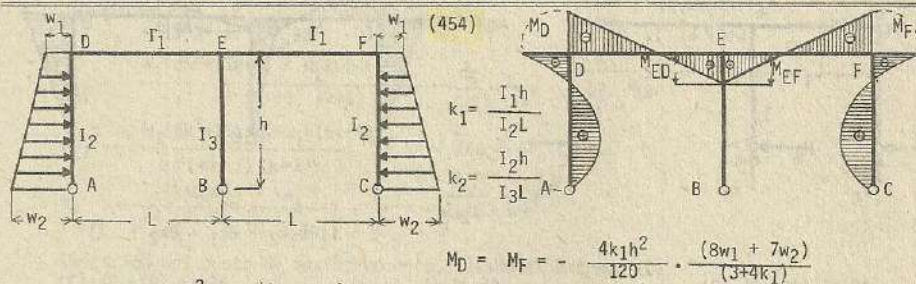
$$M_{ED} = M_{EF} = \frac{7k}{30(4k+3)} \cdot W h ; \quad M_{EB} = 0$$

Momentos por flexión o doblado en las columnas AD y CF,

$$M_y = H_A y - \frac{w}{h^2} (h y^2 - \frac{y^3}{3}) = H_C y - \frac{w}{h^2} (h y^2 - \frac{y^3}{3})$$

Momentos por flexión en las vigas DE y FE,

$$M_x = H_A h - \frac{2}{3} W h + V_A x = H_C h - \frac{2}{3} W h + V_C x$$

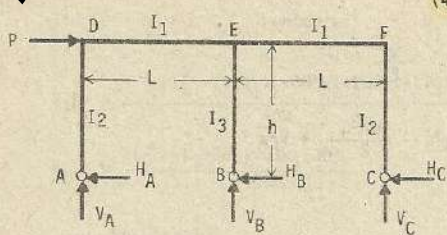


$$M_{ED} = M_{EF} = \frac{2k_1 h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k_1)}$$

Si $I_2 = I_3$, $k = \frac{I_1 h}{I_2 L}$ donde $M_D = M_F = -\frac{4k h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k)}$

$$M_{ED} = M_{EF} = \frac{2k h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k)}$$

(455)

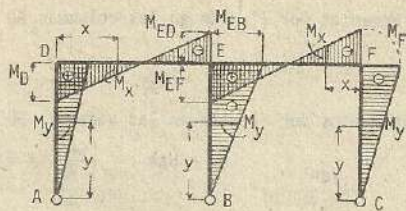


$$G = (k_1 + 2k_2 + 3)$$

$$H_A = \frac{(4k_2+3)}{4G} \cdot P ; \quad H_B = \frac{(2k_1+3)}{2G} \cdot P ; \quad H_C = \frac{(4k_2+3)}{4G} \cdot P ; \quad V_A = -\frac{Ph}{2L} ; \quad V_B = 0$$

$$V_C = \frac{Ph}{2L} ; \quad M_D = \frac{(4k_2+3)}{4G} Ph ; \quad M_{EB} = -\frac{(2k_1+3)}{2G} \cdot Ph ; \quad M_{ED} = -\frac{(2k_1+3)}{4G} \cdot Ph$$

$$M_{EF} = \frac{(2k_1+3)}{4G} \cdot Ph ; \quad M_F = -\frac{(4k_2+3)}{4G} \cdot Ph$$



$$k_1 = \frac{I_1 h}{I_2 L} ; \quad k_2 = \frac{I_1 h}{I_3 L}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$

$$H_A = H_C = \frac{4k+3}{12(k+1)} \cdot P ; \quad H_B = \frac{2k+3}{6(k+1)} \cdot P ; \quad V_A = -\frac{Ph}{2L} ; \quad V_B = 0 ; \quad V_C = \frac{Ph}{2L}$$

$$M_D = \frac{(4k+3)}{12(k+1)} \cdot Ph ; \quad M_{EB} = -\frac{(2k+3)}{6(k+1)} \cdot Ph ; \quad M_{ED} = -\frac{(2k+3)}{12(k+1)} \cdot Ph$$

$$M_{EF} = \frac{(2k+3)}{12(k+1)} \cdot Ph ; \quad M_F = -\frac{(4k+3)}{12(k+1)} \cdot Ph$$

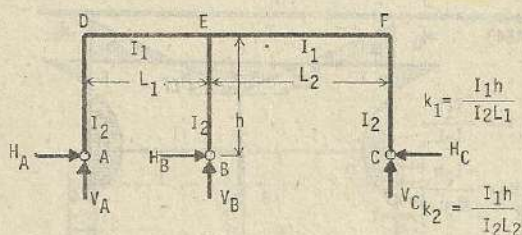
Momentos por flexión en las columnas,

$$AD \rightarrow M_y = H_A y ; \quad BE \rightarrow M_y = -H_B y ; \quad CF \rightarrow M_y = -H_C y$$

Momentos de flexión en las vigas : DE $\rightarrow M_x = H_A h + V_A x$

$$EF \rightarrow M_x = -H_C h + V_C x$$

Esfuerzos por cambio de temperatura (456)



$$k_1 = \frac{I_1 h}{I_2 L_1}$$

$$k_2 = \frac{I_1 h}{I_2 L_2}$$

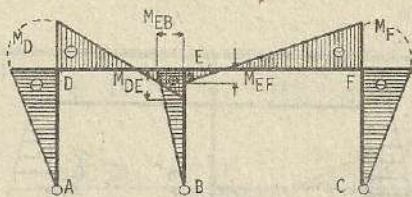
$$S = (16k_1 k_2 + 8k_1 + 8k_2 + 3)$$

$$H_A = \frac{4EetI_1(4k_1+8k_2+3)}{Sh^2} ; \quad H_B = \frac{16EetI_1(k_1 - k_2)}{Sh^2} ; \quad H_C = \frac{4EetI_1(4k_2+8k_1+3)}{Sh^2}$$

$$V_A = \frac{6EetI_1(4k_1+8k_2+3)}{ShL_1} ; \quad V_B = \frac{6EetI_1[(4k_1+8k_2+3)L_2 + (4k_2+8k_1+3)L_1]}{ShL_1L_2}$$

$$V_C = \frac{6EetI_1(4k_2+8k_1+3)}{ShL_2} ; \quad M_D = -\frac{4EetI_1(4k_1+8k_2+3)}{Sh} ; \quad M_{EB} = \frac{16EetI_1(k_1 - k_2)}{Sh}$$

$$M_{ED} = \frac{2EetI_1(4k_1+8k_2+3)}{Sh} ; \quad M_{EF} = \frac{3EetI_1(4k_2+8k_1+3)}{Sh} ; \quad M_F = -\frac{4EetI_1(4k_2+8k_1+3)}{Sh}$$

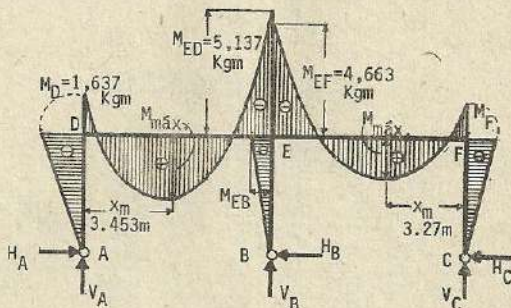
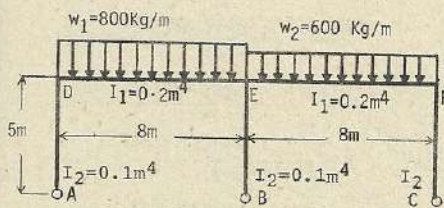


Si $L_1 = L_2 = L$, entonces $k_1 = k_2 = k$, donde $H_A = H_C = \frac{12EtI_1}{(4k+3)h^2} \cdot \delta$; $H_B = 0$

$$V_A = V_C = \frac{18EtI_1}{(4k+3)hL}; \quad V_B = \frac{36EtI_1}{(4k+3)hL}; \quad \text{Nota: En caso de descenso de temperatura, se modifican los signos de los esfuerzos}$$

$$M_D = M_F = -\frac{12EtI_1}{(4k+3)h}; \quad M_E = \frac{6EtI_1}{(4k+3)h}$$

Solución de problema numérico:



$$k = \frac{I_1 h}{I_2 L} = \frac{0.2 \times 5}{0.1 \times 8} = 1.25$$

$$H_A = \frac{[(7k+6)w_1 - kw_2]L^2}{24(4k+3)(k+1)h} = \frac{[(1.25 \times 7 + 6) \times 800 - 1.25 \times 600] \times 8 \times 8}{24 \times (4 \times 1.25 + 3) \times (1.25 + 1) \times 5} = 327 \text{ Kg}$$

$$V_A = \frac{(7k+6)w_1 - kw_2}{4(4k+3)} L = \frac{[(1.25 \times 7 + 6) \times 800 - 1.25 \times 600] \times 8}{4(4 \times 1.25 + 3)} = 2,762.5 \text{ Kg}$$

$$H_B = \frac{(w_1 - w_2)L^2}{12(k+1)h} = \frac{(800 - 600) \times 8 \times 8}{12(1.25 + 1) \times 5} = 95 \text{ Kg}; \quad V_B = \frac{(5k+3)(w_1 + w_2)L}{2(4k+3)}$$

$$= \frac{(5 \times 1.25 + 3)(800 + 600)}{2 \times (4 \times 1.25 + 3)} \times 8 = 6,475 \text{ Kg}$$

$$H_C = \frac{[(7k+6)w_2 - kw_1]L^2}{24(4k+3)(k+1)h} = \frac{[(7 \times 1.25 + 6) \times 600 - 1.25 \times 800] \times 8 \times 8}{24(4 \times 1.25 + 3)(1.25 + 1) \times 5} = 233 \text{ Kg}$$

$$V_C = \frac{[(7k+6)w_2 - kw_1]L}{4(4k+3)} = \frac{[(7 \times 1.25 + 6) \times 600 - 1.25 \times 800] \times 8}{4 \times (4 \times 1.25 + 3)} = 1,962.5 \text{ Kg}$$

$$M_D = -\frac{(7k+6) - kw_2}{24(4k+3)(k+1)} L^2 = -1,637 \text{ Kg-m}; \quad M_{EB} = \frac{(w_1 - w_2)L^2}{12(k+1)} = 474 \text{ Kg-m}$$

$$M_{ED} = -\frac{w_1(6k^2 + 13k + 6) + w_2 k(6k + 5)}{24(4k+3)(k+1)} L^2 = -5,137 \text{ Kg-m}$$

$$M_{EF} = -\frac{w_2(6k^2 + 13k + 6) + w_1 k(6k + 5)}{24(4k+3)(k+1)} L^2 = -4,663 \text{ Kg-m}$$

$$M_F = -\frac{(7k+6)w_2 - kw_1}{24(4k+3)(k+1)} L^2 = -1,163 \text{ Kg-m}$$

Ubicación del punto de máximo momento, desde el extremo (D)

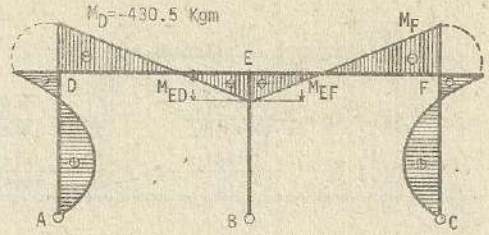
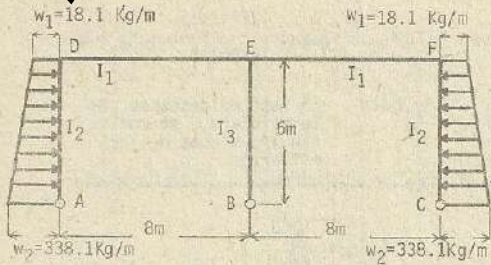
$$V_A = w_1 x_m \rightarrow x_m = \frac{V_A}{w_1} = \frac{2,762.5}{800} = 3.453 \text{ m.} \quad \therefore M_{\max} = M_D + V_A x_m - \frac{w_1 x_m^2}{2}$$

$$= 3,132.6 \text{ Kg-m.}$$

$$\text{Desde el extremo (F); } V_C = w_2 x_m \rightarrow x_m = \frac{V_C}{w_2} = \frac{1,962.5}{600} = 3.27 \text{ m.}$$

$$\therefore M_{\max} = M_F + V_C x_m - \frac{w_2 x_m^2}{2} = -1,163 + 6,417.4 - 3,207.9$$

$$= 2,046.5 \text{ Kg-m}$$



Altura útil (d_1) del miembro DE : 48 cm.
 (d_2) del miembro soporte : 44 cm.

$$\frac{d_1}{d_2} = \frac{48}{44} = 1.1 = \frac{I_1}{I_2} = \left(\frac{d_1}{d_2}\right)^3 = (1.1)^3 = 1.33$$

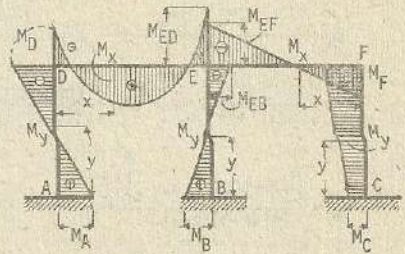
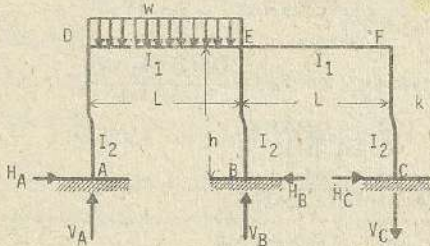
$$k_1 = \frac{I_1 h}{I_2 L} = 1.33 \times \frac{6}{8} = 1$$

$$M_D = M_F = -\frac{4k_1 h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k_1)} = -\frac{4 \times 1 \times 6 \times 6}{120} \cdot \frac{(8 \times 18.1 + 7 \times 338.1)}{(3+4 \times 1)} = -430.5 \text{ Kgm}$$

$$M_{ED} = M_{EF} = \frac{2k_1 h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k_1)} = \frac{2 \times 1 \times 6 \times 6}{120} \cdot \frac{(8 \times 18.1 + 7 \times 338.1)}{(3+4 \times 1)} = 215.3 \text{ Kgm}$$

$$M_{EB} = 0$$

(457)



$$H = (k+1)(6k^2 + 9k+1) \quad ; \quad T = (6k^2 + 9k+1)$$

$$H_A = \frac{(7k^2 + 9k+1)}{8H} \cdot \frac{wL^2}{h} \quad ; \quad H_B = \frac{(8k+1)}{8T} \cdot \frac{wL^2}{h} \quad ; \quad H_C = \frac{k^2}{8H} \cdot \frac{wL^2}{h}$$

$$V_A = \frac{(21k^3 + 56k^2 + 40k+4)wL}{8H} \quad ; \quad V_B = \frac{(5k+4)kwL}{8(k+1)} \quad ; \quad V_C = \frac{(3k^2 + 5k+1)wL}{8H}$$

$$M_A = \frac{(6k^2 + 8k+1)wL^2}{24H} \quad ; \quad M_B = \frac{(9k+1)wL^2}{24T} \quad ; \quad M_C = \frac{kwL^2}{24H}$$

$$M_D = -\frac{(15k^2 + 19k+2)wL^2}{24H} \quad ; \quad M_{ED} = -\frac{(9k^3 + 27k^2 + 19k+2)wL^2}{24H}$$

$$M_{EF} = -\frac{(9k^2 + 12k+2)kwL^2}{24H} \quad ; \quad M_{EB} = -\frac{(15k+2)wL^2}{24T} \quad ; \quad M_F = \frac{(3k+1)wL^2}{24H}$$

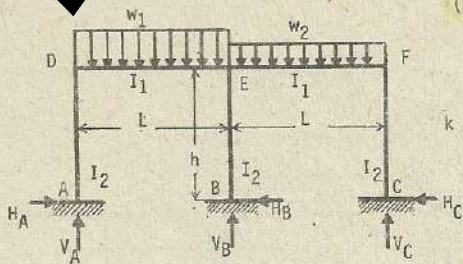
Momentos por flexión en las columnas :

$$AD \rightarrow M_y = M_A - H_A y \quad ; \quad BE \rightarrow M_y = M_B - H_B y \quad ; \quad CF \rightarrow M_y = M_C + H_C y$$

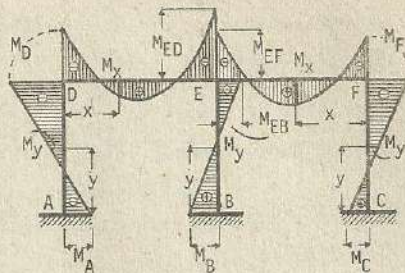
$$\text{Momentos por flexión en las vigas: } DE \rightarrow M_x = M_A - H_A h + V_A x - \frac{wx^2}{2}$$

$$EF \rightarrow M_x = M_C + H_C h - V_C x$$

(458)



$$k = \frac{I_1 h}{I_2 L}$$



otros datos (457)

$$H_A = \frac{(7k^2+9k+1)w_1L^2 - k^2w_2L^2}{8Hh} ; H_B = \frac{(8k+1)(w_1-w_2)L^2}{8Th}$$

$$H_C = \frac{(7k^2+9k+1)w_2L^2 - k^2w_1L^2}{8Hh} ; V_A = \frac{(21k^3+56k^2+40k+4)w_1L - (3k^2+5k+1)kw_1L}{8H}$$

$$V_B = \frac{(5k+4)(w_1+w_2)L}{8(k+1)} ; V_C = \frac{(21k^3+56k^2+40k+4)w_2L - (3k^2+5k+1)kw_1L}{8H}$$

$$M_A = \frac{(6k^2+8k+1)w_1L^2 + kw_2L^2}{24H} ; M_B = \frac{(9k+1)(w_1-w_2)L^2}{24T} ; M_C = \frac{(6k^2+8k+1)w_2L^2 + kw_1L^2}{24H}$$

$$M_D = -\frac{(15k^2+19k+2)w_1L^2 - (3k+1)kw_2L^2}{24H} ; M_{EB} = -\frac{(15k+2)(w_1-w_2)L^2}{24T}$$

$$M_{EF} = -\frac{(9k^2+12k+2)w_1L^2 + (9k^3+27k^2+19k+2)w_2L^2}{24H}$$

$$M_F = -\frac{(15k^2+19k+2)w_2L^2 - (3k+1)kw_1L^2}{24H}$$

$$M_{ED} = -\frac{(9k^3+27k^2+19k+2)w_1L^2 + k(9k^2+12k+2)w_2L^2}{24H}$$

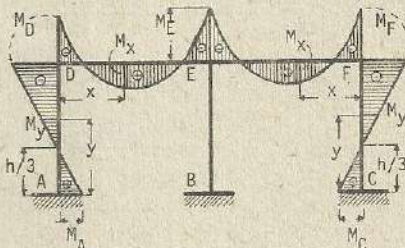
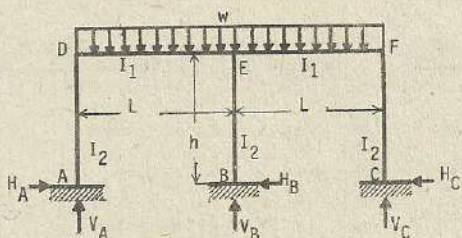
Momentos por flexión en las columnas, AD $\rightarrow M_y = M_A - H_A y$

BE $\rightarrow M_y = M_B - H_B y$; CF $\rightarrow M_y = M_C - H_C y$

Momentos por flexión en las vigas, DE $\rightarrow M_x = M_A - H_A h + V_A x - \frac{w_1 x^2}{2}$

FE $\rightarrow M_x = M_C - H_C h + V_C x - \frac{w_2 x^2}{2}$

En caso donde $w_1 = w_2 = w$



$$H_A = H_C = \frac{wL^2}{8(k+1)h} ; H_B = 0 ; V_A = V_C = \frac{(3k+4)wL}{8(k+1)} ; V_B = \frac{(5k+4)wL}{4(k+1)}$$

$$M_A = M_C = \frac{wL^2}{24(k+1)} ; M_B = 0 ; M_D = M_F = \frac{-wL^2}{12(k+1)} ; M_{ED} = M_{EF} = -\frac{(3k+2)wL^2}{24(k+1)}$$

$$M_{EB} = 0$$

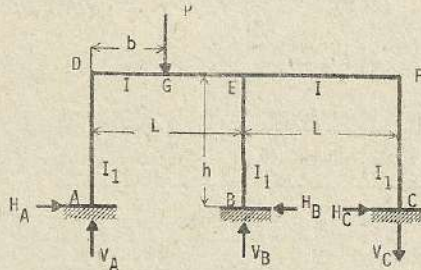
Esfuerzos de momentos por flexión en las columnas

$$AD \text{ y } CF \rightarrow M_y = M_A - H_A y = M_C - H_C y$$

$$\text{momentos en las vigas : } DE \text{ y } FE \rightarrow M_x = M_A - H_A h - V_{Ax} - \frac{wx^2}{2}$$

Ubicación del punto donde se genera el máximo momento de doblado o flexión

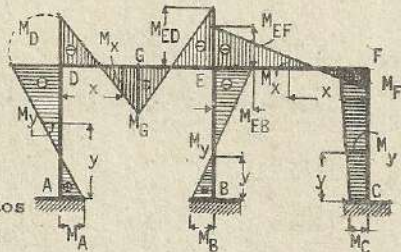
$$x_m = \frac{(5k+4)}{8(k+1)} \cdot L \rightarrow M_{m\acute{a}x} = \frac{(27k^2+40k+6)}{384(k+1)^2} \cdot wL^2$$



(459)

$$k = \frac{I_h}{I_1 L}$$

otros datos
ver (457)



$$H_A = Pb \left(\frac{b}{L} - 1 \right) \frac{\left[\frac{b}{L} (9k^2+13k+1) - (15k^2+20k+2) \right]}{2Hh}; \quad H_B = -Pb \left(\frac{b}{L} - 1 \right) \frac{\left(\frac{b}{L} + 12k+1 \right)}{2Th}$$

$$H_C = -Pb \left(\frac{b}{L} - 1 \right) \frac{\left[\frac{b}{L} (9k^2+14k+2) - (3k^2+7k+1) \right]}{2Hh}$$

$$V_A = P \left[\frac{3}{2} + \frac{b \left[\frac{b^2}{L^2} (3k^3+18k^2+22k+4) - 3 \frac{b}{L} (7k^2+11k+2) - 3k (5k^2+9k+3) \right]}{2HL} \right]$$

$$V_B = -Pb \frac{\left[\frac{b^2}{L^2} (k+4) - 6 \frac{b}{L} - 3k \right]}{2(k+1)L}; \quad V_C = -Pb \left(\frac{b}{L} - 1 \right) \frac{3 \left[\frac{b}{L} (k^3+5k^2+5k) + (k^3 - 2k) \right]}{2HL}$$

$$M_A = Pb \frac{\left[\frac{b^2}{L^2} (2k^2+2k-1) - \frac{b}{L} (6k^2+7k-1) + (4k^2 + 5) \right]}{2H}$$

$$M_B = -Pb \left(\frac{b}{L} - 1 \right) \frac{\left[-\frac{b}{L} (k+1) + (5k+1) \right]}{2T}$$

$$M_C = Pb \left(\frac{b}{L} - 1 \right) \frac{\left[\frac{b}{L} (4k^2+7k+2) - (2k^2+4k+1) \right]}{2H}$$

$$M_D = \frac{Pb \left[-\frac{b^2}{L^2} (7k^2+11k+2) + \frac{b}{L} (18k^2+26k+4) - (11k^2+15k+2) \right]}{2H}$$

$$M_{ED} = Pb \left(\frac{b}{L} - 1 \right) (k+2) \frac{\left[\frac{b}{L} (3k^2+5k+1) + k (3k+2) \right]}{2H}; \quad M_{EB} = Pb \left(\frac{b}{L} - 1 \right) \frac{\left[\frac{b}{L} (k+2) + 7k \right]}{2T}$$

$$M_{EF} = Pb \left(\frac{b}{L} - 1 \right) \frac{\left[\frac{b}{L} (k+2) (3k+4)k + k (3k^2+k-3) \right]}{2H}$$

$$M_F = -Pb \left(\frac{b}{L} - 1 \right) \frac{\left[\frac{b}{L} (5k^2 + 7k) - (k^2 + 3k) \right]}{2H}$$

$$M_G = \frac{Pb \left[\frac{b^3}{L^3} (3k^3+18k^2 + 22k+4) - 4 \frac{b^2}{L^2} (7k^2 + 11k+2) - \frac{b}{L} (15k^3+9k^2 - 17k-4) + k(12k^2+19k+5) \right]}{2H}$$

Momentos en las columnas, $AD \rightarrow M_y = M_A - H_A y$; $BE \rightarrow M_y = M_B - H_B y$

$CF \rightarrow M_y = M_C + H_C y$

Momentos en las vigas : DE $\longrightarrow M_x = M_D + V_A x \quad x \leq b$
 $M_x = M_D + V_A x - P(x-b) \quad x \geq b$
 EF $\longrightarrow M_x = M_F - V_C x$

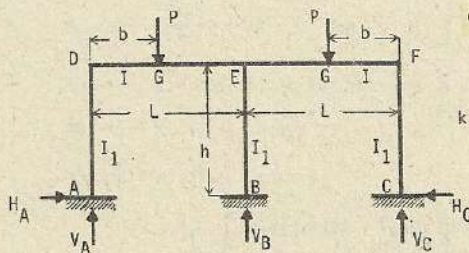
Si $b = \frac{L}{2}$; $H_A = PL \frac{(21k^2+27k+3)}{16Hh}$; $H_B = PL \frac{(24k+3)}{16Th}$; $H_C = PL \frac{3k^2}{16Hh}$

$V_A = P \frac{(39k^3+108k^2+80k+8)}{16H}$; $V_B = P \frac{(11k+8)}{16(k+1)}$; $V_C = P \frac{3(3k^3+5k^2+k)}{16H}$

$M_A = PL \frac{(6k^2+8k+1)}{16H}$; $M_B = PL \frac{(9k+1)}{16T}$; $M_C = PL \frac{k}{16H}$; $M_D = -PL \frac{(15k^2+19k+2)}{16H}$

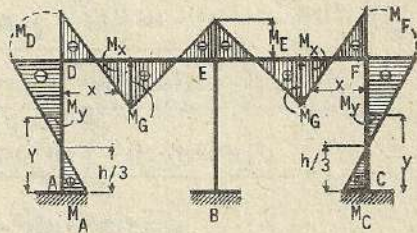
$M_{ED} = -PL \frac{(k+2)(9k^2+9k+1)}{16H}$; $M_{EF} = -PL \frac{k(9k^2+12k+2)}{16H}$; $M_{EB} = -PL \frac{(15k+2)}{16T}$

$M_F = PL \frac{k(3k+1)}{16H}$; $M_G = PL \frac{39k^3+78k^2+42k+4}{32H}$



(460)

$$k = \frac{I_h}{I_1 L}$$



$H_A = H_C = 3Pb \cdot \frac{(\frac{b}{L} - 1)^2}{2(k+1)h}$; $H_B = 0$; $V_A = V_C = P \left[-\frac{b \left[\frac{b^2}{L^2} (k+4) - 6 \frac{b}{L} - 3k \right]}{2(k+1)L} \right]$

$V_B = -Pb \frac{\left[\frac{b^2}{L^2} (k+4) - 6 \frac{b}{L} - 3k \right]}{(k+1)L}$; $M_A = M_C = Pb \cdot \frac{(\frac{b}{L} - 1)^2}{2(k+1)}$; $M_B = 0$

$M_D = M_F = -Pb \cdot \frac{(\frac{b}{L} - 1)^2}{(k+1)}$; $M_E = Pb \left(\frac{b}{L} - 1 \right) \cdot \frac{\left[\frac{b}{L} (k+2) + k \right]}{2(k+1)}$

Si $b = \frac{L}{2}$ $H_A = H_C = \frac{3}{16(k+1)} \cdot P \cdot \frac{L}{h}$; $H_B = 0$; $V_A = V_C = \frac{(5k+8)}{16(k+1)} \cdot P$

$V_B = \frac{(11k+8)}{8(k+1)} \cdot P$; $M_A = M_C = \frac{1}{16(k+1)} \cdot PL$; $M_B = 0$; $M_D = M_F = -\frac{1}{8(k+1)} PL$

$M_E = -\frac{(3k+2)}{16(k+1)} \cdot PL$; $M_G = \frac{(5k+4)}{32(k+1)} \cdot PL$

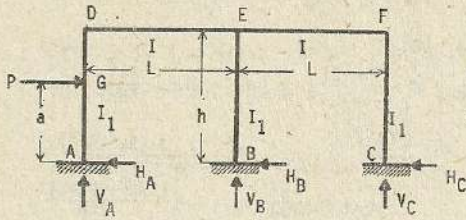
Esfuerzos de momentos por flexión o doblado en las columnas :

AD y CF $\longrightarrow M_y = M_A - H_A x$

Momentos en las vigas : DE y EF $\longrightarrow M_x = M_A - H_A h + V_A x \quad x \leq b$

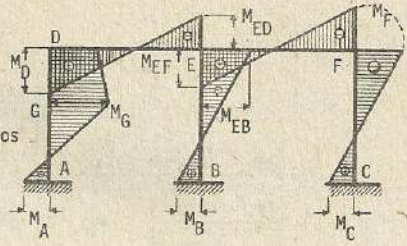
$M_x = M_A - H_A h + V_A x - P(x-b) \quad x \geq b$

(461)



$$K = \frac{I h}{I_1 L}$$

Otros datos
(457)



$$H_A = P \left[1 - \frac{a^2 \left[(72k^3 + 144k^2 + 69k + 6) - \frac{a}{h} (48k^3 + 81k^2 + 29k + 2) \right]}{6h^2H} \right]$$

$$H_B = P \frac{a^2 \left[3(12k^2 + 12k + 1) - \frac{a}{h} (24k^2 + 12k + 1) \right]}{6h^2H}$$

$$H_C = P \frac{a^2 \left[(36k^3 + 72k^2 + 30k + 3) - \frac{a}{h} (24k^3 + 45k^2 + 13k + 1) \right]}{6h^2H}$$

$$V_A = P \frac{a^2 \left[-6k^3 - 6k^2 + 3k + \frac{a}{h} (9k^3 + 13k^2 + k) \right]}{2hLH}$$

$$V_B = P \frac{a^2 \left[-(18k^2 + 27k - 9)k + \frac{a}{h} (18k^2 + 27k + 3)k \right]}{2hLH}$$

$$V_C = P \frac{a^2 \left[3(4k^2 + 7k + 2)k - \frac{a}{h} (9k^2 + 14k + 2)k \right]}{2hLH}$$

$$M_A = -P \left[1 - \frac{a^2 \left[3(18k^3 + 41k^2 + 24k + 2) - \frac{a}{h} (24k^3 + 48k^2 + 23k + 2) \right]}{6hH} \right]$$

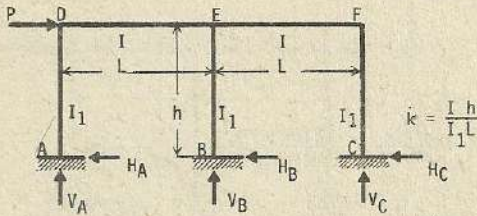
$$M_B = P \frac{a^2 \left[3(6k^2 + 7k + 1) - \frac{a}{h} (12k^2 + 9k + 1) \right]}{6hH} ; M_D = P \frac{a^2 \left[-6k^3 - 7k^2 + k + \frac{a}{h} (8k^3 + 11k^2 + k) \right]}{2hH}$$

$$M_C = P \frac{a^2 \left[(18k^3 + 39k^2 + 21k + 3) - \frac{a}{h} (12k^3 + 24k^2 + 10k + 1) \right]}{6hH}$$

$$M_{ED} = -P \frac{a^2(k+2) \left(k + k \frac{a}{h} \right) k}{2hH} ; M_{EF} = P \frac{a^2k \left[(6k^2 + 10k + 3) - \frac{a}{h} (5k^2 + 7k + 1) \right]}{2hH}$$

$$M_{EB} = -P \frac{a^2k \left[(6k + 5) - \frac{a}{h} (4k + 1) \right]}{2hH} ; M_{FF} = -P \frac{a^2k \left[(6k^2 + 11k + 3) - \frac{a}{h} (4k^2 + 7k + 1) \right]}{2hH}$$

(462)



$$(6k^2 + 9k + 1) = T$$

$$H_A = \frac{(12k^2 + 15k + 2)}{6T} \cdot P ; \quad H_B = \frac{(6k^2 + 12k + 1)}{3T} \cdot P ; \quad H_C = \frac{(12k^2 + 15k + 2)}{6T} \cdot P$$

$$V_A = -\frac{(3k+4)k}{2T} \cdot \frac{Ph}{L} ; \quad V_B = 0 ; \quad V_C = \frac{(3k+4)}{2T} \cdot \frac{Ph}{L} ; \quad M_A = -\frac{(6k^2 + 9k + 2)}{6T} \cdot Ph$$

$$M_B = \frac{(3k^2 + 6k + 1)}{3T} \cdot Ph ; \quad M_C = \frac{(6k^2 + 9k + 2)}{6T} \cdot Ph ; \quad M_D = \frac{(k+1)k}{T} \cdot Ph$$

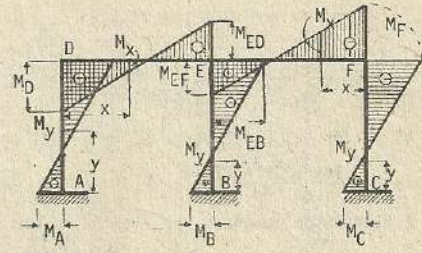
$$M_{ED} = -\frac{k(k+2)}{2T} \cdot Ph ; \quad M_{EF} = \frac{(k+2)k}{2T} \cdot Ph ; \quad M_{EB} = -\frac{(k+2)k}{T} \cdot Ph ; \quad M_F = -\frac{(k+1)k}{T} \cdot Ph$$

Momentos por doblado en las columnas, AD $\rightarrow M_y = M_A + H_A y$

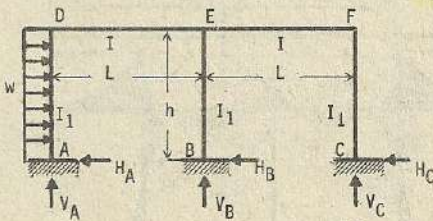
BE $\rightarrow M_y = M_B - H_B y$; CF $\rightarrow M_y = M_C - H_C y$

Momentos en las vigas : DE $\rightarrow M_x = M_A + H_A h + V_A x$

FE $\rightarrow M_x = M_C - H_C h + V_C x$



(463)



$$k = \frac{I \cdot h}{I_1 \cdot L}$$

; otros datos (457)

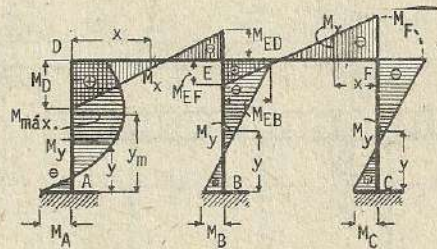
$$H_A = \frac{32k^3 + 83k^2 + 58k + 6}{8H} \cdot wh ; \quad H_B = \frac{8k^3 + 20k^2 + 13k + 1}{8H} \cdot wh ; \quad H_C = \frac{8k^3 + 17k^2 + 9k + 1}{8H} \cdot wh$$

$$V_A = -\frac{(k^2 + 5k + 5)}{8H} \cdot \frac{wh^2}{L} ; \quad V_B = \frac{(6k^2 + 9k + 1)k}{8H} \cdot \frac{wh^2}{L} = \frac{k}{8(k+1)} \cdot \frac{wh^2}{L}$$

$$V_C = \frac{(7k^2 + 14k + 6)k}{8H} \cdot \frac{wh^2}{L} ; \quad M_A = -\frac{24k^3 + 64k^2 + 47k + 6}{24H} \cdot wh^2 ; \quad M_B = \frac{12k^2 + 19k + 3}{24T} \cdot wh^2$$

$$M_C = \frac{12k^3 + 28k^2 + 18k + 3}{24H} \cdot wh^2 ; \quad M_D = \frac{(5k+7)k}{24H} \cdot wh^2 ; \quad M_{ED} = \frac{(k+2)(3k+4)k}{24H} \cdot wh^2$$

$$M_{EF} = \frac{(9k^2 + 19k + 9)k}{24H} \cdot wh^2 ; \quad M_{EB} = \frac{(12k+17)k}{24T} \cdot wh^2 ; \quad M_F = -\frac{(12k^2 + 23k + 9)k}{24H} \cdot wh^2$$



Esfuerzos de momentos por flexión en las columnas,

$$AD \longrightarrow M_y = M_A + H_A y - \frac{w y^2}{2} ; \quad BE \longrightarrow M_y = M_B - H_B y \quad ; \quad CF \longrightarrow M_y = M_C - H_C y$$

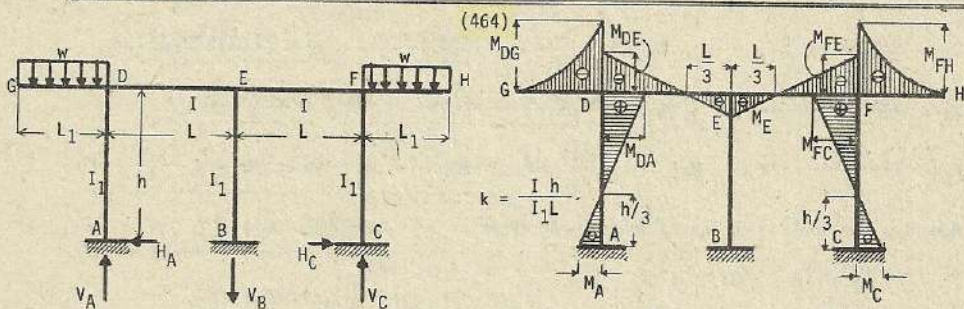
Ubicación del punto donde se genera el momento máximo,

$$y_m = \frac{16k^3 + 27k^2 + 22k + 2}{8H} \cdot h \quad \therefore \quad M_{m\acute{a}x.} = \left[\frac{(5k+7)k}{24H} + \frac{(16k^3 + 27k^2 + 22k + 2)^2}{128(k+1)^2 (6k^2 + 9k + 1)^2} \right] w h^2$$

Esfuerzos de momentos por flexión en las vigas:

$$DE \longrightarrow M_x = M_A + H_A h - \frac{w h^2}{2} + V_A x$$

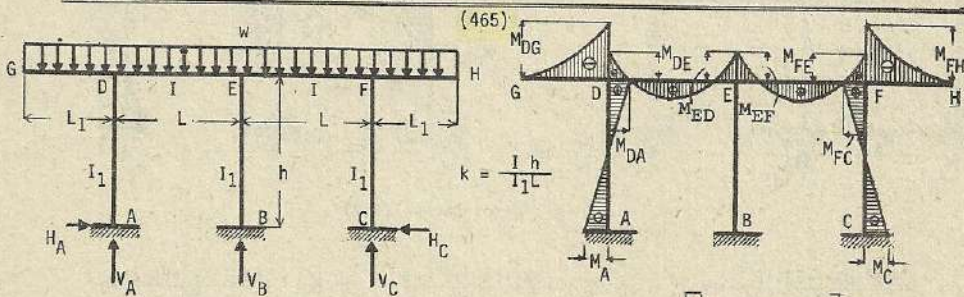
$$FE \longrightarrow M_x = M_C - H_C h + V_C x$$



$$H_A = H_C = \frac{3wL^2}{4(1+k)h} ; \quad H_B = 0 ; \quad V_A = V_C = \frac{\left[\frac{L}{4(1+k)} + 3k \right] wL^2}{4(1+k)L}$$

$$V_B = \frac{3kwL^2}{2(1+k)L} ; \quad M_A = M_C = -\frac{wL^2}{4(1+k)} ; \quad M_B = 0 ; \quad M_{DE} = M_{FE} = -\frac{k}{2(1+k)} wL^2$$

$$M_{DG} = M_{FH} = -\frac{wL^2}{2} ; \quad M_{DA} = M_{FC} = \frac{wL^2}{2(1+k)} ; \quad M_{EF} = M_{ED} = \frac{k}{4(1+k)} wL^2 \quad M_{EB} = 0$$

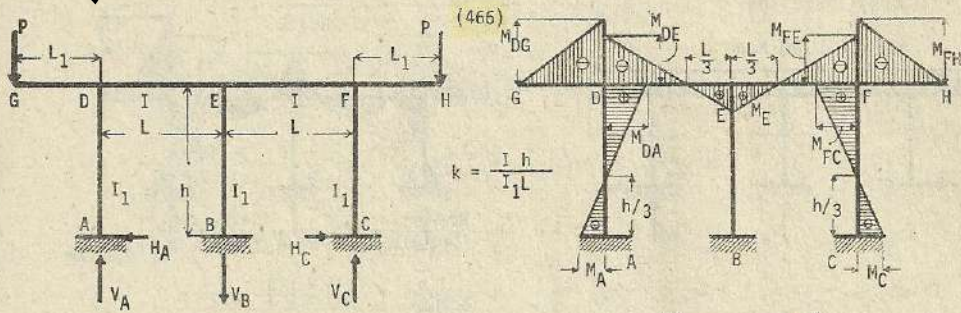


$$H_A = H_C = w \left[\frac{L^2 - 6L^2}{8(1+k)h} \right] ; \quad V_A = V_C = wL \frac{(3k+4)}{8(k+1)} + \frac{\left[\frac{L}{4(1+k)} + 3k \right] wL^2}{4(1+k)L}$$

$$V_B = wL \frac{4+5k}{4(1+k)} - \frac{wL^2}{2(1+k)L} - \frac{3k}{2(1+k)L} ; \quad M_A = M_C = \frac{wL^2}{24(1+k)} - \frac{wL^2}{4(1+k)} ; \quad M_{DG} = M_{FH} = -\frac{wL^2}{2}$$

$$M_{DE} = M_{FE} = -\frac{wL^2}{12(1+k)} - \frac{kwL^2}{2(1+k)} ; \quad M_{DA} = M_{FC} = (M_{DG} - M_{DE}) = (M_{FH} - M_{FE})$$

$$M_E = -\frac{(2+3k)}{24(1+k)} wL^2 + \frac{k}{4(1+k)} wL^2$$



$$H_A = H_C = \frac{3PL_1}{2(1+k)h} \quad ; \quad H_B = 0 \quad ; \quad V_A = V_C = \frac{\left[\frac{L}{L_1}(2+k) + 3k \right]}{2(1+k)L} PL_1$$

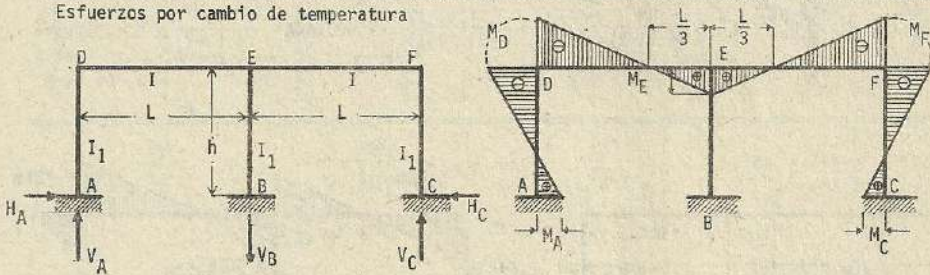
$$V_B = \frac{3kPL_1}{(1+k)L} \quad ; \quad M_A = M_C = -\frac{PL_1}{2(1+k)} \quad ; \quad M_B = 0 \quad ; \quad M_{DE} = M_{FE} = -\frac{KPL_1}{(1+k)}$$

$$M_{DG} = M_{FH} = -PL_1 \quad ; \quad M_{DA} = M_{FC} = \frac{PL_1}{(1+k)} \quad ; \quad M_{ED} = M_{EF} = \frac{KPL_1}{2(1+k)}$$

$$M_{EB} = 0$$

(467)

Esfuerzos por cambio de temperatura



$$k = \frac{I h}{I_1 L} \quad ; \quad H_A = H_C = 3EetI \frac{(4k+1)}{h^2 (k+1)k} \quad ; \quad H_B = 0 \quad ; \quad V_A = V_C = 9EetI \frac{1}{hL(k+1)}$$

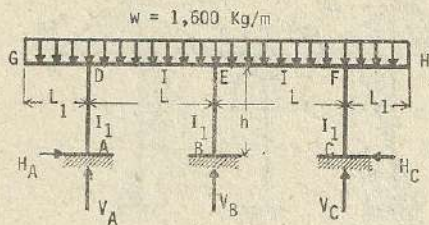
$$V_B = 18EetI \frac{1}{hL(k+1)}$$

$$M_A = M_C = 3EetI \frac{(2k+1)}{h(k+1)k} \quad ; \quad M_B = 0 \quad ; \quad M_D = M_F = -6EetI \frac{1}{h(k+1)}$$

$$M_E = 3EetI \frac{1}{h(k+1)}$$

Nota : En caso de descenso de temperatura, los signos son opuestos a las presentadas.

Problemas numéricos



$$\text{Datos : } L_1 = 2\text{m. ; } L = 4\text{m; } h = 3\text{m.}$$

$$I = 0.4 \text{ m}^4 ; \quad I_1 = 0.3 \text{ m}^4$$

$$H_A = H_C = w \left[\frac{L^2}{8(1+k)h} - \frac{6L^2}{8} \right] = 1,600 \times \frac{4 \times 4 - 6 \times 2 \times 2}{8(1+1) \times 3} = -266 \text{ Kg}$$

$$V_A = V_C = w \frac{L(3k+4)}{8(k+1)} + \frac{\left[\frac{1}{L_1} (4k+4) + 3k \right]}{4(1+k)L} w L^2 = 1,600 \times \frac{4 \times (3 \times 1 + 4)}{8(1+1)} + \frac{\left[\frac{4}{2} (4 \times 1 + 4) + 3 \times 1 \right]}{4(1+1) \times 4} \times 1,600 \times 2 \times 2 = 6,600 \text{ Kg.}$$

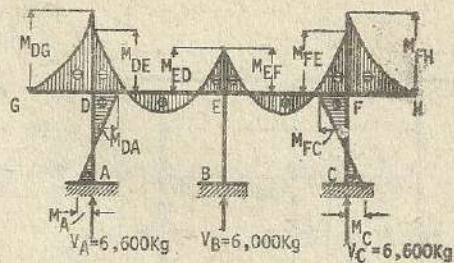
$$V_B = w \frac{(4+5k)}{4(1+k)} - w L \frac{3k}{2(1+k)L} = 1,600 \times 4 \frac{(4+5 \times 1)}{4(1+1)} - 1,600 \times 2 \times 2 \frac{3 \times 1}{2(1+1) \times 4} = 6,000 \text{ Kg.}$$

$$M_A = M_C = \frac{w L^2}{24(1+k)} - \frac{w L^2}{4(1+k)} = \frac{1,600 \times 4 \times 4}{24(1+1)} - \frac{1,600 \times 2 \times 2}{4(1+1)} = -266 \text{ Kg-m}$$

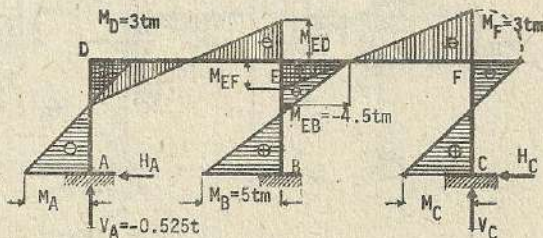
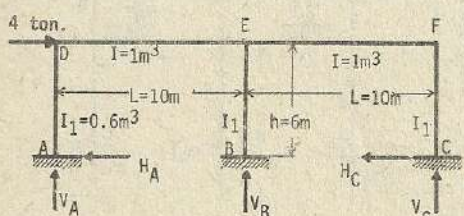
$$M_{DG} = M_{FH} = -\frac{w L^2}{2} = -\frac{1,600 \times 2 \times 2}{2} = -3,200 \text{ Kg-m ; } M_{DE} = M_{FE} = -\frac{w L^2}{12(1+k)} - \frac{k w L^2}{2(1+k)}$$

$$M_{DA} = M_{FC} = 3,200 - 2,666 = 534 \text{ Kg-m} = -\frac{1,600 \times 4 \times 4}{12(1+1)} - \frac{1 \times 1,600 \times 2 \times 2}{2(1+1)} = -2,666 \text{ Kg-m}$$

$$M_E = -\frac{(2+3k)}{24(1+k)} w L^2 + \frac{k}{4(1+k)} w L^2 = -\frac{(2+3 \times 1)}{24(1+1)} \times 1,600 \times 4 \times 4 + \frac{1}{4(1+1)} \times 1,600 \times 2 \times 2 = -1,866 \text{ Kg-m}$$



$$k = \frac{I h}{I_1 L} = \frac{0.4 \times 3}{0.3 \times 4} = 1$$



$$k = \frac{I h}{I_1 L} = \frac{1 \times 6}{0.6 \times 10} = 1 ; \quad H_A = \frac{(12k^2 + 15k + 2)}{6(6k^2 + 9k + 1)} P = \frac{(12 + 15 + 2)}{6(6 + 9 + 1)} \times 4 = 1.208 \text{ t.}$$

$$H_B = \frac{(6k^2 + 12k + 1)}{3(6k^2 + 9k + 1)} \cdot P = \frac{(6 + 12 + 1)}{3(6 + 9 + 1)} \times 4 = 1.583 \text{ t.}$$

$$H_C = \frac{(12k^2 + 15k + 2)}{6(6k^2 + 9k + 1)} \cdot P = \frac{(12 + 15 + 2)}{6(6 + 9 + 1)} \times 4 = 1.208 \text{ t.}$$

$$V_A = \frac{(3k+4)}{2(6k^2+9k+1)} \cdot \frac{Ph}{L} = -\frac{(3+4)}{2(6+9+1)} \cdot \frac{4 \times 6}{10} = -0.525 \text{ t.}$$

$$V_B = 0 ; \quad V_C = \frac{(3k+4)k}{2(6k^2+9k+1)} \cdot \frac{Ph}{L} = \frac{(3+4)}{2(6+9+1)} \times \frac{4 \times 6}{10} = 0.525 \text{ t.}$$

$$M_A = - \frac{(6k^2+9k+2)}{6(6k^2+9k+1)} Ph = - \frac{(6+9+2)}{6(6+9+1)} \times 4 \times 6 = -4.25 \text{ tm}$$

$$M_B = \frac{(3k^2+6k+1)}{3(6k^2+9k+1)} Ph = \frac{(3+6+1)}{3(6+9+1)} \times 4 \times 6 = 5 \text{ tm.}$$

$$M_C = \frac{(6k^2+9k+2)}{6(6k^2+9k+1)} Ph = \frac{(6+9+2)}{6(6+9+1)} \times 4 \times 6 = 4.25 \text{ tm}$$

$$M_D = \frac{(k+1)k}{(6k^2+9k+1)} Ph = \frac{(1+1) \times 1}{(6+9+1)} \times 4 \times 6 = 3 \text{ tm}$$

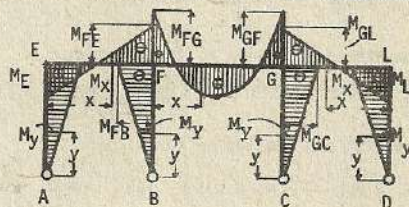
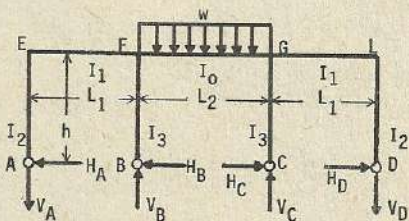
$$M_{ED} = - \frac{(k+2)k}{2(6k^2+9k+1)} Ph = - \frac{(1+2)}{2(6+9+1)} \times 4 \times 6 = -2.25 \text{ tm.}$$

$$M_{EF} = \frac{(k+2)k}{2(6k^2+9k+1)} Ph = \frac{(1+2)}{2(6+9+1)} \times 4 \times 6 = 2.25 \text{ tm.}$$

$$M_{EB} = - \frac{(k+2)k}{(6k^2+9k+1)} Ph = - \frac{(1+2)}{(6+9+1)} \times 4 \times 6 = -4.5 \text{ tm.}$$

$$M_F = - \frac{(k+1)k}{(6k^2+9k+1)} Ph = - \frac{(1+1)}{(6+9+1)} \times 4 \times 6 = -3 \text{ tm}$$

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$$k_1 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\left[9 + 12k_1 + 12k_2 + 6\alpha k_1 + 2k_1 k_2 (6+4\alpha) \right] = J$$

$$H_A = H_D = \frac{k_1 w L_2^2}{2Jh} ; \quad H_B = H_C = \frac{(3+4k_2) w L_2^2}{4Jh} ; \quad V_A = V_D = \frac{(3+2k_1) k_1 w L_2^2}{2JL_1}$$

$$V_B = V_C = \frac{\left[J + k_1 \frac{L_2}{L_1} (3+k_2) \right] w L_2}{2J}$$

$$M_E = M_L = \frac{k_1 w L_2^2}{2J}$$

$$M_{FE} = M_{GL} = - \frac{k_1 (1+k_2) w L_2^2}{J}$$

$$M_{FG} = M_{GF} = - \frac{\left[3 + 4(k_1 + k_2 + k_1 k_2) \right] w L_2^2}{4J}$$

$$M_{FB} = M_{GC} = - \frac{(3+4k_2) w L_2^2}{4J}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$; $\left[9 + 12k^2 + 24k + 2\alpha k (3+4k) \right] = U$

$$H_A = H_D = \frac{k w L_2^2}{2U h} ; \quad H_B = H_C = \frac{(4k+3) w L_2^2}{2U h} ; \quad V_A = V_D = \frac{k (3+2k) w L_2^2}{2U L_1}$$

$$V_B = V_C = \frac{\left[U + k \frac{L_2}{L_1} (3+k) \right] wL_2}{2U} ; \quad M_E = M_L = \frac{kwL_2^2}{2U} ; \quad M_{FE} = M_{GL} = - \frac{k(k+1) wL_2^2}{U}$$

$$M_{FG} = M_{BF} = - \frac{(4k^2+8k+3) wL_2^2}{4U} ; \quad M_{FB} = M_{GC} = - \frac{(4k+3) wL_2^2}{4U}$$

Si $L_1 = L_2 = L$, entonces $I_0 = I_1$ $(20k_1k_2+12k_2+18k_1+9) = V$

$$H_A = H_D = \frac{k_1wL^2}{2Vh} ; \quad H_B = H_C = \frac{(3+4k_2) wL^2}{4Vh} ; \quad V_A = V_D = \frac{k_1(3+2k_2)wL}{2V}$$

$$V_B = V_C = \frac{(22k_1k_2+12k_2+21k_1+9)wL}{2V} ; \quad M_E = M_L = \frac{k_1wL^2}{2V}$$

$$M_{FE} = M_{GL} = - \frac{k_1(1+k_2) wL^2}{V} ; \quad M_{FG} = M_{GF} = - \frac{\left[3 + 4(k_1+k_2+k_1k_2) \right] wL^2}{4V}$$

$$M_{FB} = M_{GC} = - \frac{(3+4k_2)wL^2}{4V}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$ $(20k^2+30k+9) = W$

$$H_A = H_D = \frac{kwL^2}{2Wh} ; \quad H_B = H_C = \frac{(4k+3)wL^2}{4Wh} ; \quad V_A = V_D = \frac{k(2k+3)wL}{2W}$$

$$V_B = V_C = \frac{(22k^2+33k+9)wL}{2W} ; \quad M_E = M_L = \frac{kwL^2}{2W} ; \quad M_{FE} = M_{GL} = - \frac{k(k+1)wL^2}{W}$$

$$M_{FG} = M_{BF} = - \frac{(4k^2+8k+3)wL^2}{4W} ; \quad M_{FB} = M_{GC} = - \frac{(4k+3)wL^2}{4W}$$

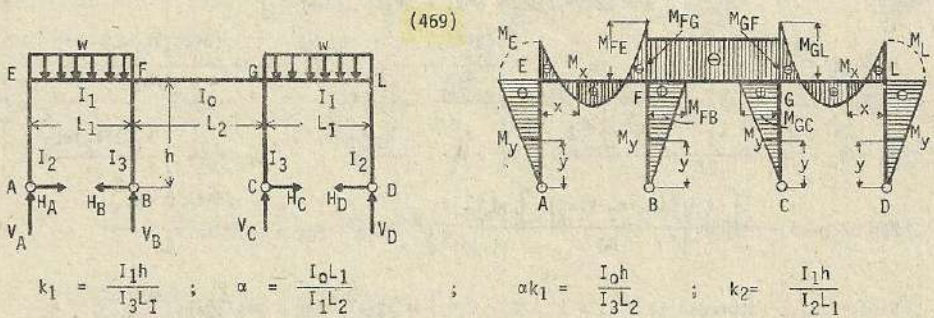
Momentos por flexión en las columnas :

AE y DL $\longrightarrow M_y = H_A y = H_D y$; BF y CG $\longrightarrow M_y = - H_B y = - H_C y$

Momentos por flexión en las vigas :

EF y GL $\longrightarrow M_x = H_A h - V_A x = H_D h - V_D x$

FG $\longrightarrow M_x = H_A h - H_B h - V_A (L_1+x) + V_B x - \frac{wx^2}{2}$



$$k_1 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

Otros datos (468)

$$H_A = H_D = \frac{(3+6k_1+2\alpha k_1)wL_1^2}{4Jh} ; \quad H_B = H_C = \frac{3(1+k_2)wL_1^2}{4Jh}$$

$$V_A = V_D = \frac{\left[9 + 15k_1 + 9k_2 + 6\alpha k_1 + 6k_1 k_2 (2+\alpha) \right] wL_1}{2J}$$

$$V_B = V_C = \frac{[9 + 9k_1 + 15k_2 + 6\alpha k_1 + 2k_1 k_2 (6+5\alpha)] wL_1}{2J} ; M_E = M_L = - \frac{(3+6k_1+2\alpha k_1)wL_1^2}{4J}$$

$$M_{FE} = M_{GL} = - \frac{(3+6k_2 + 2\alpha k_1 + 4\alpha k_1 k_2)wL_1^2}{4J} ; M_{FG} = M_{GF} = - \frac{(3k_2 + 2\alpha k_1 + 4\alpha k_1 k_2)wL_1^2}{4J}$$

$$M_{FB} = M_{GC} = \frac{3(1+k_2)wL_1^2}{4J}$$

$$\text{Si } I_2 = I_3, \text{ entonces } k_1 = k_2 = k ; H_A = H_D = \frac{(3+6k+2\alpha k)wL_1^2}{4U_h}$$

$$H_B = H_C = \frac{3(1+k)wL_1^2}{4U_h} ; V_A = V_D = \frac{[6(2+\alpha)k^2 + 6\alpha k + 24k + 9] wL_1}{2U}$$

$$V_B = V_C = \frac{[2(6+5\alpha)k^2 + 6\alpha k + 24k + 9] wL_1}{2U} ; M_E = M_L = \frac{(3+6k+2\alpha k) \cdot wL_1^2}{4U}$$

$$M_{FE} = M_{GL} = - \frac{(3+6k+2\alpha k+4\alpha k^2)wL_1^2}{4U} ; M_{FG} = M_{GF} = - \frac{(3+2\alpha+4\alpha k)kwL_1^2}{4U}$$

$$M_{FB} = M_{GC} = \frac{3(1+k)wL_1^2}{4U}$$

$$\text{Si } L_1 = L_2 = L, I_0 = I_1 ; H_A = H_D = \frac{(3+8k_1)wL^2}{4Vh} ; H_B = H_C = \frac{3(1+k_2)wL^2}{4Vh}$$

$$V_A = V_D = \frac{(18k_1 k_2 + 9k_2 + 21k_1 + 9) wL}{2V} ; V_B = V_C = \frac{(22k_1 k_2 + 15k_1 + 15k_2 + 9) wL}{2V}$$

$$M_E = M_L = - \frac{(3+8k_1)wL^2}{4V} ; M_{FE} = M_{GL} = - \frac{(3+6k_2+2k_1+4k_1 k_2) wL^2}{4V}$$

$$M_{FG} = M_{GF} = - \frac{(3k_2+2k_1+4k_1 k_2)wL^2}{4V} ; M_{FB} = M_{GC} = \frac{3(1+k_2) wL^2}{4V}$$

$$\text{Si } L_1 = L_2 = L, I_2 = I_3 ; H_A = H_D = \frac{(3+8k)wL^2}{4Wh} ; H_B = H_C = \frac{3(1+k)wL^2}{4Wh}$$

$$V_A = V_D = \frac{3(6k^2+10k+3)wL}{2W} ; V_B = V_C = \frac{(22k^2+30k+9)wL}{2W} ; M_E = M_L = - \frac{(8k+3)wL^2}{4W}$$

$$M_{FE} = M_{GL} = - \frac{(4k^2+8k+3)wL^2}{4W} ; M_{FG} = M_{GF} = \frac{(4k^2+5k)wL^2}{4W} ; M_{FB} = M_{GC} = \frac{3(1+k)wL^2}{4W}$$

Momentos por flexión en las columnas :

$$AE \text{ y } DL \longrightarrow M_y = -H_A y = -H_D y$$

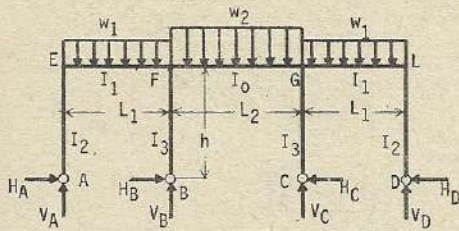
$$BF \text{ y } CG \longrightarrow M_y = H_B y = H_C y$$

Momentos por flexión en las vigas :

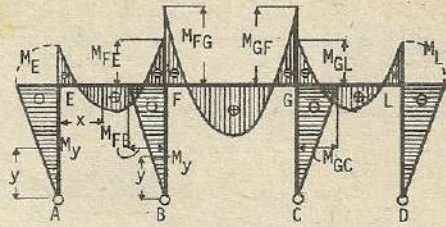
$$EF \text{ y } GL \longrightarrow M_x = -H_A h + V_A x - \frac{wx^2}{2}$$

$$= -H_D h + V_D x - \frac{wx^2}{2}$$

$$FG \longrightarrow M_x = M_{FG} = M_{GF} \quad (\text{constante})$$



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$$k_1 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{I_0 h_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \text{otros datos (468)}$$

$$H_A = H_D = \frac{w_1 L_1^2 (3+6k_1+2\alpha k_1) - 2w_2 L_2^2 k_1}{4hJ} ; \quad H_B = H_C = \frac{w_2 L_2^2 (3+4k_2) - 3w_1 L_1^2 (1+2k_2)}{4hJ}$$

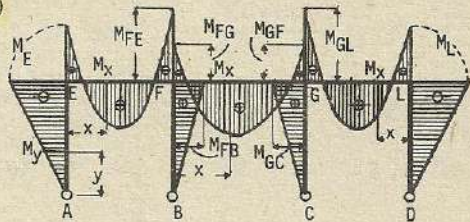
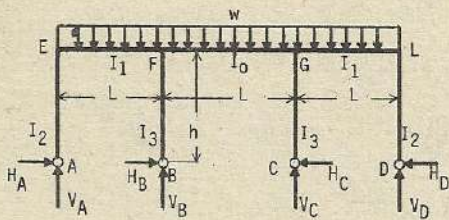
$$V_A = V_D = \frac{[9+9k_2+15k_1+6\alpha k_1+6k_1 k_2 (2+\alpha)] w_1 L_1^2 - k_1 (3+2k_2) w_2 L_2^2}{2JL_1}$$

$$V_B = V_C = \frac{w_1 L_1 [9+15k_2+9k_1+6k_1 \alpha + 2k_1 k_2 (6+5\alpha)]}{2J} + \frac{w_2 L_2 [J + k_1 \frac{L_2}{L_1} (3+k_2)]}{2J}$$

$$M_E = M_L = - \frac{w_1 L_1^2 (3+6k_1+2\alpha k_1) - 2w_2 L_2^2 k_1}{4J} ; \quad M_{FB} = M_{GC} = - \frac{w_2 L_2^2 (3+4k_2) - 3w_1 L_1^2 (1+2k_2)}{4hJ}$$

$$M_{FE} = M_{GL} = M_E + V_A L_1 - w_1 \frac{L_1^2}{2} ; \quad M_{FG} = M_{GF} = M_E + V_A L_1 - w_1 \frac{L_1^2}{2} + M_{FB}$$

(471)



$$k_1 = \frac{I_2 h}{I_3 L} ; \quad k_2 = \frac{I_1 h}{I_2 L} ; \quad \text{otros datos (468)}$$

$$H_A = H_D = \frac{(6k_1+3) \cdot wL^2}{4V} ; \quad H_B = H_C = \frac{k_2}{2V} \cdot \frac{wL^2}{h} ; \quad V_A = V_D = \frac{(9k_2+18k_1+16k_1 k_2+9) wL}{2V}$$

$$V_B = V_C = \frac{(27k_2+36k_1+44k_1 k_2+18)wL}{2V} ; \quad M_E = M_L = \frac{(6k_1+3)wL^2}{4V} ; \quad M_{FB} = M_{GC} = \frac{k_2}{2V} wL^2$$

$$M_{FE} = M_{GL} = - \frac{6(k_2+k_1) + 8k_1 k_2 + 3}{4V} \cdot wL^2 ; \quad M_{FG} = M_{GF} = - \frac{4k_2+6k_1+8k_1 k_2+3}{4V} wL^2$$

$$\text{Si } I_2 = I_3, \quad k_1 = k_2 = k$$

$$H_A = H_D = \frac{(6k+3) \cdot wL^2}{4Wh} ; \quad H_B = H_C = \frac{k}{2Wh} \cdot \frac{wL^2}{h} ; \quad V_A = V_C = \frac{(16k^2+27k+9)}{2W} \cdot wL$$

$$V_B = V_C = \frac{44k^2+63k+18}{2W} \cdot wL ; \quad M_E = M_L = - \frac{(6k+3)wL^2}{4W} ; \quad M_{FB} = M_{GC} = \frac{k}{2W} \cdot wL^2$$

$$M_{FE} = M_{GL} = - \frac{(8k^2+12k+3)}{4W} \cdot wL^2 ; \quad M_{FG} = M_{GF} = - \frac{(8k^2+10k+3)}{4W} \cdot wL^2$$

Esfuerzos de momentos por flexión en las columnas

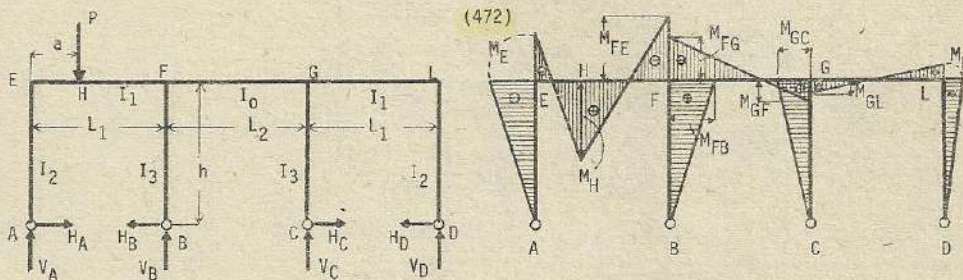
$$AE \text{ y } DL \longrightarrow M_y = -H_A y = -H_D y \quad ; \quad BF \text{ y } CG \longrightarrow M_y = H_B y = H_C y$$

Momentos en las vigas :

$$EF \text{ y } LG \longrightarrow M_x = -H_A h + V_A x - \frac{wx^2}{2}$$

$$= -H_A h + V_D x - \frac{wx^2}{2}$$

$$FG \longrightarrow M_x = -H_A h + H_D h + V_A (L+x) + V_B x - w \frac{(L+x)^2}{2}$$



$$k_1 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{I_0 - I_1}{I_1 L_2} \quad ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \quad ; \quad m = \left(1 - \frac{a}{L_1}\right)$$

$$n = 2(1+2\alpha)(k_1 + k_2) + 3(2+\alpha) \quad ; \quad \lambda_1 = \left(1 - \frac{a}{L_1}\right)(3+2\alpha k_1 + 2k_1) + 2k_1$$

$$\lambda_2 = 9 + 12k_1 + 12k_2 + 6\alpha k_1 + 2k_1 k_2 (6+4\alpha)$$

$$H_A = \frac{3Pam}{2h} \left[\frac{\lambda_1}{\lambda_2} + \frac{1+\alpha m}{n} \right] \quad ; \quad H_B = \frac{Pam}{2h} \left[\frac{3(3+2k_2) \frac{a}{L_1} + 6k_2}{\lambda_2} + \frac{3(1+\alpha m)}{n} \right]$$

$$H_C = \frac{Pam}{2h} \left[\frac{3(3+2k_2) \frac{a}{L_1} + 6k_2}{\lambda_2} - \frac{3(1+\alpha m)}{n} \right]$$

$$H_D = \frac{3Pam}{2h} \left[\frac{\lambda_1}{\lambda_2} - \frac{1+\alpha m}{n} \right] \quad ; \quad M_E = -\frac{3Pam}{2} \left[\frac{\lambda_1}{\lambda_2} + \frac{1+\alpha m}{n} \right]$$

$$M_{FE} = -\frac{Pam}{2} \left[\frac{1}{\lambda_2} (3+\alpha k_1) \left[3 \frac{a}{L_1} + 2k_2 \left(1 + \frac{a}{L_1}\right) \right] + \frac{1}{n} \left[3 \left(1 + \frac{\alpha a}{L_1}\right) + 2\alpha \left(1 + \frac{a}{L_1}\right) (k_1 + k_2) \right] \right]$$

$$M_{FG} = -\frac{P\alpha m}{2} \left[\frac{1}{\lambda_2} 2k_1 \left[(3+2k_1) \frac{a}{L_1} + 2k_1 \right] + \frac{1}{n} \left[2(3+k_1+k_2) \frac{a}{L_1} + 2(k_1+k_2) - 3 \right] \right]$$

$$M_{FB} = \frac{Pam}{2} \left[\frac{1}{\lambda_2} \left[3(3+2k_2) \frac{a}{L_1} + 6k_1 \right] + \frac{1}{n} \left[3(1+\alpha m) \right] \right]$$

$$M_{GF} = \frac{P\alpha m}{2} \left[\frac{1}{n} \left[2(3+k_1+k_2) \frac{a}{L_1} + 2(k_1+k_2) - 3 \right] - \frac{1}{\lambda_2} \left[2k_1(3+2k_2) \frac{a}{L_1} + 2k_2 \right] \right]$$

$$M_{GL} = \frac{Pam}{2} \left[-\frac{1}{n} \left[3 \left(1 + \alpha \frac{a}{L_1}\right) + 2\alpha \left(1 + \frac{a}{L_1}\right) (k_1 + k_2) \right] - \frac{1}{\lambda_2} \left[(3+2\alpha k_1) \left(3 \frac{a}{L_1} + 2k_2 \left(1 + \frac{a}{L_1}\right) \right) \right] \right]$$

$$M_{GC} = \frac{Pam}{2} \left[\frac{1}{\lambda_2} \left[3(3+2k_2) \frac{a}{L_1} + 6k_2 \right] - \frac{1}{n} \left[3(1+\alpha m) \right] \right]$$

$$M_L = -\frac{3Pam}{2} \left[\frac{1}{\lambda_2} \left[m(3+2\alpha k_1+2k_1) + 2k_1 \right] + 2k_2 \right] - \frac{1}{n} (1+\alpha m); \quad V_A = Pm + \frac{M_{FE} - M_E}{L_1}$$

$$V_B = P \frac{a}{L_1} + \frac{M_E - M_{FE}}{L_1} + \frac{M_{GF} - M_{FG}}{L_2}; \quad V_C = \frac{M_{GF} - M_{FG}}{L_2} + \frac{M_{GL} - M_L}{L_1}; \quad V_D = \frac{M_{GL} - M_L}{L}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I_1$, o sea $k_1 = k_2 = k$, $\alpha = 1$

$$H_A = \frac{3Pa \left(1 - \frac{a}{L}\right)}{2h} \left[\frac{3(1+2k) - (3+4k) \frac{a}{L}}{(20k^2+30k+9)} + \frac{2 - \frac{a}{L}}{3(4k+3)} \right]$$

$$H_B = \frac{Pa \left(1 - \frac{a}{L}\right)}{2h} \left[\frac{3(3+2) \frac{a}{L} + 6k}{(20k^2+30k+9)} + \frac{\left(2 - \frac{a}{L}\right)}{(4k+3)} \right]$$

$$H_C = \frac{Pa \left(1 - \frac{a}{L}\right)}{2h} \left[\frac{3(3+2k) \frac{a}{L} + 6k}{(20k^2+30k+9)} - \frac{\left(2 - \frac{a}{L}\right)}{(4k+3)} \right]$$

$$H_D = \frac{3Pa \left(1 - \frac{a}{L}\right)}{2h} \left[\frac{3(1+2k) - (3+4k) \frac{a}{L}}{(20k^2+30k+9)} - \frac{\left(2 - \frac{a}{L}\right)}{3(4k+3)} \right]$$

$$M_E = -\frac{3Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{3(1+2k) - (3+4k) \frac{a}{L}}{(20k^2+30k+9)} + \frac{\left(2 - \frac{a}{L}\right)}{3(4k+3)} \right]$$

$$M_{FE} = -\frac{Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{\frac{a}{L} (3+2k)^2 + 2k(3+2k)}{(20k^2+30k+9)} + \frac{1 + \frac{a}{L}}{3} \right]$$

$$M_{FG} = -\frac{Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{2k \left[(3+2k) \frac{a}{L} + 2k \right]}{(20k^2+30k+9)} + \frac{(6+4k) \frac{a}{L} + 4k - 3}{3(4k+3)} \right]$$

$$M_{FB} = \frac{Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{3(3+2k) \cdot \frac{a}{L} + 6k}{(20k^2+30k+9)} + \frac{2 - \frac{a}{L}}{4k+3} \right]$$

$$M_{GF} = \frac{Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{(6+4k) \frac{a}{L} + 4k - 3}{3(4k+3)} - \frac{2k \left[(3+2k) \frac{a}{L} + 2k \right]}{(20k^2+30k+9)} \right]$$

$$M_{GL} = \frac{Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{1 + \frac{a}{L}}{3} - \frac{\frac{a}{L} (3+2k)^2 + 2k(3+2k)}{(20k^2+30k+9)} \right]$$

$$M_{GC} = \frac{Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{3(3+2k) \frac{a}{L} + 6k}{(20k^2+30k+9)} - \frac{2 - \frac{a}{L}}{4k+3} \right]$$

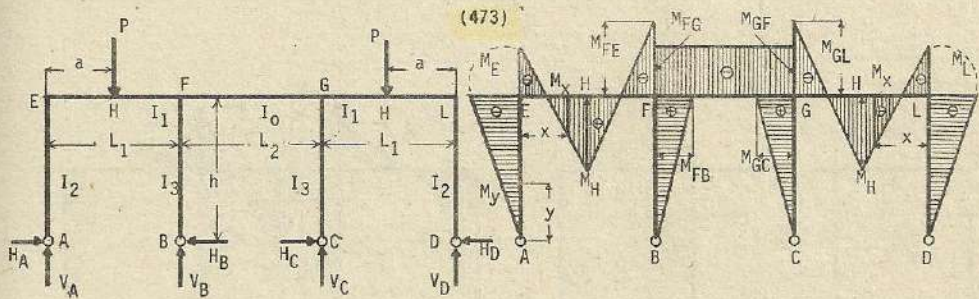
$$M_L = -\frac{3Pa \left(1 - \frac{a}{L}\right)}{2} \left[\frac{3(1+2k) - \frac{a}{L} (3+4k)}{(20k^2+30k+9)} - \frac{2 - \frac{a}{L}}{3(4k+3)} \right]$$

$$V_A = P \left(1 - \frac{a}{L}\right) + \frac{Pa \left(1 - \frac{a}{L}\right)}{2L} \left[\frac{-\frac{a}{L} (4k^2+24k+18) - (4k^2-12k-9)}{(20k^2+30k+9)} + \frac{9 - \left(1 + \frac{a}{L}\right) (4k+6)}{3(4k+3)} \right]$$

$$V_B = P \frac{a}{L} + \frac{Pa \left(1 - \frac{a}{L}\right)}{2L} \left[\frac{\frac{a}{L} (4k^2 + 24k + 18) + (4k^2 - 12k - 9)}{20k^2 + 30k + 9} + \frac{(4k+6) \left(1 + 3 \frac{a}{L}\right) + 8k - 15}{3(4k+3)} \right]$$

$$V_C = \frac{Pa \left(1 - \frac{a}{L}\right)}{2L} \left[\frac{-\frac{a}{L} (4k^2 + 24k + 18) - (4k^2 - 12k - 9)}{20k^2 + 30k + 9} + \frac{(4k+6) \left(1 + 3 \frac{a}{L}\right) + 8k - 15}{3(4k+3)} \right]$$

$$V_D = \frac{Pa \left(1 - \frac{a}{L}\right)}{2L} \left[\frac{-\frac{a}{L} (4k^2 + 24k + 18) - (4k^2 - 12k - 9)}{(20k^2 + 30k + 9)} + \frac{\left(1 + \frac{a}{L}\right) (4k+6) - 9}{3(4k+3)} \right]$$



$$k_1 = \frac{I_1 h}{I_3 L_1}; \quad \alpha = \frac{I_0 L_1}{I_1 L_2}; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2}; \quad k_2 = \frac{I_1 h}{I_2 L_1}; \quad \text{otros datos (468)}$$

$$H_A = H_D = \frac{3Pa \left(1 - \frac{a}{L_1}\right)}{h} \left[\frac{\left(1 - \frac{a}{L_1}\right) (3 + 2\alpha k_1 + 2k_1) + 2k_1}{J} \right]$$

$$H_B = H_C = \frac{3Pa \left(1 - \frac{a}{L_1}\right)}{h} \left[\frac{(3 + 2k_2) \frac{a}{L_1} + 2k_2}{J} \right]$$

$$M_E = M_L = -3Pa \left(1 - \frac{a}{L_1}\right) \left[\frac{\left(1 - \frac{a}{L_1}\right) (3 + 2\alpha k_1 + 2k_1) + 2k_1}{J} \right]$$

$$M_{FE} = M_{GL} = -Pa \left(1 - \frac{a}{L_1}\right) \left[\frac{(3 + 2\alpha k_1) \left[\frac{3a^2}{L_1} + 2k_2 \left(1 + \frac{a}{L_1}\right) \right]}{J} \right]$$

$$M_{FG} = M_F = -Pa \left(1 - \frac{a}{L_1}\right) \left[\frac{2k_1 \left[(3 + 2k_2) \frac{a}{L_1} + 2k_2 \right]}{J} \right]; \quad M_{FB} = M_{GC} = 3Pa \left(1 - \frac{a}{L_1}\right) \left[\frac{(3 + 2k_2) \frac{a}{L_1} + 2k_2}{J} \right]$$

$$V_B = V_C = P \frac{a}{L_1} - \frac{Pa \left(1 - \frac{a}{L_1}\right)}{L_1} \left[\frac{-\frac{a}{L_1} \left[18 + 12\alpha k_1 + 6(k_1 + k_2) + 4\alpha k_1 k_2 \right] + (9 + 12k_1 + 6\alpha k_1 - 6k_2 - 4\alpha k_1 k_2)}{J} \right]$$

$$\text{Si } L_1 = L_2 = L, \quad I_0 = I_1, \quad I_2 = I_3, \quad k_1 = k_2 = k, \quad \alpha = 1$$

$$H_A = H_D = \frac{3Pa \left(1 - \frac{a}{L}\right) \left[3(1+2k) - (3+4k) \frac{a}{L} \right]}{hW}; \quad H_B = H_C = \frac{3Pa \left(1 - \frac{a}{L}\right) \left[(3+2k) \frac{a}{L} + 2k \right]}{hW}$$

$$M_E = M_L = -3Pa \left(1 - \frac{a}{L}\right) \frac{\left[3(1+2k) - (3+4k) \frac{a}{L} \right]}{W}; \quad M_{FE} = M_{GL} = -Pa \left(1 - \frac{a}{L}\right) \frac{\left[\frac{a}{L} (3+2k)^2 + 2k(3+2k) \right]}{W}$$

$$M_{FG} = M_{GF} = -Pa \left(1 - \frac{a}{L}\right) \frac{2k \left[(3+2k) \cdot \frac{a}{L} + 2k \right]}{W}; \quad M_{FB} = M_{GC} = 3Pa \left(1 - \frac{a}{L}\right) \frac{\left[(3+2k) \cdot \frac{a}{L} + 2k \right]}{W}$$

Esfuerzos de momentos por flexión en las columnas

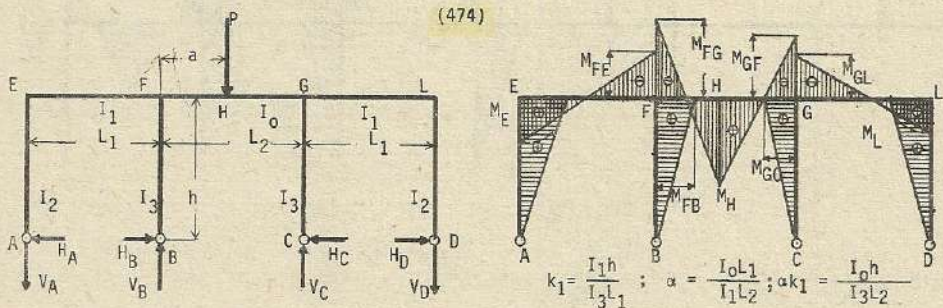
$$AE \text{ y } DL \longrightarrow M_y = H_{Ay} = -H_{Dy}$$

$$BF \text{ y } CG \longrightarrow M_y = H_{By} = H_{Cy}$$

$$\text{Momentos en las vigas: } EF \text{ y } LG \longrightarrow M_x = -H_A h + V_A x \quad x \leq a$$

$$M_x = -H_A h + V_A x - P(x-a) \quad x \geq a$$

$$FG \longrightarrow M_x = -Pa \left(1 - \frac{a}{L}\right) \frac{2k \left[(3+2k) \cdot \frac{a}{L} + 2k \right]}{W}$$



$$H_A = \frac{3Pa \left(1 - \frac{a}{L_2}\right)}{2h} \left[\frac{2k_1}{J} + \frac{1 - 2 \frac{a}{L_2}}{K} \right] \quad \text{otros datos (468)} \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$H_B = \frac{Pa \left(1 - \frac{a}{L_2}\right)}{2h} \left[\frac{3(3+4k_2)}{J} + \frac{3 \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]; \quad H_C = \frac{Pa \left(1 - \frac{a}{L_2}\right)}{2h} \left[\frac{3(3+4k_2)}{J} + \frac{3 \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$H_D = \frac{3Pa \left(1 - \frac{a}{L_2}\right)}{2h} \left[\frac{2k_1}{J} - \frac{1 - 2 \frac{a}{L_2}}{K} \right]; \quad M_E = \frac{3}{2} Pa \left(1 - \frac{a}{L_2}\right) \left[\frac{2k_1}{J} + \frac{1 - 2 \frac{a}{L_2}}{K} \right]$$

$$M_{FE} = -\frac{Pa \left(1 - \frac{a}{L_2}\right)}{2} \left[\frac{12k_1 (1+2k_2)}{J} + \frac{\left[3 + 2(k_1+k_2)\right] \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$M_{FG} = -\frac{Pa \left(1 - \frac{a}{L_2}\right)}{2} \left[\frac{3(3+4k_1+4k_1 k_2+4k_2)}{J} - \frac{2(3+k_1+k_2) \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$M_{FB} = -\frac{Pa \left(1 - \frac{a}{L_2}\right)}{2} \left[\frac{3(3+4k_2)}{J} + \frac{3 \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$M_{GL} = -\frac{Pa \left(1 - \frac{a}{L_2}\right)}{2} \left[\frac{12k_1 (1+k_2)}{J} - \frac{\left[3 + 2(k_1+k_2)\right] \left(1 + 2 \frac{a}{L_2}\right)}{K} \right]$$

$$K = 2(1+2a)(k_1+k_2) + 3(2+a)$$

$$M_{GF} = - \frac{Pa \left(1 - \frac{a}{L_2}\right)}{2} \left[\frac{3(3 + 4k_1 + 4k_1k_2 + 4k_2)}{J} - \frac{3(3 + k_1 + k_2) \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$M_{GC} = - \frac{Pa \left(1 - \frac{a}{L_2}\right)}{2} \left[\frac{3(3 + 4k_2)}{J} - \frac{3 \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]; \quad M_L = \frac{3}{2} Pa \left(1 - \frac{a}{L_2}\right) \left[\frac{2k_1}{J} - \frac{1 - 2 \frac{a}{L_2}}{K} \right]$$

$$V_A = P \frac{a}{2L_1} \left(1 - \frac{a}{L_2}\right) \left[\frac{6(3k_1 + 2k_1k_2)}{J} + \frac{2(3 + k_1 + k_2) \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$V_B = P \left(1 - \frac{a}{L_2}\right) + P \frac{a}{2} \left(1 - \frac{a}{L_2}\right) \left[\frac{6(3k_1 + 2k_1k_2)}{J} \frac{1}{L_1} + \frac{2(3 + k_1 + k_2) \left(1 - 2 \frac{a}{L_2}\right) \left(\frac{1}{L_1} + \frac{1}{L_2}\right)}{K} \right]$$

$$V_C = P \frac{a}{L_2} + P \frac{a}{2} \left(1 - \frac{a}{L_2}\right) \left[\frac{6(3k_1 + 2k_1k_2)}{J} \frac{1}{L_1} - \frac{2(3 + k_1 + k_2) \left(1 - 2 \frac{a}{L_2}\right) \left(\frac{1}{L_1} + \frac{1}{L_2}\right)}{K} \right]$$

$$V_D = P \frac{a}{2L_1} \left[\frac{6(3k_1 + 2k_1k_2)}{J} - \frac{2(3 + k_1 + k_2) \left(1 - 2 \frac{a}{L_2}\right)}{K} \right]$$

$$\text{Si } a = \frac{L}{2}; \quad H_A = H_D = \frac{3PL_2k_1}{4Jh}; \quad H_B = H_C = \frac{3PL_2(3+4k_2)}{8Jh}; \quad M_E = M_L = \frac{3PL_2k_1}{4J}$$

$$M_{FE} = M_{GL} = - \frac{3PL_2(1+k_2)k_1}{2J}; \quad M_{FG} = M_{GF} = \frac{3PL_2(3+4k_1k_2+4k_2)}{8J}$$

$$M_{FB} = M_{GC} = - \frac{3PL_2(3+4k_2)}{8J}; \quad V_A = V_D = \frac{3PL_2(3k_1+2k_1k_2)}{4JL_1}$$

$$V_B = V_C = \frac{P}{2} + \frac{3PL_2(3k_1+2k_1k_2)}{4JL_1}$$

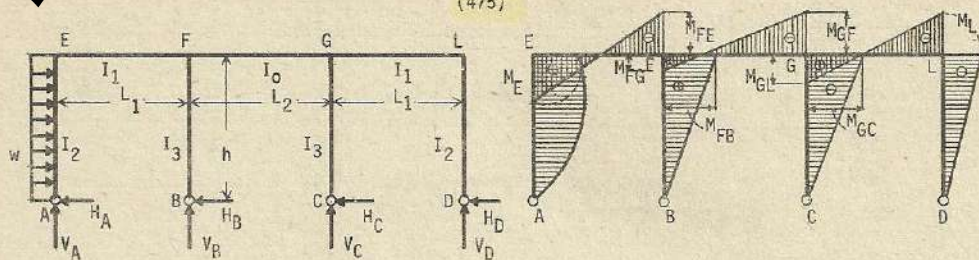
$$\text{Si } L_1 = L_2 = L, \quad k_1 = k_2 = k, \quad a = 1$$

$$H_A = H_D = \frac{3PLk}{4Wh}; \quad H_B = H_C = \frac{3PL(3+4k)}{8Wh}; \quad M_E = M_L = - \frac{3PLk}{4W}$$

$$M_{FE} = M_{GL} = - \frac{3PL(1+k)k}{2W}; \quad M_{FG} = M_{GF} = - \frac{3PL(4k^2+8k+3)}{8W}; \quad M_{FB} = M_{GC} = - \frac{3PL(3+4k)}{8W}$$

$$V_A = V_D = \frac{3P(2k+3)k}{4W}; \quad V_B = V_C = \frac{P}{2} + \frac{3P(2k+3)k}{4W}; \quad M_H = \frac{PL(28k^2+36k+9)}{8W}$$

(475)



$$k_1 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \text{otros datos (468) (474)}$$

$$(8\alpha k_1 k_2 + 12k_1 k_2 + 12k_2 + 12k_1 + 6\alpha k_1 + 9) = X$$

$$H_A = \frac{wh}{2} + \frac{wh}{8} \left[\frac{6 + (2\alpha + 1)(4k_1 - k_2)}{K} - \frac{6k_2 + 2(3+2\alpha)k_1 k_2}{X} \right]$$

$$H_B = \frac{wh}{8} \left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} - \frac{3k_2}{X} \right] ; \quad H_C = \frac{wh}{8} \left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} + \frac{3k_2}{X} \right]$$

$$H_D = \frac{wh}{8} \left[\frac{6 + (2\alpha + 1)(4k_1 - k_2)}{K} + \frac{6k_2 + 2(3+2\alpha)k_1 k_2}{X} \right]$$

$$V_A = \frac{wh^2}{8L_1} \left[\frac{3(3k_2 + 2k_1 k_2 + 2\alpha k_1 k_2)}{X} - \frac{12 + 12k_1 \alpha + 4k_1 + 4k_2 - 3\alpha k_2}{K} \right]$$

$$V_B = \frac{wh^2}{8L_1} \left[\frac{12(1-\alpha \frac{L_1}{L_2}) + k_2(4-3\alpha - 22\alpha \frac{L_1}{L_2}) + 4k_1(1+3\alpha + 2\alpha \frac{L_1}{L_2})}{K} - \frac{9k_2 + 2(3+3\alpha)k_1 k_2}{X} \right]$$

$$V_C = \frac{wh^2}{8I_1} \left[\frac{9k_2 + 2(3+2\alpha)k_1 k_2}{X} - \frac{12(1-\alpha \frac{L_1}{L_2}) + k_2(4-3\alpha - 22\alpha \frac{L_1}{L_2}) + 4k_1(1+3\alpha + 2\alpha \frac{L_1}{L_2})}{K} \right]$$

$$V_D = \frac{wh^2}{8L_1} \left[\frac{12 + 4k_1(1+3\alpha) + k_2(4-3\alpha)}{K} + \frac{9k_2 + 2(3+3\alpha)k_1 k_2}{X} \right]$$

$$M_E = \frac{wh^2}{8} \left[\frac{6 + (2\alpha + 1)(4k_1 - k_2)}{K} - \frac{6k_2 + 2(3+2\alpha)k_1 k_2}{X} \right]$$

$$M_{FE} = \frac{wh^2}{8} \left[\frac{6 + 4\alpha k_1 + k_2(5-\alpha)}{K} - \frac{3k_2 + 2\alpha k_1 k_2}{X} \right] ; \quad M_{FB} = \frac{wh^2}{8} \left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} - \frac{3k_2}{X} \right]$$

$$M_{GF} = -\frac{wh^2}{8} \left[\frac{6\alpha + 11\alpha k_2 - 4\alpha k_1}{K} - \frac{2\alpha k_1 k_2}{X} \right] ; \quad M_{GL} = \frac{wh^2}{8} \left[\frac{6 + 4\alpha k_1 + k_2(5-\alpha)}{K} + \frac{3k_2 + 2k_1 k_2}{X} \right]$$

$$M_{GC} = -\frac{wh^2}{8} \left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} + \frac{3k_2}{X} \right] ; \quad M_{FG} = \left[\frac{6\alpha + 11\alpha k_2 - 4\alpha k_1}{K} + \frac{2\alpha k_1 k_2}{X} \right]$$

$$M_L = -\frac{wh^2}{8} \left[\frac{6 + (2\alpha + 1)(4k_1 - k_2)}{K} + \frac{6k_2 + 2(3+2\alpha)k_1 k_2}{X} \right]$$

$$K = 2(1+2\alpha)(k_1 + k_2) + 3(2+\alpha)$$

$$L_1 = L_2 = L, \quad I_2 = I_3, \quad I_0 = I_1, \quad \text{donde } k_1 = k_2 = k \quad \alpha = 1 \quad ; \quad 20k^2 + 30k + 9 = W$$

$$H_A = \frac{wh}{2} + \frac{wh}{8} \left[\frac{(3k+2)}{(4k+3)} - \frac{10k^2+6k}{W} \right]; \quad H_B = \frac{wh}{8} \left[\frac{(5k+4)}{(4k+3)} - \frac{3k}{W} \right]$$

$$H_C = \frac{wh}{8} \left[\frac{(5k+4)}{(4k+3)} + \frac{3k}{W} \right]; \quad H_D = \frac{wh}{8} \left[\frac{(3k+2)}{(4k+3)} + \frac{10k^2+6k}{W} \right]$$

$$V_A = \frac{wh^2}{8L} \left[\frac{3(4k^2+3k)}{W} - \frac{(17k+12)}{3(4k+3)} \right]; \quad V_B = \frac{wh^2}{8L} \left[\frac{k}{(4k+3)} - \frac{3(4k^2+3k)}{W} \right]$$

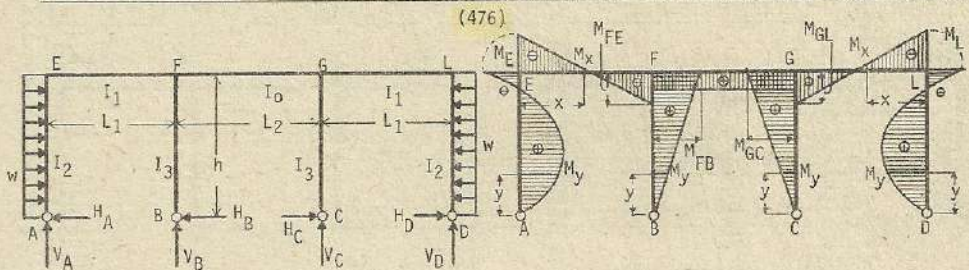
$$V_C = \frac{wh^2}{8L} \left[\frac{3(4k^2+3k)}{W} - \frac{k}{(4k+3)} \right]; \quad V_D = \frac{wh^2}{8L} \left[\frac{(17k+12)}{3(4k+3)} + \frac{3(4k^2+3k)}{W} \right]$$

$$M_E = \frac{wh^2}{8} \left[\frac{(3k+2)}{(4k+3)} - \frac{10k^2+6k}{W} \right]; \quad M_{FE} = \frac{wh^2}{8} \left[\frac{(8k+6)}{3(4k+3)} - \frac{(2k^2+3k)}{W} \right]$$

$$M_{FG} = \frac{wh^2}{8} \left[\frac{(7k+6)}{3(4k+3)} + \frac{2k^2}{W} \right]; \quad M_{FB} = \frac{wh^2}{8} \left[\frac{(5k+4)}{(4k+3)} - \frac{3k}{W} \right]; \quad M_{GF} = \frac{wh^2}{8} \left[\frac{(7k+6)}{3(4k+3)} - \frac{2k^2}{W} \right]$$

$$M_{GL} = \frac{wh^2}{8} \left[\frac{(8k+6)}{3(4k+3)} + \frac{(2k^2+3k)}{W} \right]; \quad M_{GC} = \frac{wh^2}{8} \left[\frac{(5k+4)}{(4k+3)} + \frac{3k}{W} \right]$$

$$M_L = -\frac{wh^2}{8} \left[\frac{(3k+2)}{(4k+3)} + \frac{(10k^2+6k)}{W} \right]$$



$$k_1 = \frac{I_1 h}{I_3 L_1}; \quad \alpha = \frac{I_0 L_1}{I_1 L_2}; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2}; \quad k_2 = \frac{I_1 h}{I_2 L_1}; \quad \text{otros datos (475)}$$

$$H_A = H_D = \frac{3(2k_1 k_2 \alpha + 3k_1 k_2 + 3k_2 + 2\alpha k_1 + 4k_1 + 3)}{2X} \cdot wh; \quad H_B = H_C = -\frac{3k_2}{4X} \cdot wh$$

$$V_A = V_D = \frac{3(2\alpha k_1 k_2 + 2k_1 k_2 + 3k_2)}{4X} \cdot \frac{wh^2}{L_1}; \quad V_B = V_C = -\frac{3(2\alpha k_1 k_2 + 2k_1 k_2 + 3k_2)}{4X} \cdot \frac{wh^2}{L_1}$$

$$M_E = M_L = -\frac{(2\alpha k_1 k_2 + 3k_1 k_2 + 3k_2)}{2X} \cdot wh^2; \quad M_{FE} = M_{GL} = \frac{2\alpha k_1 k_2 + 3k_2}{4X} \cdot wh^2$$

$$M_{FG} = M_{GF} = \frac{\alpha k_1 k_2}{2X} \cdot wh^2; \quad M_{FB} = M_{GC} = -\frac{3k_2}{4X} \cdot wh^2$$

Esfuerzos de momentos por doblado en las columnas :

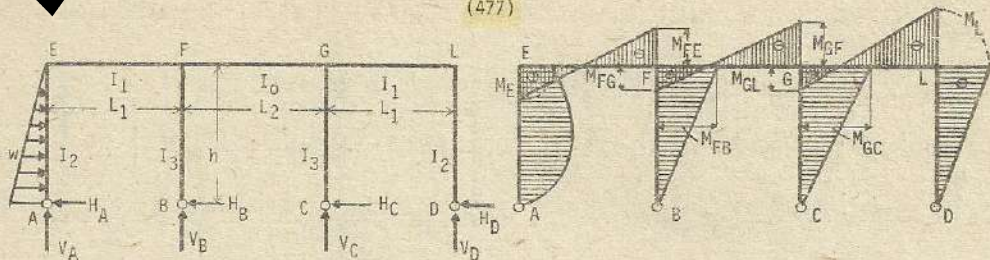
$$AE \text{ y } DL \longrightarrow M_y = H_A y - \frac{w y^2}{2}; \quad BF \text{ y } CG \longrightarrow M_y = H_B y$$

$$\text{Momentos en las vigas : } EF \text{ y } LG \longrightarrow M_x = H_A h - \frac{wh^2}{2} + V_A x$$

$$= H_D h - \frac{wh^2}{2} + V_D x$$

$$FG : \quad M = \frac{\alpha k_1 k_2}{2X} \cdot wh^2 \quad (\text{constante})$$

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$$k_1 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \text{Otros datos (468)}$$

(474)

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$$H_A = \frac{2}{3} w + \frac{w}{60} \left[\frac{30 + (2\alpha + 1)(20k_1 - 7k_2)}{K} - \frac{14k_2(3 + 3k_1 + 2\alpha k_1)}{X} \right]$$

$$H_B = \frac{w}{60} \left[\frac{30(1 + \alpha) + 27(1 + 2\alpha)k_2}{K} - \frac{21k_2}{X} \right] ; \quad H_C = \frac{w}{60} \left[\frac{30(1 + \alpha) + 27(1 + 2\alpha)k_2}{K} + \frac{21k_2}{X} \right]$$

$$H_D = \frac{w}{60} \left[\frac{30 + (2\alpha + 1)(20k_1 - 7k_2)}{K} + \frac{14k_2(3 + 3k_1 + 2\alpha k_1)}{X} \right]$$

$$V_A = -\frac{wh}{60L_1} \left[\frac{60(1 + \alpha k_1) + 20(k_1 + k_2) - 21\alpha k_2}{K} - \frac{7k_2 \{9 + 6k_1(1 + \alpha)\}}{X} \right]$$

$$V_B = \frac{wh}{60L_1} \left\{ \frac{60(1 + \alpha k_1) + 20(k_1 + k_2) - 21\alpha k_2 - \left[\frac{20(3 - 2k_1)\alpha + 122\alpha k_2}{K} \right] \frac{L_1}{L_2}}{K} - \frac{7k_2 \{9 + 6k_1(1 + \alpha)\}}{X} \right\}$$

$$V_C = -\frac{wh}{60L_1} \left\{ \frac{60(1 + \alpha k_1) + 20(k_1 + k_2) - 21\alpha k_2 - \left[\frac{20(3 - 2k_1)\alpha + 122\alpha k_2}{K} \right] \frac{L_1}{L_2}}{K} + \frac{7k_2 \{9 + 6k_1(1 + \alpha)\}}{X} \right\}$$

$$V_D = \frac{wh}{60L_1} \left[\frac{60(1 + \alpha k_1) + 20(k_1 + k_2) - 21\alpha k_2}{K} + \frac{7k_2 \{9 + 6k_1(1 + \alpha)\}}{X} \right]$$

$$M_E = \frac{wh}{60} \left[\frac{30 + (2\alpha + 1)(20k_1 - 7k_2)}{K} - \frac{14k_2(3 - 3k_2 - 2\alpha k_1)}{X} \right]$$

$$M_{FE} = -\frac{wh}{60} \left[\frac{10(3 + 2k_2) + 20\alpha k_1 + 7k_2(1 - \alpha)}{K} - \frac{7k_2(3 + 2\alpha k_1)}{X} \right]$$

$$M_{FG} = \frac{wh}{60} \left[\frac{10(3 - 2k_1)\alpha + 61\alpha k_2}{K} + \frac{14\alpha k_1 k_2}{X} \right] ; \quad M_{GC} = -\frac{wh}{60} \left[\frac{30(1 + \alpha) + 27k_2(1 + \alpha)}{K} + \frac{21k_2}{X} \right]$$

$$M_{GL} = \frac{wh}{60} \left[\frac{10(3 + 2k_2) + 20\alpha k_1 + 7k_2(1 - \alpha)}{K} + \frac{7k_2(3 + 2\alpha k_1)}{X} \right]$$

$$M_L = -\frac{wh}{60} \left[\frac{30 + (2\alpha + 1)(20k_1 - 7k_2)}{K} + \frac{14k_2(3 + 3k_1 + 2\alpha k_1)}{X} \right]$$

$$M_{FB} = \frac{wh}{60} \left[\frac{30(\alpha + 1) + 27k_2(1 + 2\alpha)}{K} - \frac{21k_2}{X} \right] ; \quad M_{GF} = -\frac{wh}{60} \left[\frac{10(3 - 2k_1)\alpha + 61\alpha k_2}{K} - \frac{14\alpha k_1 k_2}{X} \right]$$

$$K = 2(1 + 2\alpha)(k_1 + k_2) + 3(2 + \alpha)$$

$$X = (8\alpha k_1 k_2 + 12k_1 k_2 + 12k_1 + 12k_2 + 6\alpha k_1 + 9)$$

$$w = (20k_2^2 + 30k_2 + 9)$$

$$\text{Si } I_1 = I_2 = L, \quad I_2 = I_3, \quad I_0 = I_1, \text{ entonces } k_1 = k_2 = k \quad \alpha = 1$$

$$H_A = \frac{2}{3}w + \frac{w}{60} \left[\frac{13k+10}{4k+3} - \frac{14k(5k+3)}{W} \right]; \quad H_B = \frac{w}{60} \left[\frac{27k+20}{4k+3} - \frac{21k}{W} \right]$$

$$H_C = \frac{w}{60} \left[\frac{27k+20}{4k+3} + \frac{21k}{W} \right]; \quad H_D = \frac{w}{60} \left[\frac{13k+10}{4k+3} + \frac{14k(5k+3)}{W} \right]$$

$$V_A = -\frac{wh}{60L} \left[\frac{79k+60}{3(4k+3)} - \frac{7k(12k+9)}{W} \right]; \quad V_B = \frac{wh}{60L} \left[-\frac{k}{(4k+3)} - \frac{7k(12k+9)}{W} \right]$$

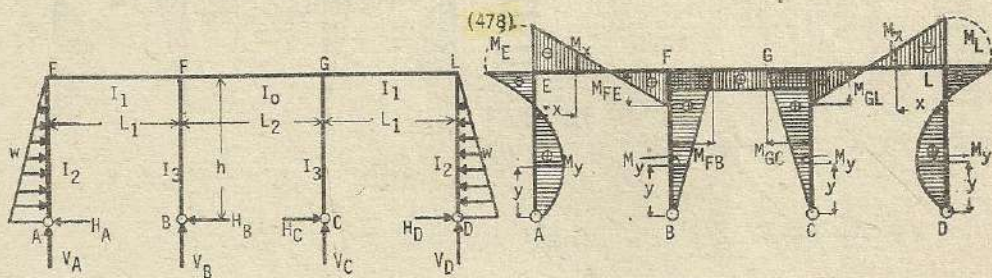
$$V_C = -\frac{wh}{60L} \left[-\frac{k}{(4k+3)} + \frac{7k(12k+9)}{W} \right]; \quad V_D = \frac{wh}{60L} \left[\frac{79k+60}{3(4k+3)} + \frac{7k(12k+9)}{W} \right]$$

$$M_E = \frac{wh}{60} \left[\frac{13k+10}{4k+3} - \frac{14k(5k+3)}{W} \right]; \quad M_{FE} = \frac{wh}{60} \left[\frac{40k+30}{3(4k+3)} - \frac{7k(2k+3)}{W} \right]$$

$$M_{FG} = \frac{wh}{60} \left[\frac{41k+30}{3(4k+3)} + \frac{14k^2}{W} \right]; \quad M_{FB} = \frac{wh}{60} \left[\frac{27k+20}{4k+3} - \frac{21k}{W} \right]$$

$$M_{GF} = \frac{wh}{60} \left[\frac{41k+30}{3(4k+3)} - \frac{14k^2}{W} \right]; \quad M_{GL} = \frac{wh}{60} \left[\frac{40k+30}{3(4k+3)} + \frac{7k(2k+3)}{W} \right]$$

$$M_{GC} = -\frac{wh}{60} \left[\frac{27k+20}{4k+3} + \frac{21k}{W} \right]; \quad M_L = -\frac{wh}{60} \left[\frac{13k+10}{4k+3} + \frac{14k(5k+3)}{W} \right]$$



$$k_1 = \frac{I_1 h}{I_3 L_1}; \quad \alpha = \frac{I_0 L_1}{I_1 L_2}; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2}; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$(8\alpha k_1 k_2 + 12k_1 k_2 + 12k_1 + 12k_2 + 6\alpha k_1 + 9) = X; \quad (20k^2 + 30k + 9) = W$$

$$H_A = H_D = \frac{22\alpha k_1 k_2 + 33k_1 k_2 + 33k_2 + 20\alpha k_1 + 40k_1 + 30}{5X} w; \quad H_B = H_C = -\frac{7k_2}{10X} w$$

$$V_A = V_D = \frac{(14\alpha k_1 + 14k_1 + 21) k_2}{10X} \cdot \frac{wh}{L_1}; \quad V_B = V_C = -\frac{(14\alpha k_1 + 14k_1 + 21) k_2}{10X} \cdot \frac{wh}{L_1}$$

$$M_E = M_L = -\frac{7(2\alpha k_1 + 3k_1 + 3) k_2 \cdot wh}{15X}; \quad M_{FE} = M_{GL} = \frac{7(2\alpha k_1 + 3) k_2 \cdot wh}{30X}$$

$$M_{FG} = M_{GF} = \frac{7\alpha k_1 k_2 \cdot wh}{15X}; \quad M_{FB} = M_{GC} = -\frac{7k_2 \cdot wh}{10X}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I$, donde $k_1 = k_2 = k$, $\alpha = 1$

$$H_A = H_D = \frac{55k^2 + 93k + 30}{5W} w ; \quad H_B = H_C = -\frac{7k}{10W} w ; \quad V_A = V_D = \frac{(28k + 21)k}{10W} \cdot \frac{wh}{L}$$

$$V_B = V_C = -\frac{(28k + 21)k}{10W} \cdot \frac{wh}{L} ; \quad M_E = M_L = -\frac{7(5k + 3)kwh}{15W} ; \quad M_{FE} = M_{GL} = \frac{7(2k + 3)kwh}{30W}$$

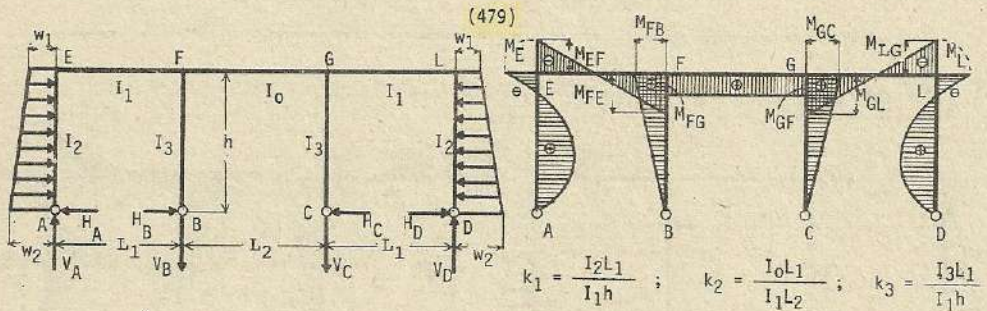
$$M_{FG} = M_{GF} = -\frac{7k^2wh}{15W} ; \quad M_{FB} = M_{GC} = \frac{7kwh}{10W}$$

Momentos por flexión en las columnas :

$$AE \text{ y } DL \longrightarrow M_y = H_A y + w \cdot \frac{y^3}{3h^2} - \frac{wy^2}{h} ; \quad BF \text{ y } CG \longrightarrow M_y = -H_B y$$

$$\text{Momentos en las vigas : } EF \text{ y } LG \longrightarrow M_x = M_E + V_A x$$

$$FG \longrightarrow M_x = -\frac{7k^2wh}{15W} \quad (\text{constante})$$



$$C = \frac{h^2}{120} \cdot (8w_1 + 7w_2) ; \quad D = (4 + 3k_1)(4 + 2k_2 + 3k_3) - 4$$

$$M_E = M_L = -\frac{4(3 + 2k_2 + 3k_3)}{D} \cdot C ; \quad M_{FE} = M_{GL} = \frac{2(2k_2 + 3k_3)}{D} \cdot C ; \quad M_{FG} = M_{GF} = \frac{4k_2 C}{D}$$

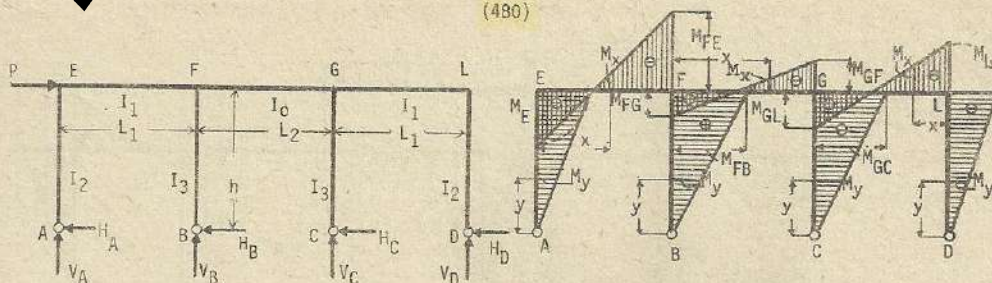
$$M_{FB} = M_{GC} = \frac{6k_3 C}{D} ; \quad \text{Si } L_1 = L_2 = L ; \quad I_0 = I_1, \text{ entonces } k_1 = \frac{I_2 L}{I_1 h} ; \quad k_2 = 1 ; \quad k_3 = \frac{I_3 L}{I_1 h}$$

$$D = 3(4 + 3k_1)(2 + k_3) - 4 ; \quad C = \frac{h^2}{120} (8w_1 + 7w_2)$$

$$M_E = M_L = -\frac{4(5 + 3k_3)}{D} \cdot C ; \quad M_{FE} = M_{GL} = \frac{2(2 + 3k_3)}{D} \cdot C$$

$$M_{FG} = M_{GF} = \frac{4C}{D} ; \quad M_{FB} = M_{GC} = \frac{6k_3 C}{D}$$

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$$k_1 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \text{otros datos (474)}$$

$$H_A = H_D = \frac{P}{2} \cdot \frac{(3+2k_1+4\alpha k_1)}{K} ; \quad H_B = H_C = \frac{P}{2} \cdot \frac{[3(1+\alpha)+2k_2(1+2\alpha)]}{2K}$$

$$V_A = -\frac{Ph}{L_1} \cdot \frac{(3+k_1+k_2+3\alpha k_1)}{K} ; \quad V_B = \frac{Ph [3+k_1+k_2+3\alpha k_1 - (3\alpha+4\alpha k_2-2\alpha k_1) \cdot \frac{L_1}{L_2}]}{KL_1}$$

$$V_C = -\frac{Ph [3+k_1+k_2+3\alpha k_1 - (3\alpha+4\alpha k_2-2\alpha k_1)] \frac{L_1}{L_2}}{KL_1} ; \quad V_D = \frac{Ph (3+k_1+k_2+3\alpha k_1)}{KL_1}$$

$$M_E = \frac{Ph}{2} \cdot \frac{(3+2k_1+4\alpha k_1)}{K} ; \quad M_{FE} = -\frac{Ph}{2} \cdot \frac{(3+2k_2+2\alpha k_1)}{K} ; \quad M_{FG} = \frac{Ph}{2} \cdot \frac{(3\alpha+4\alpha k_2-2\alpha k_1)}{K}$$

$$M_{FB} = \frac{Ph}{2} \cdot \frac{[3(1+\alpha)+2k_2(1+2\alpha)]}{K} ; \quad M_{GF} = -\frac{Ph}{2} \cdot \frac{(3\alpha+4\alpha k_2-2\alpha k_1)}{K}$$

$$M_{GL} = \frac{Ph}{2} \cdot \frac{(3+2k_2+2\alpha k_1)}{K} ; \quad M_{GC} = -\frac{Ph}{2} \cdot \frac{[3(1+\alpha)+2k_2(1+2\alpha)]}{K} ; \quad M_L = -\frac{Ph}{2} \cdot \frac{(3+2k_1+4\alpha k_1)}{K}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I_1$, $k_1 = k_2 = k$, $\alpha = 1$

$$H_A = H_D = \frac{P(2k+1)}{2(4k+3)} ; \quad H_B = H_C = \frac{P(k+1)}{(4k+3)} ; \quad V_A = -\frac{Ph(5k+3)}{3L(4k+3)} ; \quad V_B = \frac{Phk}{L(4k+3)}$$

$$V_C = -\frac{Phk}{L(4k+3)} ; \quad V_D = \frac{Ph(5k+3)}{3L(4k+3)} ; \quad M_E = \frac{Ph(2k+1)}{2(4k+3)} ; \quad M_{FE} = -\frac{Ph(2k+3)}{6(4k+3)}$$

$$M_{FG} = \frac{Ph(2k+3)}{6(4k+3)} ; \quad M_{FB} = \frac{Ph(k+1)}{(4k+3)} ; \quad M_{BF} = -\frac{Ph(2k+3)}{6(4k+3)} ; \quad M_{GC} = -\frac{Ph(k+1)}{(4k+3)}$$

$$M_{GL} = \frac{Ph(2k+3)}{6(4k+3)} ; \quad M_L = -\frac{Ph(2k+1)}{2(4k+3)}$$

Momentos por flexión en las columnas :

$$AE \longrightarrow M_y = H_A x ; \quad BF \longrightarrow M_y = H_B y ; \quad CG \longrightarrow M_y = -H_C y ; \quad DC \longrightarrow M_y = -H_D y$$

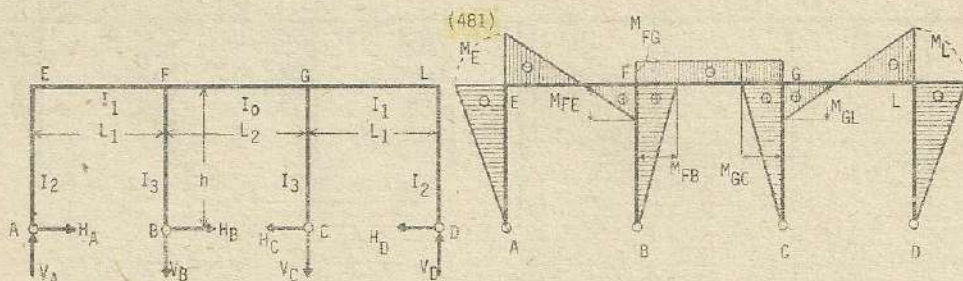
Momentos en las vigas :

$$EF \longrightarrow M_x = H_A h - V_A x$$

$$FG \longrightarrow M_x = (H_A + H_D) h - V_A(L_1+x) + V_B x$$

$$LG \longrightarrow M_x = -H_D h + V_D x$$

Esfuerzos por cambio de temperatura (caso por ascenso de temperatura)



$$k_1 = \frac{I_1 h}{I_2 L_1} ; \quad k_2 = \frac{I_1 h}{I_3 L_1} ; \quad \alpha = \frac{L_2}{L_1} ; \quad \beta = \frac{I_0}{I_1} ; \quad V' = (20k_1 k_2 + 12k_1 + 18k_2 + 9)$$

$$H_A = H_D = \frac{2 \left[2 \frac{\alpha}{\beta} (1+k_2) + \frac{4}{3} k_2 \right] + \alpha \frac{\alpha}{\beta}}{\left(\frac{2}{3} k_1 + 1 \right) \left[2 \frac{\alpha}{\beta} (1+k_2) + \frac{4}{3} k_2 \right] - \left[\frac{2}{3} k_2 + \frac{\alpha}{\beta} \left(\frac{2}{3} k_2 + 1 \right) \right]} \cdot \frac{E \alpha t I_1}{h^2 L_1}$$

$$H_B = H_C = -\frac{1}{h} \left[H_A h \left(1 + \frac{\alpha}{\beta} \right) - V_A L_1 \left(\frac{2}{3} + \frac{\alpha}{\beta} \right) \right]$$

$$V_A = V_B = V_C = V_D = \frac{\frac{4}{3} k_2 + \frac{\alpha}{\beta} \left[\alpha \frac{2}{3} k_1 + 1 \right] + 2 \left(\frac{2}{3} k_2 + 1 \right)}{\left(\frac{2}{3} k_1 + 1 \right) \left[2 \frac{\alpha}{\beta} (1+k_2) + \frac{4}{3} k_2 \right] - \left[\frac{2}{3} k_2 + \frac{\alpha}{\beta} \left(\frac{2}{3} k_2 + 1 \right) \right]} \cdot \frac{E \alpha t I_1}{h L_1}$$

Si $L_1 = L_2 = L$; $I_0 = I_1$, $\alpha = 1$, $\beta = 1$

$$H_A = H_D = \frac{15(4k_2+3)}{V'} \cdot \frac{E \alpha t I_1}{h^2} ; \quad H_B = H_C = \frac{15(2k_1+3)}{V'} \cdot \frac{E \alpha t I_1}{h^2}$$

$$V_A = V_B = V_C = V_D = \frac{9(2k_1+8k_2+9)}{V'} \cdot \frac{E \alpha t I_1}{h L} ; \quad M_E = M_L = -\frac{15(4k_2+3)}{V'} \cdot \frac{E \alpha t I_1}{h}$$

$$M_{FE} = M_{GL} = \frac{6(3k_1+2k_2+6)}{V'} \cdot \frac{E \alpha t I_1}{h} ; \quad M_{FB} = M_{GC} = -\frac{15(2k_1+3)}{V'} \cdot \frac{E \alpha t I_1}{h}$$

$$M_{FG} = M_{GF} = -\frac{12(k_1-k_2)+9}{V'} \cdot \frac{E \alpha t I_1}{h}$$

Si $L_1 = L_2 = L$; $I_0 = I_1$; $I_2 = I_3$, $k_1 = k_2 = k$; $(20k^2+30k+9) = W$

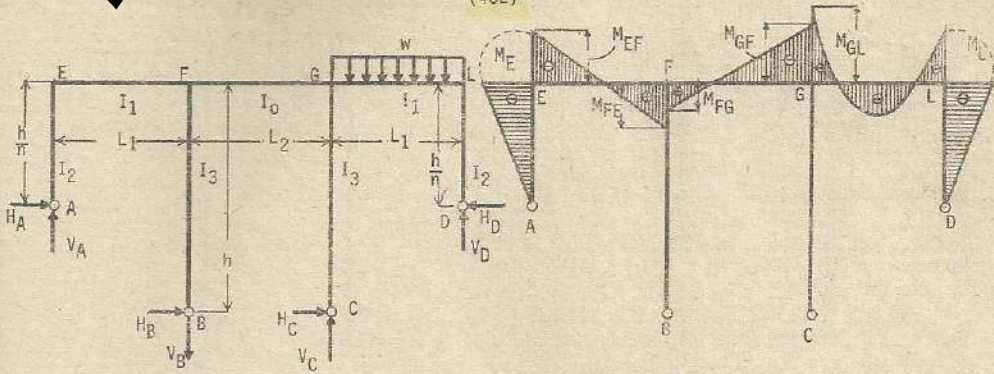
$$H_A = H_D = \frac{15(4k+3)}{W} \cdot \frac{E \alpha t I_1}{h^2} ; \quad H_B = H_C = \frac{15(2k+3)}{W} \cdot \frac{E \alpha t I_1}{h^2}$$

$$V_A = V_B = V_C = V_D = \frac{9(10k+9)}{W} \cdot \frac{E \alpha t I_1}{h L} ; \quad M_E = M_L = -\frac{15(4k+3)}{W} \cdot \frac{E \alpha t I_1}{h}$$

$$M_{FE} = M_{GL} = \frac{6(5k+6)}{W} \cdot \frac{E \alpha t I_1}{h} ; \quad M_{FB} = M_{GC} = -\frac{15(2k+3)}{W} \cdot \frac{E \alpha t I_1}{h} ; \quad M_{FG} = M_{GF} = -\frac{9}{W} \cdot \frac{E \alpha t I_1}{h}$$

Nota : En caso de descenso de temperatura, los signos de los esfuerzos son opuestos a las presentadas.

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$$k_1 = \frac{I_2 L_1^n}{I_1 h} \quad ; \quad k_2 = \frac{I_0 L_1}{I_1 L_2} \quad ; \quad k_3 = \frac{I_3 L_1}{I_1 h} \quad ; \quad k_4 = \frac{I_3}{n I_2}$$

$$\alpha = 6 \left[2(n^2 + k_4)(1 + 2k_2) + k_3(2 + 3k_2) + 2n^2 + 2n \right] \quad ; \quad \beta = (4 + 3k_1)(4 + 2k_2 + 3k_3) - 4$$

$$M_E = \frac{w L_1^2}{8} \left[\frac{6k_3(n+1+k_2)}{\alpha} - \frac{k_1(6+2k_2+3k_3)}{\beta} \right]$$

$$M_{FB} = \frac{w L_1^2}{8} \left[\frac{6k_3(n+1+k_2)}{\alpha} - \frac{3k_3(2+k_1)}{\beta} \right] \quad ; \quad M_{GF} = -\frac{w L_1^2 k_2}{4} \left[3 \left[\frac{2(n^2+k_4) - k_3(n-1)}{\alpha} \right] + \frac{(2+k_1)}{\beta} \right]$$

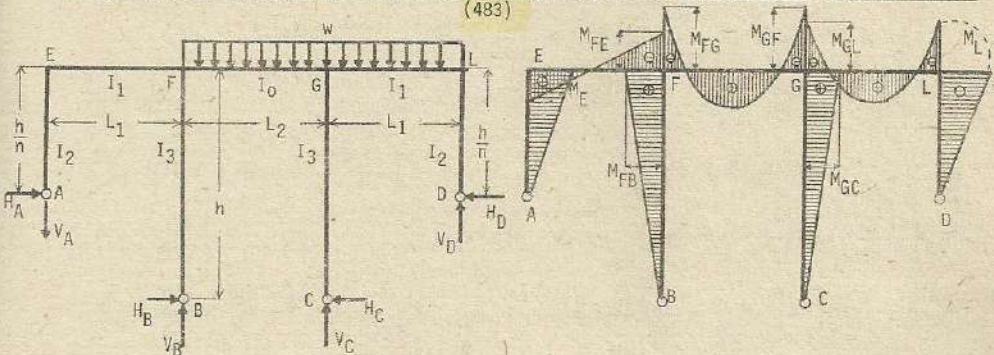
$$M_{GL} = -\frac{w L_1^2}{8} \left[\frac{12k_2(n^2+k_4) + 6k_3(n^2+n+k_2)}{\alpha} + \frac{(2+k_1)(2k_2+3k_3)}{\beta} \right]$$

$$M_{GC} = -\frac{w L_1^2}{8} \left[\frac{6k_3(n+1+k_2)}{\alpha} + \frac{3k_3(2+k_1)}{\beta} \right] \quad ; \quad M_L = -\frac{w L_1^2}{8} \left[\frac{6k_3(n+1+k_2)}{\alpha} + \frac{k_1(6+2k_2+3k_3)}{\beta} \right]$$

$$M_{FG} = \frac{w L_1^2 k_2}{4} \left[\frac{6(n^2+k_4) - 3k_3(n-1)}{\alpha} - \frac{(2-k_1)}{\beta} \right]$$

$$M_{FE} = \frac{w L_1^2}{8} \left[\frac{6[2k_2(n^2+k_4) + k_3(n^2+n+k_2)]}{\alpha} - \frac{(2+k_1)(2k_2+3k_3)}{\beta} \right]$$

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$$k_1 = \frac{I_2 L_1^n}{I_1 h} \quad ; \quad k_2 = \frac{I_0 L_1}{I_1 L_2} \quad ; \quad k_3 = \frac{I_3 L_1}{I_1 h} \quad ; \quad k_4 = \frac{I_3}{n I_2}$$

$$\alpha = 6[2(n^2+K_4)(1+2K_2) + K_3(2+3K_2) + 2nK_3(n+1)] ; \quad \beta = (4+3K_1)(4+2K_2+3K_3) - 4$$

$$M_{FE} = \frac{wL_1^2}{8} \left[\frac{6[2K_2(n^2+K_4)+K_3(n^2+n+K_2)]}{\alpha} - \frac{(2+K_1)(2K_2+3K_3)}{\beta} \right] - \frac{wL_2^2}{8} \frac{(1+K_1)}{\beta}$$

$$M_{FG} = \frac{wL_1^2 K_2}{4} \left[\frac{6(n^2+K_4)-3K_3(n-1)}{\alpha} - \frac{(2+K_1)}{\beta} \right] - \frac{wL_2^2}{4\beta} [K_3(4+3K_1)+4(1+K_1)]$$

$$M_{FB} = \frac{wL_1^2}{8} \left[\frac{6nK_3(n+1+K_2)}{\alpha} - \frac{3K_3(2+K_1)}{\beta} \right] + \frac{wL_2^2 K_3(4+3K_1)}{4\beta}$$

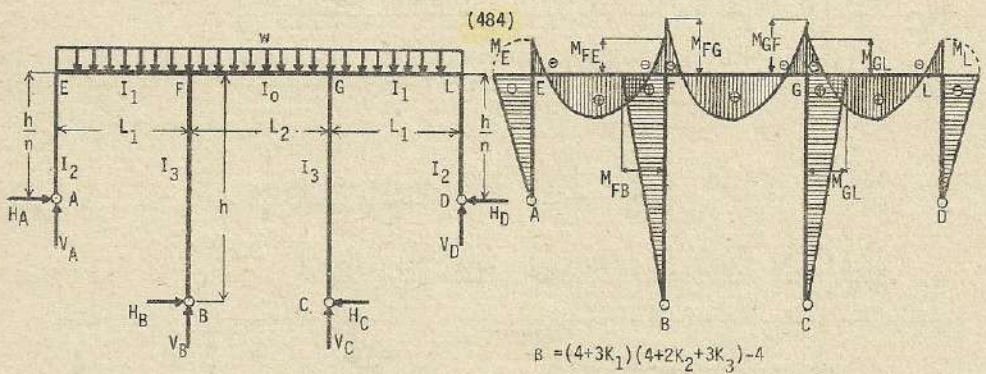
$$M_E = -\frac{wL_1^2}{8} \left[\frac{6K_3(n+1+K_2)}{\alpha} - \frac{K_1(6+2K_2+3K_3)}{\beta} \right] + \frac{K_1 wL_2^2}{2\beta}$$

$$M_{GF} = -\frac{wL_1^2 K_2}{4} \left[\frac{3[2(n^2+K_4) - K_3(n-1)]}{\alpha} + \frac{(2+K_1)}{\beta} \right] - \frac{wL_2^2}{4\beta} \cdot [K_3(4+3K_1)+4(1+K_1)]$$

$$M_{GL} = -\frac{wL_1^2}{8} \left[\frac{12K_2(n^2+K_4)+6K_3(n^2+n+K_2)}{\alpha} + \frac{(2+K_1)(2K_2+3K_3)}{\beta} \right] - \frac{wL_2^2(1+K_1)}{\beta}$$

$$M_{GC} = -\frac{wL_1^2}{8} \left[\frac{6nK_3(n+1+K_2)}{\alpha} + \frac{3K_3(2+K_1)}{\beta} \right] + \frac{wL_2^2 K_3(4+3K_1)}{4\beta}$$

$$M_L = -\frac{wL_1^2}{8} \left[\frac{6K_3(n+1+K_2)}{\alpha} + \frac{K_1(6+2K_2+3K_3)}{\beta} \right] + \frac{K_1 wL_2^2}{2\beta}$$



$$\beta = (4+3K_1)(4+2K_2+3K_3) - 4$$

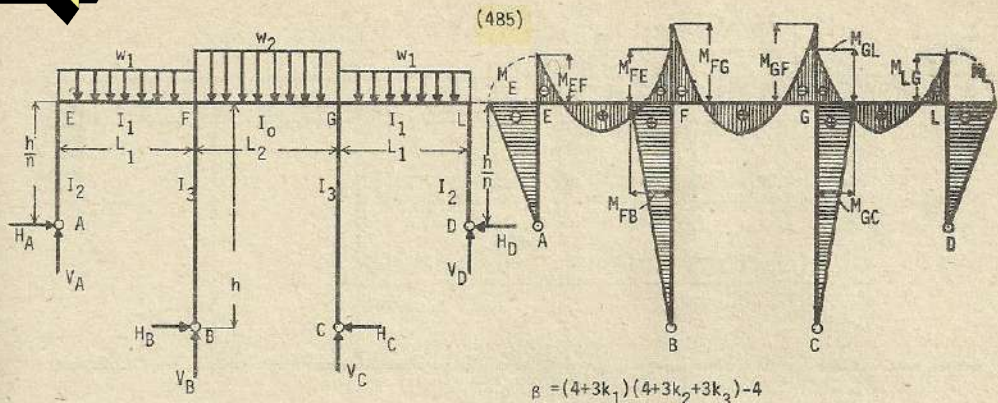
$$K_1 = \frac{I_2 L_1^3 n}{I_1 h} ; \quad K_2 = \frac{I_0 L_1}{I_1 L_2} ; \quad K_3 = \frac{I_3 L_1}{I_1 h} ; \quad K_4 = \frac{I_3}{n I_2}$$

$$M_E = -\frac{wK_1}{4\beta} [L_1^2(6+2K_2+3K_3) - 2L_2^2] ; \quad M_{FE} = -\frac{w}{4\beta} [L_1^2(2+K_1)(2K_2+3K_3) + 4(1+K_1)L_2^2]$$

$$M_{FG} = -\frac{w}{4\beta} [2K_2(2+K_1)L_1^2 + L_2^2[K_3(4+3K_1) + 4(1+K_1)]]$$

$$M_{FB} = -\frac{wK_3}{4\beta} [3L_1^2(2+K_1) - L_2^2(4+3K_1)] ; \quad M_{GF} = M_{FG} ; \quad M_{GL} = M_{FE}$$

$$M_{GC} = M_{FB} ; \quad M_L = M_E$$

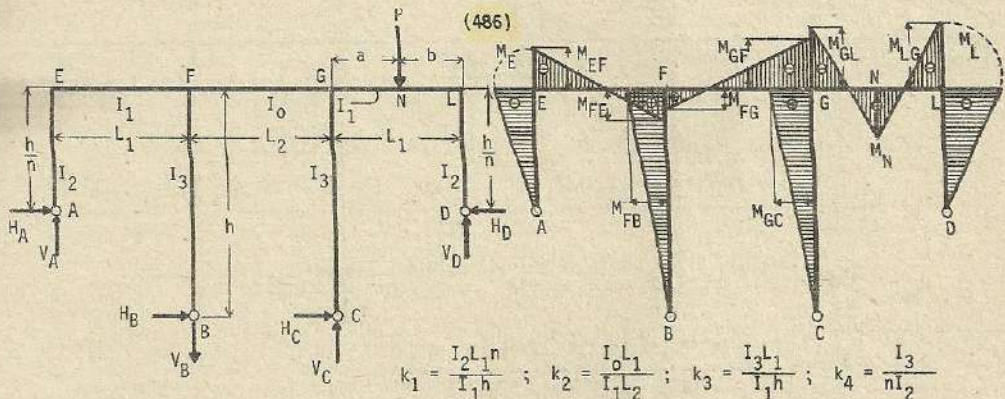


$$k_1 = \frac{I_2 L_1 n}{I_1 h} ; k_2 = \frac{I_0 L_1}{I_1 L_2} ; k_3 = \frac{I_3 L_1}{I_1 h} ; k_4 = \frac{I_3}{n I_2}$$

$$M_E = -\frac{k_1}{4\beta} \left[w_1 L_1^2 (6+2k_2+3k_3) - 2w_2 L_2^2 \right] ; M_{FE} = -\frac{1}{4\beta} \left[w_1 L_1^2 (2k_2+3k_3)(2+k_1) + 4w_2 L_2^2 (1+k_1) \right]$$

$$M_{FG} = -\frac{1}{4\beta} \left[2w_1 L_1^2 k_2 (2+k_1) + w_2 L_2^2 [4(1+k_1) + k_3(4+3k_1)] \right] ; M_{GF} = M_{FG} ; M_{GL} = M_{FE}$$

$$M_{FB} = -\frac{k_3}{4\beta} \left[3w_1 L_1^2 (2+k_1) - w_2 L_2^2 (4+3k_1) \right] ; M_{GC} = M_{FB} ; M_L = M_E$$



$$\alpha = 6 \left[2(h^2+k_4)(1+2k_2+k_3(2+3k_2)+2nk_3(n+1)) \right] ; \beta = (4+3k_1)(4+2k_2+3k_3) - 4$$

$$M_E = \frac{3k_1 P a b}{2L_1^2} \left[\frac{2k_4 [a(3k_2+n+2)+b(2b+1)]}{\alpha} - \frac{a(4+2k_2+3k_3)+2b}{\beta} \right]$$

$$M_{FE} = \frac{P a b}{2L_1^2} \left[\frac{12k_2(n^2+k_4)(a+2b)+6k_3[b(3k_2+2n^2+n)+an(n+2)]}{\alpha} - \frac{(2k_2+3k_3)[2a+b(4+3k_1)]}{\beta} \right]$$

$$M_{FG} = \frac{P a b k_2}{L_1^2} \left[\frac{6(n^2+k_4)(2b+a)+9k_3(b-an)}{\alpha} - \frac{b(4+3k_1)+2a}{\beta} \right]$$

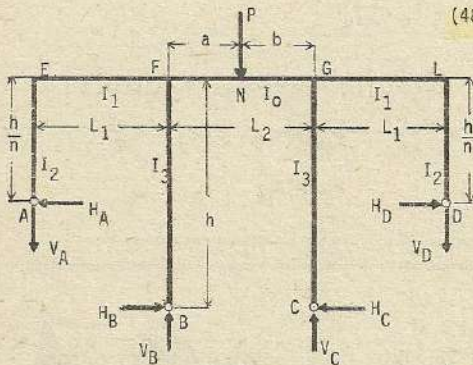
$$M_{FB} = \frac{3P a b k_3}{2L_1^2} \left[\frac{2n[a(2+3k_2+n)+b(2n+1)]}{\alpha} - \frac{b(4+3k_1)+2a}{\beta} \right]$$

$$M_{GF} = -\frac{Pabk_2}{L_1^2} \left[\frac{6(n^2+k_4)(2b+a)+9k_3(b-an)}{\alpha} + \frac{b(4+3k_1)+2a}{\beta} \right]$$

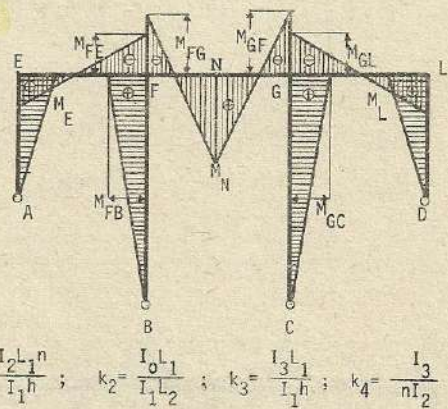
$$M_{GL} = -\frac{Pab}{2L_1^2} \left[\frac{12k_2(a+2b)(n^2+k_4)+6k_3[b(3k_2+2n^2+n)+an(n+2)]}{\alpha} + \frac{(2k_2+3k_3)[2a+b(4+3k_1)]}{\beta} \right]$$

$$M_{GC} = -\frac{3Pabk_3}{2L_1^2} \left[\frac{2n[a(2+3k_2+n)+b(2n+1)]}{\alpha} + \frac{b(4+3k_1)+2a}{\beta} \right]$$

$$M_D = -\frac{3k_1Pab}{2L_1^2} \left[\frac{2k_4[a(2+3k_2+n)+b(2n+1)]}{\alpha} + \frac{a(4+2k_2+3k_3)+2b}{\beta} \right]$$



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$$k_1 = \frac{I_2 L_1 n}{I_1 h} ; k_2 = \frac{I_0 L_1}{I_1 L_2} ; k_3 = \frac{I_3 L_1}{I_1 h} ; k_4 = \frac{I_3}{n I_2}$$

$$\alpha = 6[2(n^2+k_4)(1+2k_2)+k_3(2+3k_2+2n^2+2n)] ; \beta = (4+3k_1)(4+2k_2+3k_3)-4$$

$$M_E = \frac{3Pab}{L_2^2} \left[\frac{k_3(b-a)(2n+1)}{\alpha} + \frac{k_1(a+b)}{\beta} \right] ; M_{FB} = \frac{3Pabk_3}{2L_2^2} \left[\frac{2n(b-a)(2n+1)}{\alpha} + \frac{(4+3k_1)(a+b)}{\beta} \right]$$

$$M_{FG} = -\frac{3Pab}{2L_2^2} \left[\frac{4(b-a)[(n^2+k_4)+k_3(n^2+n+1)]}{\alpha} + \frac{(a+b)[4(1+k_1)+k_3(4+3k_1)]}{\beta} \right]$$

$$M_{FE} = -\frac{3Pab}{L_2^2} \left[\frac{(b-a)[2n^2+2k_4+k_3(n+2)]}{\alpha} + \frac{2(a+b)(1+k_1)}{\beta} \right]$$

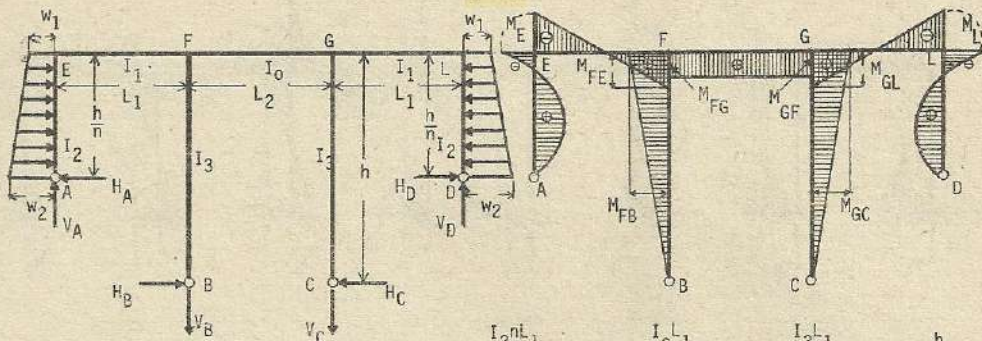
$$M_{GF} = -\frac{3Pab}{2L_2^2} \left[\frac{(a+b)[4(1+k_1)+k_3(4+3k_1)]}{\beta} - \frac{4(b-a)[(n^2+k_4)+k_3(n^2+n+a)]}{\alpha} \right]$$

$$M_{GL} = -\frac{3Pab}{L_2^2} \left[\frac{2(a+b)(1+k_1)}{\beta} - \frac{(b-a)[2n^2+2k_4+k_3(n+2)]}{\alpha} \right]$$

$$M_{GC} = \frac{3Pabk_3}{2L_2^2} \left[\frac{(a+b)(4+3k_1)}{\beta} - \frac{2n(b-a)(2n+1)}{\alpha} \right]$$

$$M_L = \frac{3Pab}{L_2^2} \left[\frac{k_1(a+b)}{\beta} - \frac{k_3(b-a)(2n+1)}{\alpha} \right]$$

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$$K_1 = \frac{I_2 n L_1}{I_1 h} ; K_2 = \frac{I_0 L_1}{I_1 L_2} ; K_3 = \frac{I_3 L_1}{I_1 h} ; h' = \frac{h}{n}$$

$$\beta = (4+3K_1)(4+2K_2+3K_3) - 4 ; \gamma = \frac{h'^2}{120} (15w_1 + 7mh') ; mh' = w_2 - w_1$$

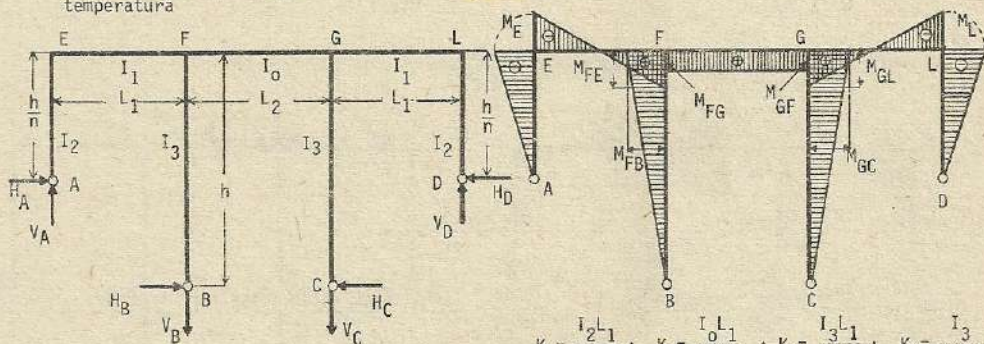
$$\text{reemplazando término } (mh') ; \gamma = \frac{h'^2}{120} (8w_1 + 7w_2)$$

$$M_E = M_L = - \frac{4(3+2K_2+3K_3)\gamma}{\beta} ; M_{FG} = M_{GF} = \frac{4K_2\gamma}{\beta} ; M_{FE} = M_{GL} = \frac{2(2K_2+3K_3)\gamma}{\beta}$$

$$M_{FB} = M_{GC} = \frac{6K_3\gamma}{\beta}$$

Esfuerzos por cambio de temperatura

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$$K_1 = \frac{I_2 L_1}{I_1 h} ; K_2 = \frac{I_0 L_1}{I_1 L_2} ; K_3 = \frac{I_3 L_1}{I_1 h} ; K_4 = \frac{I_3}{I_2 n}$$

$$h' = \frac{h}{n} ; u = \frac{I_2 n}{h} ; \delta_1 ; \text{Leve deformación de } L_1, \text{ dilatación (+), contracción (-)}$$

$$\delta_2 ; \text{Leve deformación de } L_2, \text{ dilatación (+), contracción (-)}$$

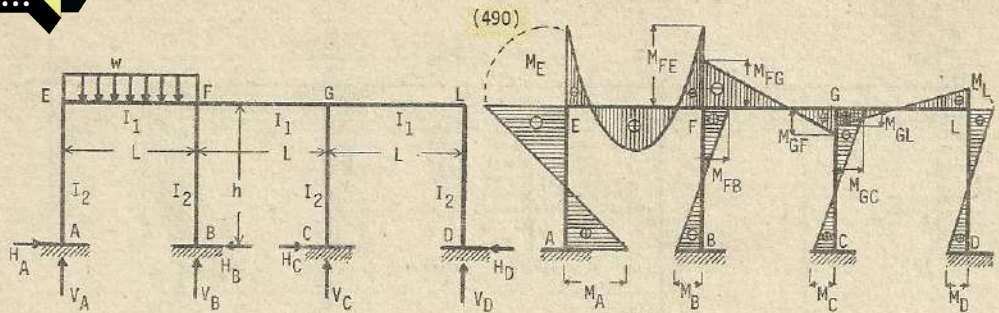
$$s_1 = E\delta_1 ; s_2 = E\delta_2 ; \beta = (4+3K_1)(4+2K_2+3K_3) - 4$$

$$M_E = M_L = - \frac{3\mu}{\beta h} [3K_3 s_2 + 2n^2 (3+2K_2+3K_3) (2s_1 + s_2)]$$

$$M_{FE} = M_{GL} = \frac{3\mu}{\beta h} [n^2 (2K_2+3K_3) (2s_1 + s_2) + 6(K_4 + K_3) s_2]$$

$$M_{FG} = M_{GF} = \frac{3\mu k_2}{\beta h} [2n^2 (2s_1 + s_2) - (4K_4 + 3K_3) s_2]$$

$$M_{FB} = M_{GC} = \frac{3\mu k_4}{\beta h} [3n_2 K_1 (2s_1 + s_2) + [(4+3K_1)(2+K_2) - 2] s_2]$$



$$K = \frac{I_1 h}{I_2 L} : (36k^2 + 41k + 4) = Y ; (5k^2 + 10k + 4) = Z$$

$$V_A = \frac{(624k^4 + 2,099k^3 + 2,244k^2 + 816k + 64)}{8YZ} \cdot wL ; V_B = \frac{(936k^4 + 2,681k^3 + 2,484k^2 + 832k + 64)}{8YZ} \cdot wL$$

$$V_C = -\frac{(144k^4 + 339k^3 + 180k^2 + 16k)}{8YZ} \cdot wL ; V_D = \frac{(24k^2 + 79k + 44)k^2}{8YZ} \cdot wL$$

$$H_A = \frac{(117k^3 + 213k^2 + 95k + 8)}{4YZ} \cdot \frac{wL^2}{h} ; H_B = \frac{(198k^3 + 385k^2 + 186k + 16)}{8YZ} \cdot \frac{wL^2}{h}$$

$$H_C = \frac{(18k^2 + 5k + 2)k}{8YZ} \cdot \frac{wL^2}{h} ; H_D = \frac{(27k^2 + 23k + 3)k}{4YZ} \cdot \frac{wL^2}{h}$$

$$M_B = -\frac{(213k^3 + 415k^2 + 198k + 16)}{24YZ} \cdot wL^2 ; M_A = \frac{(219k^3 + 396k^2 + 178k + 16)}{24YZ} \cdot wL^2$$

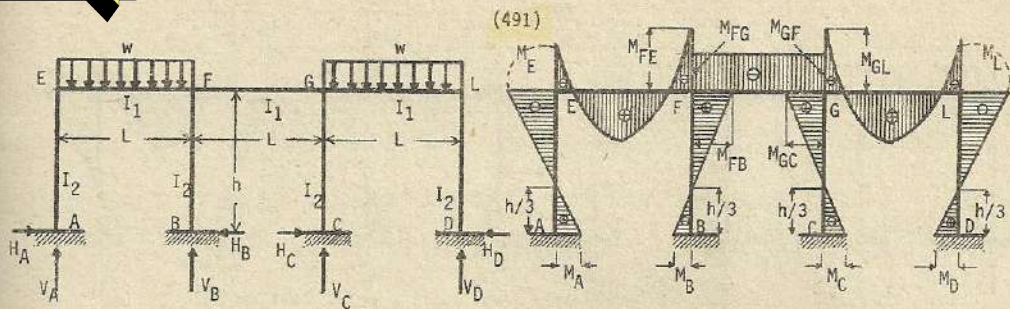
$$M_C = -\frac{(3k^3 - 25k^2 - 10k)}{24YZ} \cdot wL^2 ; M_D = \frac{(69k^3 + 76k^2 + 18k)}{24YZ} \cdot wL^2$$

$$M_E = -\frac{(483k^3 + 882k^2 + 392k + 32)}{24YZ} \cdot wL^2 ; M_{FE} = -\frac{(144k^4 + 483k^3 + 519k^2 + 196k + 16)}{12YZ} \cdot wL^2$$

$$M_{FB} = \frac{(381k^3 + 740k^2 + 360k + 32)}{24YZ} \cdot wL^2 ; M_{FG} = -\frac{(288k^4 + 585k^3 + 298k^2 + 32k)}{24YZ} \cdot wL^2$$

$$M_{GF} = \frac{(72k^4 + 195k^3 + 110k^2 + 16k)}{24YZ} \cdot wL^2 ; M_{GC} = -\frac{(51k^3 + 40k^2 + 16k)}{24YZ} \cdot wL^2$$

$$M_{GL} = \frac{(36k^2 + 72k + 35)k^2}{12YZ} \cdot wL^2 ; M_L = -\frac{(93k + 62)k^2}{24YZ} \cdot wL^2$$



$$Z = (5k^2 + 10k + 4) \quad k = \frac{I_1 h}{I_2 L}$$

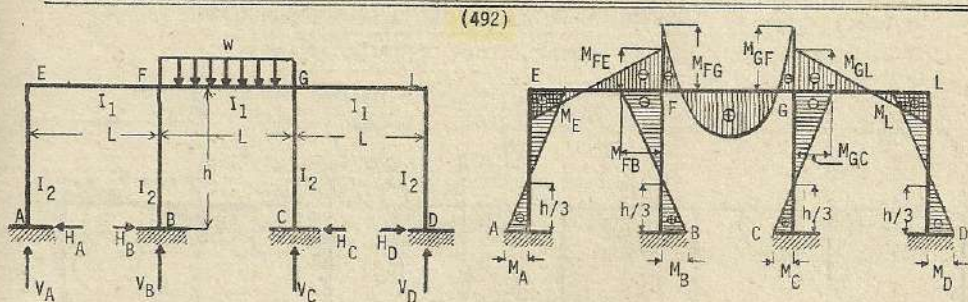
$$V_A = V_D = \frac{(9k^2 + 20k + 8)}{4Z} \cdot wL ; \quad V_B = V_C = \frac{(11k^2 + 20k + 8)}{4Z} \cdot wL ; \quad H_A = H_D = \frac{(1+2k)}{2Z} \cdot \frac{wL^2}{h}$$

$$H_B = H_C = \frac{(2+3k)}{4Z} \cdot \frac{wL^2}{h} ; \quad M_A = M_D = \frac{(1+2k)}{6Z} \cdot wL^2 ; \quad M_B = M_C = -\frac{(2+3k)}{12Z} \cdot wL^2$$

$$M_E = M_L = -\frac{(1+2k)}{3Z} \cdot wL^2 ; \quad M_{FE} = M_{GL} = -\frac{(3k^2 + 8k + 4)}{12Z} \cdot wL^2 ; \quad M_{FG} = M_{GF} = -\frac{(2+3k)k}{12Z} \cdot wL^2$$

$$M_{FB} = M_{GC} = \frac{(2+3k)}{6Z} \cdot wL^2 ; \quad \text{Considerando como } (x) \text{ la distancia, desde los puntos } (F) \text{ y } (G), \text{ las ubicaciones de los m\u00e1ximos momentos de } (EF) \text{ y } (GL), \text{ tenemos :}$$

$$x_m = \frac{(11k^2 + 20k + 8)}{4Z} \cdot L \quad \therefore \quad M_{\text{m\u00e1x.}} = \frac{(243k^4 + 760k^3 + 832k^2 + 384k + 64)}{96 Z^2} \cdot wL^2$$



$$k = \frac{I_1 h}{I_2 L} ; \quad V_A = V_D = \frac{k(2+k)}{4Z} \cdot wL ; \quad V_B = V_C = \frac{(11k^2 + 22k + 8)}{4Z} \cdot wL$$

$$H_A = H_D = \frac{k}{4Z} \cdot \frac{wL^2}{h} ; \quad H_B = H_C = \frac{(1+k)}{2Z} \cdot \frac{wL^2}{h} ; \quad M_A = M_D = -\frac{k}{12Z} \cdot wL^2$$

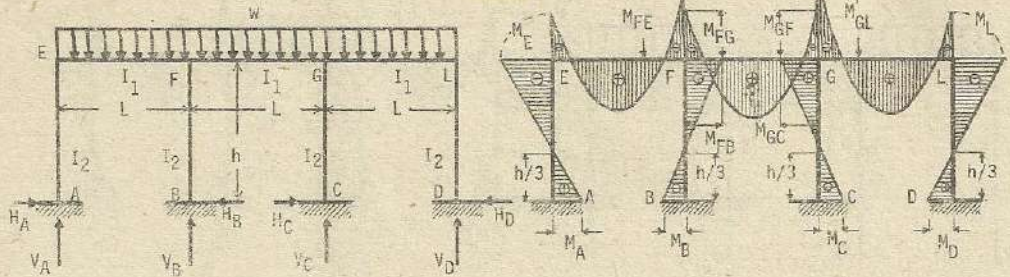
$$M_B = M_C = -\frac{(1+k)}{6Z} \cdot wL^2 ; \quad M_E = M_L = \frac{k}{6Z} \cdot wL^2 ; \quad M_{FE} = M_{GL} = -\frac{k(4+3k)}{12Z} \cdot wL^2$$

$$M_{FG} = -\frac{(4+8k+3k^2)}{12Z} \cdot wL^2 ; \quad M_{FB} = M_{GC} = -\frac{(1+k)}{3Z} \cdot wL^2 ;$$

$$M_{\text{m\u00e1x.}} = \frac{(9k^2 + 14k + 4)}{24 Z} \cdot wL^2 \quad (\text{respecto al punto central de la viga } FG)$$

$$Z = (5k^2 + 10k + 4)$$

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$$k = \frac{I_1 h}{I_2 L} ; Z = (5k^2 + 10k + 4)$$

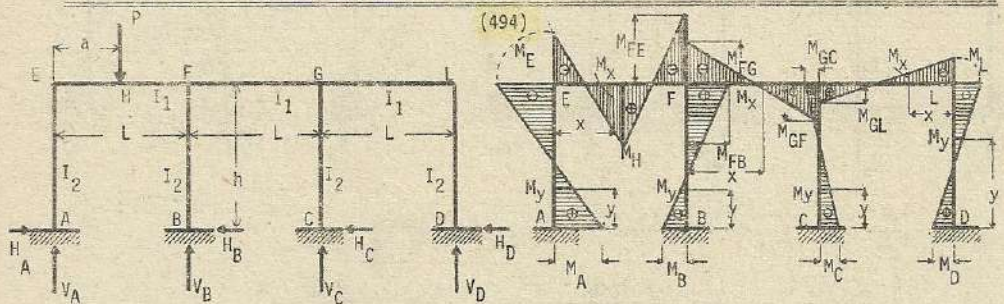
$$H_A = H_D = \frac{(3k+2)}{4Z} \cdot \frac{wL^2}{h} ; H_B = H_C = \frac{k}{4Z} \cdot \frac{wL^2}{h} ; V_A = V_D = \frac{4k^2+9k+4}{2Z} \cdot wL$$

$$V_B = V_C = \frac{11k^2+21k+8}{2Z} \cdot wL ; M_A = M_D = \frac{3k+2}{12Z} \cdot wL^2 ; M_B = M_C = -\frac{k}{12Z} \cdot wL^2$$

$$M_E = M_L = -\frac{3k+2}{6Z} \cdot wL^2 ; M_{FE} = M_{GL} = -\frac{3k^2+6k+2}{6Z} \cdot wL^2 ; M_{FG} = M_{GF} = -\frac{3k^2+8k+2}{6Z} \cdot wL^2$$

$$M_{FB} = M_{GC} = \frac{k}{6Z} \cdot wL^2 ; M_{\text{máx.}} = \frac{3k^2+10k+4}{24Z} \cdot wL^2 \quad (\text{Respecto al punto central de la viga FG})$$

Quando en los pórticos de figuras simétricas, accionan cargas igualmente simétricas, Los esfuerzos de momentos por flexión de las columnas, habitualmente se encierran dentro de una línea recta, y el punto de inflexión se ubica a 1/3 de la altura, desde la base. Esto es, los esfuerzos de momentos de la parte superior de la columna, es el doble que el de la parte inferior, siendo contrario sus signos.



$$K = \frac{I_1 h}{I_2 L} ; A = \left(1 - \frac{a}{L}\right) ; Y = (36k^2 + 41k + 4) ; Z = (5k^2 + 10k + 4)$$

$$H_A = 3PaA \left[\frac{(84k^3 + 160k^2 + 76k + 6) - \frac{a}{L} (51k^3 + 107k^2 + 57k + 4)}{YZh} \right]$$

$$H_B = 3PaA \left[\frac{(96k^3 + 166k^2 + 62k + 4) + \frac{a}{L} (6k^3 + 53k^2 + 62k + 8)}{2YZh} \right]$$

$$H_C = 3PaA \left[\frac{-(24k^3 + 84k^2 + 54k + 4) + \frac{a}{L} (66k^3 + 173k^2 + 110k + 8)}{2YZh} \right]$$

$$H_D = 3PaA \left[\frac{(24k^3 + 35k^2 + 18k + 2)}{Y Z h} - \frac{a}{L} \frac{(21k^3 + 47k^2 + 33k + 4)}{h} \right]$$

$$V_A = P - \frac{Pa}{2 Y Z L} \left[3(152k^3 + 354k^2 + 234k + 40)k + 3(229k^3 + 498k^2 + 288k + 32) \frac{a}{L} - (96k^4 - 619k^3 + 1,048k^2 + 576k + 64) \left(\frac{a}{L} \right)^2 \right]$$

$$V_B = \frac{Pa}{2 Y Z L} \left[3(152k^3 + 354k^2 + 234k + 40)k + 3(229k^3 + 498k^2 + 288k + 32) \frac{a}{L} - (96k^4 + 619k^3 + 1,048k^2 + 576k + 64) \left(\frac{a}{L} \right)^2 \right] + \frac{12kaA \left[k \left(1 + \frac{a}{L} \right) + \left(2 \frac{a}{L} - \frac{3}{4} \right) \right]}{Y L} P$$

$$V_C = PaA \frac{[(144k^3 + 298k^2 + 78k) + (144k^3 + 441k^2 + 384k + 48) \frac{a}{L}]}{2 Y Z L}$$

$$V_D = 3PaA \frac{[(8k^3 + 48k^2 + 54k + 24) + (8k^3 - 13k^2 - 64k - 48) \frac{a}{L}]}{2 Y Z L}$$

$$M_A = PaA \frac{[(148k^3 + 270k^2 + 116k + 4) - (77k^3 + 144k^2 + 54k - 8) \frac{a}{L}]}{2 Y X}$$

$$M_E = -PaA \frac{[(356k^3 + 690k^2 + 340k + 32) - (229k^3 + 498k^2 + 288k + 32) \frac{a}{L}]}{2 Y Z}$$

$$M_{FE} = -PaA \frac{[(48k^3 + 144k^2 + 122k + 26)k + (48k^4 + 195k^3 + 275k^2 + 144k + 16) \frac{a}{L}]}{Y Z}$$

$$M_{FG} = -PaA \frac{[(96k^3 + 116k^2 - 38k - 36) + (k + 2)(96k^2 + 161k + 52) \frac{a}{L}]}{2 Y Z}$$

$$M_{FB} = PaA \frac{[(172k^2 + 282k + 88)k + (37k^3 + 176k^2 + 184k + 32) \frac{a}{L}]}{2 Y Z}$$

$$M_B = PaA \frac{[(116k^3 + 216k^2 + 98k + 12) - (19k^3 + 17k^2 - 2k + 8) \frac{a}{L}]}{2 Y Z}$$

$$M_{GF} = PaA \frac{[(24k^3 + 34k^2 - 46k - 36) + (k + 2)(24k^2 + 79k + 44) \frac{a}{L}]}{2 Y Z}$$

$$M_{GL} = PaA \frac{[(12k^3 + 31k^2 + 36k + 18) + (12k^3 + 10k^2 - 37k - 36) \frac{a}{L}]}{Y Z}$$

$$M_{GC} = PaA \frac{[-(28k^2 + 118k + 72) + (107k^2 + 276k + 160) \frac{a}{L}]}{2 Y Z}$$

$$M_C = PaA \frac{[-(44k^3 + 134k^2 + 90k + 12) + (91k^3 + 243k^2 + 170k + 24) \frac{a}{L}]}{2 Y Z}$$

$$M_L = -PaA \frac{[(76k^2 + 90k + 36) - (56k^2 + 118k + 72) \frac{a}{L}]}{2 Y Z}$$

$$M_D = PaA \frac{[(68k^3 + 120k^2 + 72k + 12) - (67k^3 + 164k^2 + 126k + 24) \frac{a}{L}]}{2 Y Z}$$

$$\text{Si } a = \frac{L}{2} : H_A = \frac{3PL(117k^3 + 213k^2 + 95k + 8)}{8 Y Z h} ; H_B = \frac{3PL(198k^3 + 385k^2 + 186k + 16)}{16 Y Z h}$$

$$H_C = \frac{3PLk(18k^2 + 5k + 2)}{16 Y Z h} ; H_D = \frac{3PLk(27k^2 + 23k + 3)}{8 Y Z h}$$

$$V_A = \frac{1,152k^4 + 4,037k^3 + 4,436k^2 + 1,632k + 128}{16YZ} P$$

$$V_B = \frac{2,088k^4 + 5,783k^3 + 5,156k^2 + 1,680k + 128}{16YZ} P ; V_C = \frac{3k(144k^3 + 339k^2 + 180k + 16)}{16YZ} P$$

$$V_D = \frac{3k^2(24k^2 + 79k + 44)}{16YZ} P ; M_A = \frac{(219k^3 + 396k^2 + 178k + 16)}{16YZ} PL$$

$$M_E = -\frac{(483k^3 + 882k^2 + 392k + 32)}{16YZ} PL ; M_{FE} = -\frac{(144k^4 + 483k^3 + 519k^2 + 196k + 16)}{8YZ} PL$$

$$M_{FG} = -\frac{k(288k^3 + 585k^2 + 298k + 32)}{16YZ} PL ; M_{FB} = \frac{(381k^3 + 740k^2 + 360k + 32)}{16YZ} PL$$

$$M_B = \frac{(213k^3 + 415k^2 + 198k + 16)}{16YZ} PL ; M_{GF} = \frac{k(72k^3 + 195k^2 + 110k + 16)}{16YZ} PL$$

$$M_{GL} = \frac{k(36k^3 + 103k^2 + 71k)}{8YZ} PL ; M_{GC} = \frac{k(51k^2 + 40k + 16)}{16YZ} PL$$

$$M_C = \frac{(3k^3 - 25k^2 - 10k)}{16YZ} PL ; M_L = -\frac{(93k^2 + 62k)}{16YZ} PL ; M_D = \frac{(69k^3 + 76k^2 + 18k)}{16YZ} PL$$

Momentos por flexión en las columnas:

$$AE \rightarrow M_y = M_A - H_A y ; BF \rightarrow M_y = M_B + H_B y$$

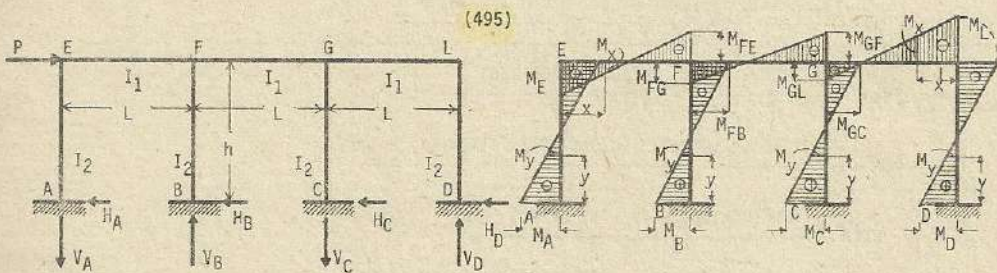
$$CG \rightarrow M_y = M_C + H_C y ; DL \rightarrow M_y = M_D - H_D y$$

Momentos en las vigas : EF $\rightarrow M_x = M_E + V_A x$ $x \leq a$

$$M_x = M_E + V_A x - P(x-a) \quad x \geq a$$

$$FG \rightarrow M_x = M_{FG} + (V_A + V_B - P)x$$

$$GL \rightarrow M_x = M_L + V_D x$$



$$k = \frac{I_1 h}{I_2 L} ; Y = (36k^2 + 41k + 4)$$

$$H_A = \frac{(9k^2 + 8k + 1)}{Y} P ; H_B = \frac{(18k^2 + 25k + 2)}{2Y} P ; H_C = \frac{(18k^2 + 25k + 2)}{2Y} P$$

$$H_D = \frac{(9k^2 + 8k + 1)}{Y} P ; V_A = \frac{3k(5k + 4)}{2Y} \cdot \frac{Ph}{L} ; V_B = \frac{9k^2}{2Y} \cdot \frac{Ph}{L} ; V_C = \frac{9k^2}{2Y} \cdot \frac{Ph}{L}$$

$$V_D = \frac{3k(5k + 4)}{2Y} \cdot \frac{Ph}{L} ; M_A = -\frac{(9k^2 + 10k + 2)}{2Y} Ph ; M_B = -\frac{(9k^2 + 13k + 2)}{2Y} Ph$$

$$M_C = \frac{(9k^2+13k+2)}{2Y} \cdot Ph \quad ; \quad M_D = \frac{(9k^2+10k+2)}{2Y} \cdot Ph \quad ; \quad M_E = \frac{3(3k+2)k}{2Y} \cdot Ph$$

$$M_{FE} = -\frac{3(k+1)k}{Y} \cdot Ph \quad ; \quad M_{FG} = \frac{3(k+2)k}{2Y} \cdot Ph \quad ; \quad M_{FB} = \frac{3(3k+4)k}{2Y} \cdot Ph$$

$$M_{GF} = \frac{3(k+2)k}{2Y} \cdot Ph \quad ; \quad M_{GL} = \frac{3(k+1)k}{Y} \cdot Ph \quad ; \quad M_{GC} = -\frac{3(3k+4)k}{Y} \cdot Ph$$

$$M_L = -\frac{3(3k+2)}{2Y} \cdot Ph$$

Momentos por flexión en las columnas:

$$AE \rightarrow M_y = M_A + H_A y \quad ; \quad BF \rightarrow M_y = M_B + H_B y$$

$$CG \rightarrow M_y = M_C - H_C y \quad ; \quad DL \rightarrow M_y = M_D - H_D y$$

Momentos en las vigas:

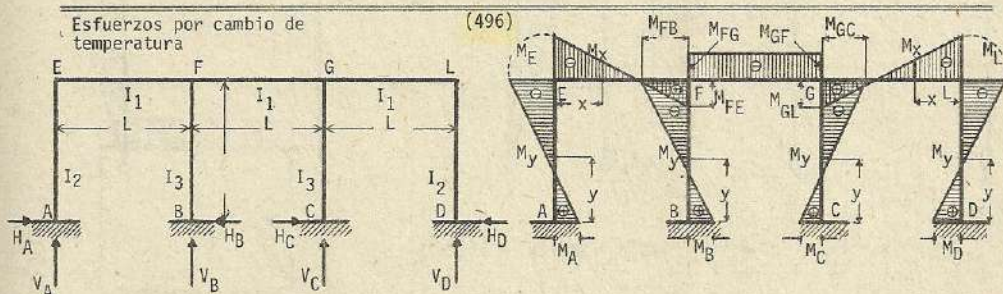
$$EF \rightarrow M_x = M_E - V_A x$$

$$FG \rightarrow M_x = M_{FG} - (V_A - V_B) x$$

$$GL \rightarrow M_x = M_L + V_D x$$

Esfuerzos por cambio de temperatura

(496)



$$k_1 = \frac{I_1 h}{I_2 L} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L} \quad ; \quad C = (4k_1 + 6k_2 + 5k_1 k_2 + 4) \quad ; \quad Z = (5k_1^2 + 10k_1 + 4)$$

$$H_A = H_D = 3E\epsilon t I_1 \frac{(19k_1 + 6k_2 + 20k_1 k_2 + 4)}{h^2 k_1 C} \quad ; \quad H_B = H_C = 6E\epsilon t I_1 \frac{(k_1 + 9k_2 + 5k_1 k_2 + 1)}{h^2 k_1 C}$$

$$V_A = V_B = V_C = V_D = 9E\epsilon t I_1 \frac{(k_1 + 4k_2 + 6)}{hLC} \quad ; \quad M_A = M_D = 3E\epsilon t I_1 \frac{(9k_1 + 6k_2 + 10k_1 k_2 + 4)}{hk_1 C}$$

$$M_B = M_C = 3E\epsilon t I_1 \frac{(2k_1 + 8k_2 + 5k_1 k_2 + 2)}{hk_2 C} \quad ; \quad M_E = M_L = -30E\epsilon t I_1 \frac{(k_2 + 1)}{hC}$$

$$M_{FE} = M_{GL} = 3E\epsilon t I_1 \frac{(3k_1 + 2k_2 + 8)}{hC} \quad ; \quad M_{FG} = M_{GF} = -6E\epsilon t I_1 \frac{(1 + k_1 - k_2)}{hC}$$

$$M_{FB} = M_{GC} = -15E\epsilon t I_1 \frac{(2 + k_1)}{hC}$$

Si $I_2 = I_3$, $k_1 = k_2 = k$

$$H_A = H_D = 3E\epsilon t I_1 \frac{(20k^2 + 25k + 4)}{h^2 k Z} \quad ; \quad H_B = H_C = 6E\epsilon t I_1 \frac{(5k^2 + 10k + 1)}{h^2 k Z}$$

$$V_A = V_B = V_C = V_D = 9E\epsilon t I_1 \frac{(5k + 6)}{hL Z} \quad ; \quad M_A = M_D = 3E\epsilon t I_1 \frac{(10k^2 + 15k + 4)}{hk Z}$$

$$M_B = M_C = 3EetI_1 \frac{(5k^2 + 10k + 2)}{hkZ} ; \quad M_E = M_L = 30EetI_1 \frac{(1+k)}{hZ}$$

$$M_{FE} = M_{GL} = 3EetI_1 \frac{(5k+8)}{hZ} ; \quad M_{FG} = M_{GF} = -15EetI_1 \frac{(k+2)}{hZ}$$

$$M_{FB} = M_{GC} = -6EetI_1 \frac{1}{hZ}$$

Esfuerzos de momentos por flexión o doblado en las columnas :

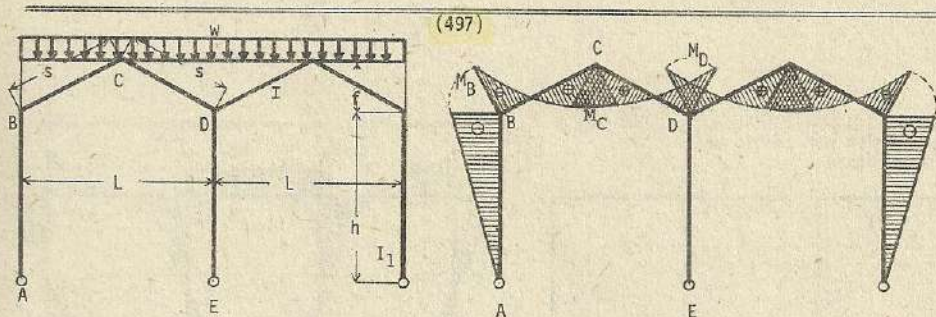
$$AE \text{ y } DL \longrightarrow M_y = M_A - H_A y = M_D + H_D y ; \quad BF \text{ y } CG \longrightarrow M_y = M_B - H_B y = M_C - H_C y$$

Momentos en las vigas :

$$EF \text{ y } LG \longrightarrow M_x = M_E + V_A x = M_L + V_D x$$

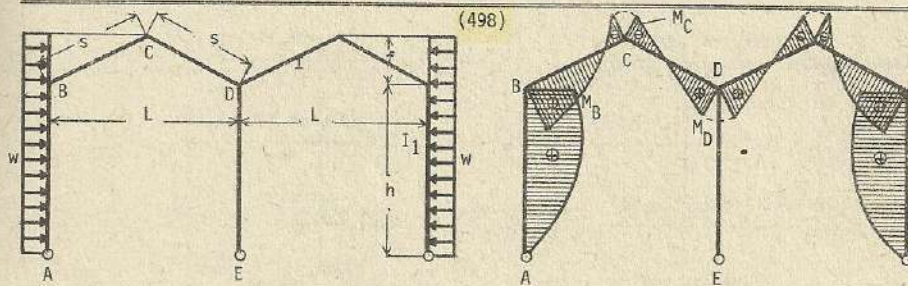
$$FG \longrightarrow M = M_{FG} = M_{GF} \quad (\text{constante})$$

Nota : El problema desarrollado corresponde al caso de ascenso de la temperatura, en caso contrario, los signos serán contrarios a las presentadas.



$$k = \frac{I_1 h}{I_1 s} ; \quad n = \frac{f}{h} ; \quad \delta = 4(3+2k) + n(12+7n)$$

$$M_B = \frac{wL^2(1+n)}{\delta} ; \quad M_C = \frac{wL^2}{16} \cdot \frac{8(1+k) - n(2+3n)}{\delta} ; \quad M_D = -\frac{wL^2}{8} \cdot \frac{8(1+k) + n(2+n)}{\delta}$$



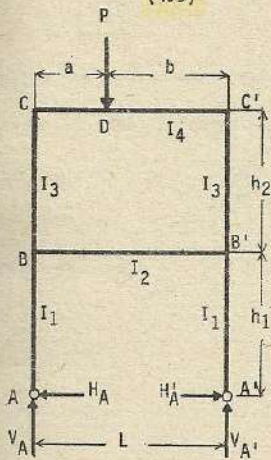
$$k = \frac{I_1 h}{I_1 s} ; \quad n = \frac{f}{h} ; \quad \alpha = 6+7n ; \quad \gamma = 6(1+k)+n ; \quad \delta = 4(3+2k) + n(12+7n)$$

$$\beta = 2(3+n) + 5k$$

$$M_B = \frac{wf^2}{8} \left[\frac{26+37n}{\delta} \right] + \frac{wn^2}{2} \left[\frac{na-2k}{\delta} \right] ; \quad M_C = -\frac{wf^2}{8} \left[\frac{40+n(21+4n)+31k}{\delta} \right] + \frac{wn^2}{2} \left[\frac{4na+k(2+5n)}{\delta} \right]$$

$$M_D = \frac{wf^2}{8} \left[\frac{2(19+17k) + n(13+2n)}{\delta} \right] + \frac{wh^2}{4} \left[\frac{2n\gamma+h(2+3n)}{\delta} \right]$$

(499)



$$M_{C'} = -\frac{Pa(1-\alpha)\lambda}{2} \left[\frac{2(3k_1+2k_1k_2+3k_2)}{m} - \frac{1-2\alpha}{n} \right]$$

$$k_1 = \frac{I_2 h_1}{I_1 L} ; \quad k_2 = \frac{I_2 h_2}{I_3 L} ; \quad \alpha = \frac{a}{L} ; \quad \lambda = \frac{I_2}{I_4}$$

$$m = (3k_1 + 2k_1k_2 + 3k_2)(k_2 + 2\lambda) + k_1k_2 ; \quad n = 1 + 6k_2\lambda$$

$$V_A = P(1-\alpha) ; \quad V_{A'} = P\alpha ; \quad H_A = H_{A'} = \frac{M_{AB}}{h_1}$$

$$M_{BA} = M_{B'A'} = \frac{3Pa(1-\alpha)}{2} \cdot \frac{k_2\lambda}{m}$$

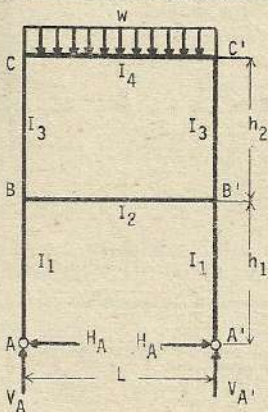
$$M_{BC} = \frac{Pa(1-\alpha)}{2} \left[\frac{k_2(3+2k_1)}{m} - \frac{1-2\alpha}{n} \right]$$

$$M_{B'C'} = \frac{Pa(1-\alpha)}{2} \left[\frac{k_2(3+2k_1)}{m} + \frac{1-2\alpha}{n} \right]$$

$$M_{BB'} = M_{BA} - M_{BC} ; \quad M_{B'B} = M_{B'A'} - M_{B'C'}$$

$$M_C = -\frac{Pa(1-\alpha)\lambda}{2} \left[\frac{2(3k_1+2k_1k_2+3k_2)}{m} + \frac{1-2\alpha}{n} \right]$$

(500)



$$k_1 = \frac{I_2 h_1}{I_1 L} ; \quad k_2 = \frac{I_2 h_2}{I_3 L} ; \quad \text{otros datos (499)}$$

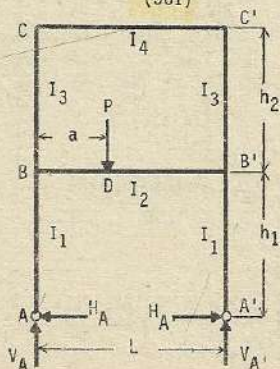
$$V_A = V_{A'} = \frac{wL}{2} ; \quad H_A = H_{A'} = \frac{M_{BA}}{h_1}$$

$$M_{BA} = M_{B'A'} = \frac{wL^2}{4} \cdot \frac{k_2\lambda}{m} ; \quad M_{BB'} = M_{B'B} = -\frac{wL^2}{6} \cdot \frac{k_1k_2\lambda}{m}$$

$$M_{BC} = M_{B'C'} = M_{BA} - M_{BB'}$$

$$M_C = M_{C'} = -\frac{wL^2}{6} \cdot \frac{\lambda(3k_1+2k_1k_2+3k_2)}{m}$$

(501)



$$k_1 = \frac{I_2 h_1}{I_1 L} ; \quad k_2 = \frac{I_2 h_2}{I_3 L} ; \quad \text{otros datos (499)}$$

$$V_A = P(1-\alpha) ; \quad V_{A'} = P\alpha ; \quad H_A = H_{A'} = \frac{M_{BA}}{h_1}$$

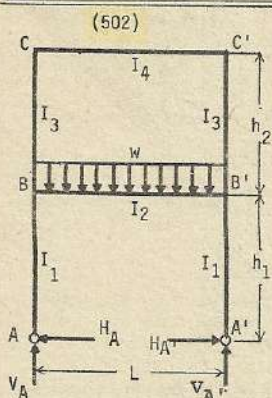
$$M_{BA} = M_{B'A'} = -\frac{3Pa(1-\alpha)}{2} \cdot \frac{k_2(k_2+2\lambda)}{m}$$

$$M_{BC} = \frac{Pa(1-\alpha)}{2} \left[\frac{2k_1(2k_2+3\lambda)}{m} + \frac{1-2\alpha}{n} \right]$$

$$M_{B'C'} = \frac{Pa(1-\alpha)}{2} \left[\frac{2k_1(2k_2+3\lambda)}{m} - \frac{1-2\alpha}{n} \right]$$

$$M_{BB'} = M_{BA} - M_{BC} ; \quad M_{B'B} = M_{B'A'} - M_{B'C'}$$

$$M_C = -\frac{Pa(1-\alpha)}{2} \left[\frac{2k_1k_2}{m} - \frac{1-2\alpha}{n} \right]; \quad M_{C'} = -\frac{Pa(1-\alpha)}{2} \left[\frac{2k_1k_2}{m} + \frac{1-2\alpha}{n} \right]$$



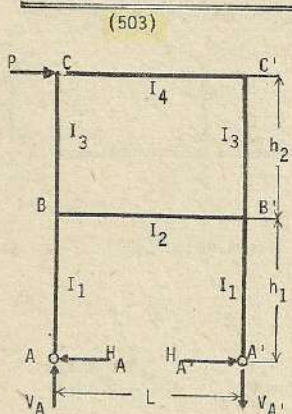
$$k_1 = \frac{I_2 h_1}{I_1 L}; \quad k_2 = \frac{I_2 h_2}{I_3 L}; \quad \text{otros datos (499)}$$

$$V_A = V_{A'} = \frac{wL}{2}; \quad H_A = H_{A'} = \frac{M_{BA}}{h_1}$$

$$M_{BA} = M_{B'A'} = -\frac{wL^2}{4} \cdot \frac{k_2(k_2+2\lambda)}{m}; \quad M_{BB'} = M_{B'B} = M_{BA} - M_{BC}$$

$$M_{BC} = M_{B'C'} = \frac{wL^2}{6} \cdot \frac{k_1(2k_2+3\lambda)}{m}$$

$$M_C = M_{C'} = -\frac{wL^2}{6} \cdot \frac{k_1 k_2}{m}$$



$$k_1 = \frac{I_2 h_1}{I_1 L}; \quad k_2 = \frac{I_2 h_2}{I_3 L}; \quad \text{otros datos (499): } \beta = \frac{h_1}{h_2}$$

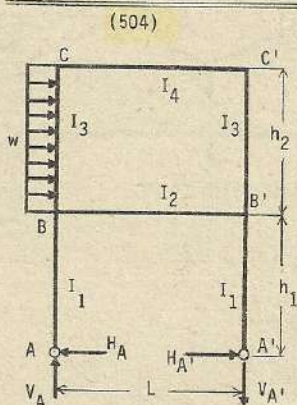
$$V_A = V_{A'} = -\frac{ph_2(1+\beta)}{L}; \quad H_A = -H_{A'} = \frac{P}{2}; \quad M_{BA} = P \frac{h}{2}$$

$$M_{B'A'} = -P \frac{h}{2}; \quad M_{BC} = -\frac{Ph_2}{2} \cdot \frac{(3k_2+\lambda-\beta)}{n}$$

$$M_{B'C'} = \frac{Ph_2}{2} \cdot \frac{(3k_2+\lambda-\beta)}{n}; \quad M_{B'B} = M_{B'A'} - M_{BC}$$

$$M_{B'B} = M_{B'A'} - M_{B'C'}; \quad M_C = \frac{Ph_2}{2} \cdot \frac{(3k_2+1+\beta)}{n}$$

$$M_{C'} = -\frac{Ph_2}{2} \cdot \frac{(3k_2+1+\beta)}{n}$$



$$k_1 = \frac{I_2 h_1}{I_1 L}; \quad k_2 = \frac{I_2 h_2}{I_3 L}; \quad \beta = \frac{h_1}{h_2}$$

Otros datos (499)

$$V_A = -V_{A'} = -\frac{wh_2^2(1+2\beta)}{2L}; \quad H_A = \frac{M_{BA}}{h_1}; \quad H_{A'} = \frac{M_{B'A'}}{h}$$

$$M_{BA} = \frac{wh_2^2}{8} \left[4\beta - \frac{k_2(k_2+3\lambda)}{m} \right]; \quad M_{B'A'} = -\frac{wh_2^2}{8} \left[4\beta + \frac{k_2(k_2+3\lambda)}{m} \right]$$

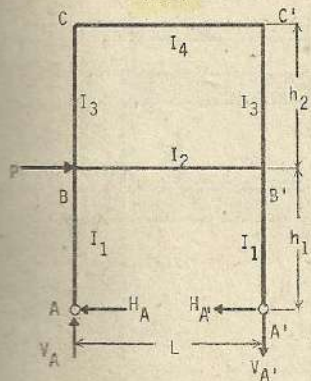
$$M_{BB'} = \frac{wh_2^2}{4} \left[\frac{4k_2(1+3\beta) + \lambda(1+2\beta)}{n} + \frac{k_1 k_2 (k_2+3\lambda)}{3m} \right]$$

$$M_{B'B} = -\frac{wh_2^2}{4} \left[\frac{4k_2(1+3\beta) + \lambda(1+2\beta)}{n} - \frac{k_1 k_2 (k_2+3\lambda)}{3m} \right]$$

$$M_{BC} = M_{BA} - M_{BB'}; \quad M_{B'C'} = M_{B'A'} - M_{B'B}$$

$$M_C = \frac{wh_2^2}{4} \left[\frac{2k_2+1+2\beta}{n} - \frac{k_2(6k_1+2k_1k_2+3k_2)}{6m} \right]; \quad M_{C'} = -\frac{wh_2^2}{4} \left[\frac{2k_2+1+2\beta}{n} - \frac{k_2(6k_1+2k_1k_2+3k_2)}{6m} \right]$$

(505)



$$k_1 = \frac{I_2 h_1}{I_1 L} ; \quad k_2 = \frac{I_2 h_2}{I_3 L} ; \quad \text{otros datos (499)}$$

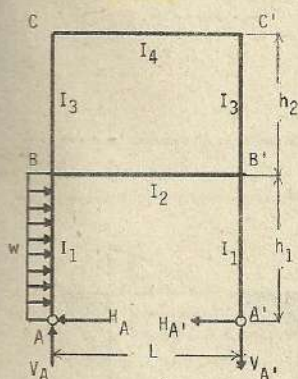
$$V_A = V_{A'} = -P \frac{h_1}{L} ; \quad H_A = -H_{A'} = -\frac{P}{2}$$

$$M_{BA} = -M_{B'A'} = +P \frac{h_1}{2} ; \quad M_{BC} = -M_{B'C'} = P \frac{h_1}{2n}$$

$$M_{BB'} = -M_{B'B} = P \frac{h_1}{2} \cdot \frac{6k_2 + \lambda}{n}$$

$$M_C = -M_{B'B} = P \frac{h_1}{2n} = M_{BC} = -M_{B'C'}$$

(506)



$$k_1 = \frac{I_2 h_1}{I_1 L} ; \quad k_2 = \frac{I_2 h_2}{I_3 L} ; \quad \text{otros datos (499)}$$

$$V_A = -V_{A'} = -w \frac{h_1^2}{2L} ; \quad H_A = \frac{wh_1}{2} + \frac{M_{BA}}{h_1} ; \quad H_{A'} = \frac{M_{B'A'}}{h_1}$$

$$M_{BB'} = \frac{wh_1^2}{8} \left[\frac{2(6k_2 + \lambda)}{n} - \frac{k_1 k_2 (k_2 + 2\lambda)}{m} \right]$$

$$M_{B'B} = -\frac{wh_1^2}{8} \left[\frac{2(6k_2 + \lambda)}{n} + \frac{k_1 k_2 (k_2 + 2\lambda)}{m} \right]$$

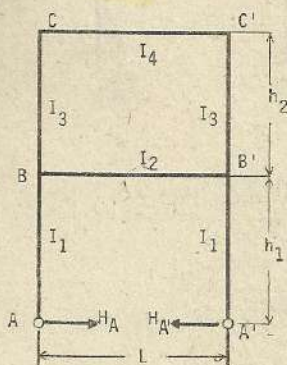
$$M_{BC} = \frac{wh_1^2}{8} \left[\frac{2}{n} - \frac{k_1 (2k_2 + 3\lambda)}{m} \right] ; \quad M_{BA} = M_{BB'} + M_{BC}$$

$$M_{B'C'} = -\frac{wh_1^2}{8} \left[\frac{2}{n} + \frac{k_1 (2k_2 + 3\lambda)}{m} \right] ; \quad M_{B'A'} = M_{B'B} + M_{B'C'}$$

$$M_C = \frac{wh_1^2}{8} \left[\frac{2}{n} + \frac{k_1 k_2}{m} \right] ; \quad M_{C'} = -\frac{wh_1^2}{8} \left[\frac{2}{n} - \frac{k_1 k_2}{m} \right]$$

Esfuerzos debido al cambio de temperatura

(507)



$$k_1 = \frac{I_2 h_1}{I_1 L} ; \quad k_2 = \frac{I_2 h_2}{I_3 L} ; \quad \text{otros datos (499)}$$

$$H_A = H_{A'} = \frac{M_{BA}}{h_1} ; \quad M_{BB'} = M_{B'B} = -\frac{3EetI_2}{h_1} \cdot \frac{k_2 (2\lambda + k_2)}{m}$$

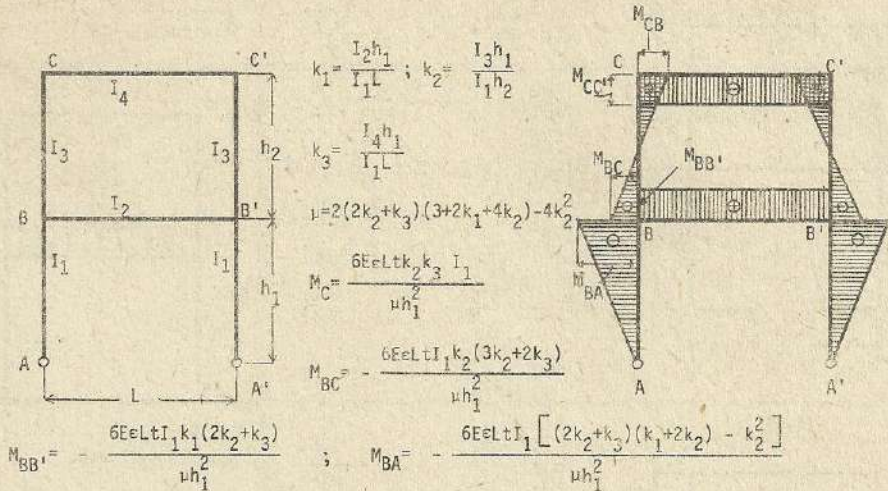
$$M_{BA} = M_{B'A'} = -\frac{3EetI_2}{h_1} \cdot \frac{k_2 (k_2 + 2) + \lambda (2k_2 + 3)}{m}$$

$$M_{BC} = M_{B'C'} = -\frac{3EetI_2}{h_1} \cdot \frac{(2k_2 + 3\lambda)}{m}$$

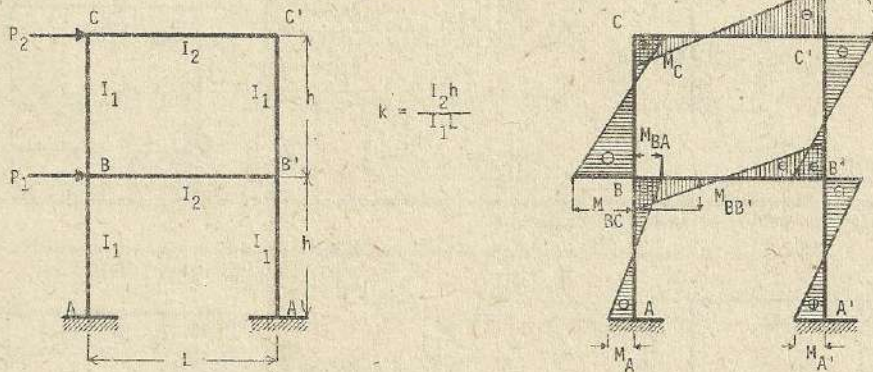
$$M_C = M_{C'} = \frac{3EetI_2}{h_1} \cdot \frac{k_2}{m}$$

Cuidado : En caso de descenso de temperatura los signos de los esfuerzos son contrarios a las presentadas.

Los esfuerzos por cambio de temperatura pueden también resolverse según :



(508)



$$M_A = -\frac{(3k+1)hP_2 + (18k^2+12k+1)h(P_1+P_2)}{2(36k^2+18k+1)}$$

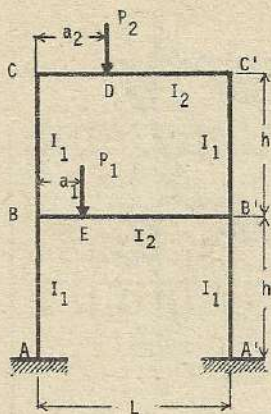
$$M_{BA} = \frac{-(3k+1)hP_2 + 6k(3k+1)h(P_1+P_2)}{2(36k^2+18k+1)} ; M_{BB'} = \frac{6k(3k+1)hP_2 + 3k(6k+1)h(P_1+P_2)}{2(36k^2+18k+1)}$$

$$M_{BC} = -\frac{(3k+1)(6k+1)hP_2 - 3kh(P_1+P_2)}{2(36k^2+18k+1)} ; M_C = \frac{9k(2k+1)hP_2 + 3kh(P_1+P_2)}{2(36k^2+18k+1)}$$

Si $P_1 = P_2 = P$; $k = I$, $(\frac{h}{L} = \frac{I_1}{I_2})$

$$M_A = -\frac{3}{5} Ph ; M_{BA} = \frac{2}{5} Ph ; M_{B'B} = \frac{3}{5} Ph ; M_{BC} = -\frac{1}{5} Ph ; M_C = \frac{3}{10} Ph$$

(509)



$$k = \frac{I_2 h}{I_1 L}$$

$$N_1 = \frac{6}{L} \frac{P_1 a_1 (L - a_1)}{L^2}$$

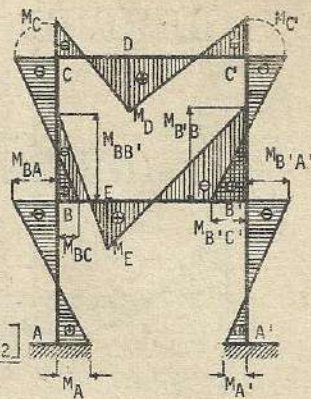
$$N_2 = \frac{6}{L} \frac{P_2 a_2 (L - a_2)}{L^2}$$

$$M_A = M_{A'} = \frac{(k+2)N_1 - N_2}{3(k^2+6k+7)}$$

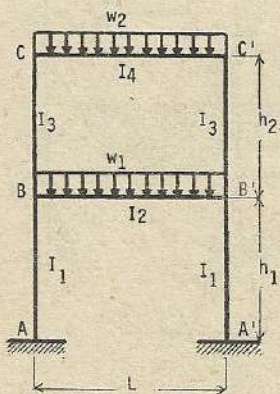
$$M_{BA} = M_{B'A'} = -\frac{2[(k+2)N_1 - N_2]}{3(k^2+6k+7)}$$

$$M_{BB'} = M_{B'B} = -\frac{(4k+3)N_1 + kN_2}{3(k^2+6k+7)} ; M_{BC} = M_{B'C'} = \frac{(2k+3)N_1 + (k+2)N_2}{3(k^2+6k+7)}$$

$$M_C = M_{C'} = -\frac{kN_1 + (2k+7)N_2}{3(k^2+6k+7)}$$



(510)



$$K_1' = \frac{I_1}{h_1} ; K_2' = \frac{I_2}{L} ; K_3' = \frac{I_3}{h_2} ; K_4' = \frac{I_4}{L}$$

$$\alpha = \frac{K_3'}{K_1'} ; \beta_1 = \frac{K_2'}{K_1'} ; \beta_2 = \frac{K_4'}{K_3'}$$

$$A = 12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)$$

$$M_{AB} = \frac{(2+\beta_2)w_1 L^2 - w_2 L^2}{A} ; M_{BA} = \frac{2(2+\beta_2)w_1 L^2 - 2w_2 L^2}{A}$$

$$M_{BB'} = -\frac{(4+2\beta_2+3\alpha+2\alpha\beta_2)w_1 L^2 + \beta_1 w_2 L^2}{A}$$

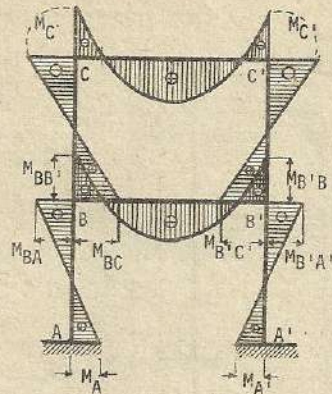
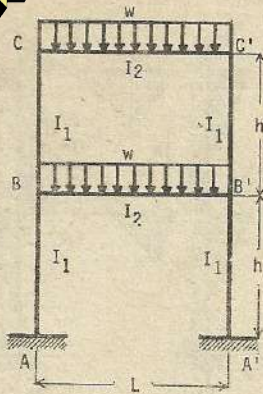
$$M_{BC} = \frac{(3\alpha+2\alpha\beta_2)w_1 L^2 + (2+\beta_1)w_2 L^2}{A} ; M_{CB} = \frac{\alpha\beta_2 w_1 L^2 + (4+2\beta_1+3\alpha)w_2 L^2}{A}$$

$$M_{CC'} = \frac{\alpha\beta_2 w_1 L^2 + (4+2\beta_1+3\alpha)w_2 L^2}{A}$$

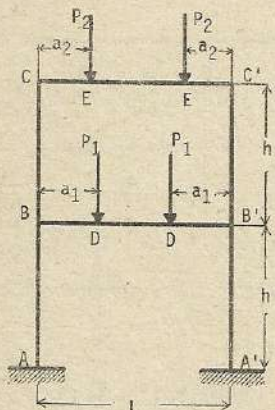
$$\text{St } w_1 = w_2 = w, \beta_1 = \beta_2 = \beta, \alpha = 1 ; B = 12(7+6\beta+\beta^2)$$

$$M_{AB} = \frac{(1+\beta)wL^2}{B} ; M_{BA} = \frac{2(1+\beta)wL^2}{B} ; M_{BB'} = -\frac{(7+5\beta)wL^2}{B}$$

$$M_{BC} = \frac{(5+3\beta)wL^2}{B} ; M_{CB} = \frac{(7+3\beta)wL^2}{B} ; M_{CC'} = -\frac{(7+3\beta)wL^2}{B}$$

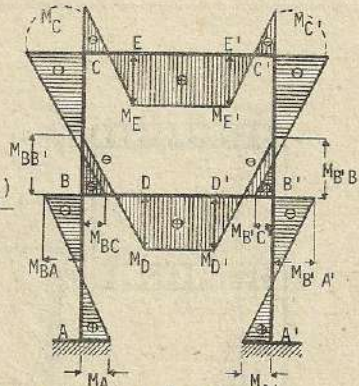


Si $k = 1$, $\frac{h}{L} = \frac{I_1}{I_2}$: $M_A = \frac{1}{84} wL^2$; $M_{BA} = -\frac{2}{84} wL^2$; $M_{BB'} = -\frac{6}{84} wL^2$
 $M_{BC} = \frac{4}{84} wL^2$; $M_C = -\frac{5}{84} wL^2$



(511)

$k = 1$
 $M_1 = 3P_1 a_1 \frac{(L - a_1)}{L}$
 $M_2 = 3P_2 a_2 \frac{(L - a_2)}{L}$
 $M_A = \frac{3P_1 a_1 (L - a_1) - P_2 a_2 (L - a_2)}{14L}$
 $M_{BA} = -2M_A$



$M_{BB'} = -\frac{11P_1 a_1 (L - a_1) + P_2 a_2 (L - a_2)}{14L}$; $M_{BC} = \frac{5P_1 a_1 (L - a_1) + 3P_2 a_2 (L - a_2)}{14L}$

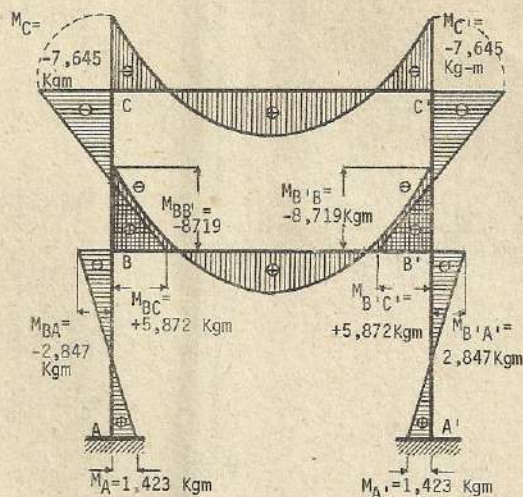
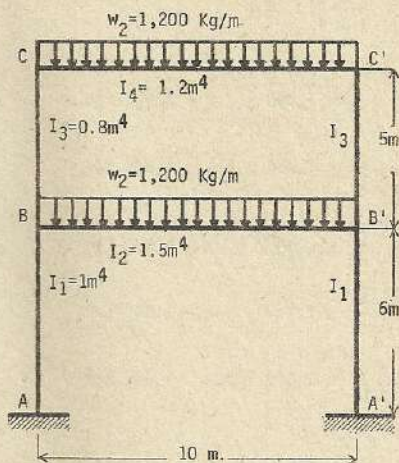
$M_C = -\frac{P_1 a_1 (L - a_1) + 9P_2 a_2 (L - a_2)}{14L}$

Si $P_2 = P_1 = P$, $a_2 = a_1 = \frac{L}{3}$

$M_A = \frac{2}{63} PL$; $M_{BA} = -\frac{4}{63} PL$; $M_{BB'} = -\frac{12}{63} PL$; $M_{BC} = \frac{8}{63} PL$

$M_C = -\frac{10}{63} PL$

Aplicación numérica



$$k_1^- = \frac{I_1}{h_1} = \frac{1}{6} ; k_2^- = \frac{I_2}{L} = \frac{1.5}{10} ; k_3^- = \frac{I_3}{h_2} = \frac{0.8}{5} ; k_4^- = \frac{I_4}{L} = \frac{1.2}{10}$$

$$\beta_1 = \frac{k_2^-}{k_1^-} = \frac{1.5}{10} \times \frac{6}{1} = 0.9 ; \beta_2 = \frac{k_4^-}{k_3^-} = \frac{1.2}{10} \times \frac{5}{0.8} = \frac{3}{4} ; \alpha = \frac{k_3^-}{k_1^-} = \frac{0.8}{5} \times \frac{6}{1} = \frac{4.8}{5}$$

$$M_A = \frac{(2+\beta_2)w_1L^2 - w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{(2+\frac{3}{4}) \times 1,200 \times 10 \times 10 - 1,200 \times 10 \times 10}{12(4+2 \times 0.9 + 2 \times \frac{3}{4} + 0.9 \times \frac{3}{4} + 3 \times \frac{4.8}{5} + 2 \times \frac{4.8}{5} \times \frac{3}{4})} = \frac{21,000}{147.54}$$

$$= 1,423 \text{ kg m}$$

$$M_{BA} = \frac{2(2+\beta_2)w_1L^2 - 2w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{2(2+\frac{3}{4})1,200 \times 10^2 - 2 \times 1,200 \times 10^2}{147.54} = 2,847 \text{ kgm}$$

$$M_{BB'} = - \frac{(4+2\beta_2+3\alpha+2\alpha\beta_2)w_1L^2 + \beta_1w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = - \frac{(4+2 \times \frac{3}{4} + 3 \times \frac{4.8}{5} + 2 \times \frac{4.8}{5} \times \frac{3}{4}) \times 1200 \times 10^2 - 0.9 \times 12 \times 10^4}{147.54}$$

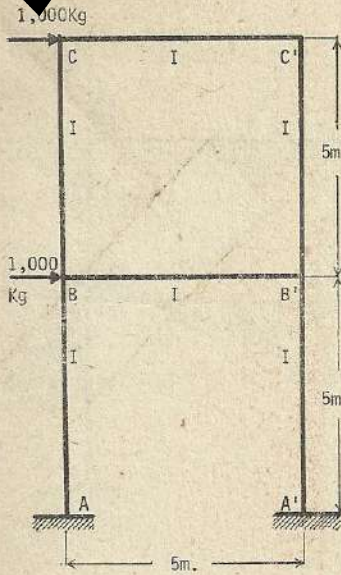
$$= - 8,719 \text{ Kg m}$$

$$M_{BC} = \frac{(3\alpha+2\alpha\beta_2)w_1L^2 + (2+\beta_1)w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{(3 \times \frac{4.8}{5} + 2 \times \frac{4.8}{5} \times \frac{3}{4}) \times 1200 \times 10^2 + (2+0.9) \times 1200 \times 10^2}{147.54}$$

$$= 5,872 \text{ kg m}$$

$$M_{CB} = \frac{\alpha\beta_2w_1L^2 + (4+2\beta_1+3\alpha)w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{\frac{4.8}{5} \times \frac{3}{4} \times 1200 \times 10^2 + (4+2 \times 0.9 + 3 \times \frac{4.8}{5}) \times 1,200 \times 10^2}{147.54}$$

$$= 7,645 \text{ Kg m}$$



$$k = \frac{I_1 h}{I_1 L} = \frac{I \times 5}{I \times 5} = 1$$

$$M_A = -\frac{3}{5} Ph = -\frac{3}{5} \times 1,000 \times 5 = -3,000 \text{ Kgm}$$

$$M_{BA} = -\frac{2}{5} Ph = \frac{2}{5} \times 1,000 \times 5 = 2,000 \text{ Kgm}$$

$$M_{BB'} = \frac{3}{5} Ph = \frac{3}{5} \times 1,000 \times 5 = 3,000 \text{ Kgm}$$

$$M_{BC} = -\frac{1}{5} Ph = -\frac{1}{5} \times 1,000 \times 5 = -1,000 \text{ Kgm}$$

$$M_C = \frac{3}{10} Ph = \frac{3}{10} \times 1,000 \times 5 = 1,500 \text{ Kgm}$$

(512)

$$k'_1 = \frac{I_1}{h_1} ; \quad k'_2 = \frac{I_2}{L} ; \quad k'_3 = \frac{I_3}{h_2} ; \quad k'_4 = \frac{I_4}{L}$$

$$\beta_1 = \frac{k'_2}{k'_1} \quad \beta_2 = \frac{k'_4}{k'_3} \quad \alpha = \frac{k'_3}{k'_1}$$

$$C = 14 \left[4(1 + \beta_1 + \beta_2 + \beta_1 \beta_2) + \alpha(3 + 4\beta_2) \right]$$

$$M_{BA} = \frac{4(1 + \beta_2) w_1 L^2 - 2w_2 L^2}{C}$$

$$M_{BB'} = \frac{[4(1 + \beta_2) + \alpha(3 + 4\beta_2)] w_1 L^2 + 2\beta_1 w_2 L^2}{C}$$

$$M_{B'B} = \frac{[4(1 + 1.5\beta_1 + \beta_2 + 1.5\beta_1 \beta_2) + \alpha(3 + 4\beta_2)] w_1 L^2 - \beta_1 w_2 L^2}{C}$$

$$M_{BC} = \frac{\alpha(3 + 4\beta_2) w_1 L^2 + 2(1 + \beta_1) w_2 L^2}{C} ; \quad M_{CB} = \frac{2\alpha\beta_2 w_1 L^2 + (4 + 4\beta_1 + 3\alpha) w_2 L^2}{C}$$

$$M_{C'C} = \frac{[4(1 + \beta_1 + 1.5\beta_2 + 1.5\beta_1 \beta_2) + \alpha(3 + 6\beta_2)] w_2 L^2 - \alpha\beta_2 w_1 L^2}{C}$$

$$M_A = \frac{1}{2} M_{BA} ; \quad M_{CC'} = -M_{CB}$$

